# NCERT Solutions for Class-XI Maths

## Chapter-4 Exercise-4.1

# NCERT Math Class 11

**1.** Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1+3+3^2+\ldots+3^{n-1}=\frac{(3^n-1)}{2}$$

**1.** Let the given statement be $P(n)$, i.e.,

$$P(n):1+3+3^2+\ldots+3^{n-1}=\frac{(3^n-1)}{2}$$

For $n=1$, we have

$$P(1):=\frac{(3'-1)}{2}=\frac{3-1}{2}=\frac{2}{2}=1$$ , which is true.

Let $P(k)$ be true for some positive integer $k$, i.e.,

$$1+3+3^2+\ldots+3^{k-1}=\frac{(3^k-1)}{2}$$

We shall now prove that $P(k+1)$ is true.

Consider

$$1+3+3^2+\ldots+3^{k-1}+3^{(k+1)-1}$$

$$=\left(1+3+3^2+\ldots+3^{k-1}\right)+3^k$$

$$=\frac{(3^k-1)}{2}+3^k$$

$$=\frac{(3^k-1)+2.3^k}{2}$$

$$=\frac{(1+2)3^k-1}{2}$$

$$=\frac{3.3^k-1}{2}$$

$$=\frac{3^{k+1}-1}{2}$$

Using (i)

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., N.

**2.** Prove the following by using the principle of mathematical induction for all $n \in N$:

**2.** m Let the given statement be P(n), as

$$P(n): 1^3 + 2^3 + 3^3 \ldots\ldots\ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

First, we check if it is true for n = 1,

$$P(1): 1^3 = 1 = \left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1^2 = 1$$

∴ It is true for n = 1.

Now we assume that it is true for some positive integer k, such that

$$P(k): 1^3 + 2^3 + 3^3 \ldots\ldots\ldots + k^3 = \left(\frac{k(k+1)}{2}\right)^2 \quad \ldots\ldots\ldots\ldots(1)$$

We shall prove that P(k + 1)is true,

Solving the left hand side with n = k + 1,

$$1^3 + 2^3 + 3^3 \ldots\ldots\ldots + k^3 + (k+1)^3$$

$$\Rightarrow \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$$

$$\Rightarrow \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$\Rightarrow (k+1)^2\left(\frac{k^2}{4} + (k+1)\right)$$

$$\Rightarrow (k+1)^2\left(\frac{k^2+4k+4}{4}\right)$$

$$\Rightarrow (k+1)^2\left(\frac{(k+2)^2}{4}\right)$$

$$\Rightarrow \left(\frac{\{(k+1)+1\}(k+1)}{2}\right)^2 \quad [\text{As } k + 2 = (k + 1) + 1]$$

Which is equal to the Right hand side for n = k + 1.We proved that P(k + 1) is true.

Hence by principle of mathematical induction it is true for all n ϵ N.

**3.** Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+\ldots+\frac{1}{(1+2+3+\ldots n)}=\frac{2n}{(n+1)}$$

**3.** Let the given statement be $P(n)$, i.e.,

$$P(n):1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots n}=\frac{2n}{n+1}$$

For $n=1$, we have

$P(1):1=\frac{2.1}{1+1}=\frac{2}{2}=1$, which is true.

Let $P(k)$ be true for some positive integer $k$, i.e.,

$$1+\frac{1}{1+2}+\ldots+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+k}=\frac{2k}{k+1}$$

We shall now prove that $P(k+1)$ is true.

Consider

$$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+k}+\frac{1}{1+2+3+\ldots+k+(k+1)}$$

$$=\left(1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots k}\right)+\frac{1}{1+2+3+\ldots+k+(k+1)}$$

$$=\frac{2k}{k+1}+\frac{1}{1+2+3+\ldots+k+(k+1)}$$

[Using (i)

$$=\frac{2k}{k+1}+\frac{1}{\left(\frac{(k+1)(k+1+1)}{2}\right)}$$

$$\left[1+2+3+\ldots+n=\frac{n(n+1)}{2}\right]$$

$$=\frac{2k}{(k+1)}+\frac{2}{(k+1)(k+2)}$$

$$=\frac{2}{(k+1)}\left(k+\frac{1}{k+2}\right)$$

$$=\frac{2}{(k+1)}\left(\frac{k^2+2k+1}{k+2}\right)$$

$$=\frac{2}{(k+1)}\left[\frac{(k+1)^2}{k+2}\right]$$

$$=\frac{2(k+1)}{(k+2)}$$

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., $N$.

**4.** Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1.2.3+2.3.4+...+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$$

**4.** Let the given statement be P(n), as

$$P(n):1.2.3+2.3.4+..........+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$$

First, we check if it is true for n = 1,

$$P(1):1.2.3=6=\frac{1(2)(3)(4)}{4}=6;$$

∴ It is true for n = 1.

Now we assume that it is true for some positive integer k, such that

$$P(k):1.2.3+2.3.4+..........+k(k+1)(k+2)=\frac{k(k+1)(k+2)(k+3)}{4} \quad .............(1)$$

We shall prove that P(k + 1)is true,

Solving the left hand side with n = k + 1

$$1.2.3+2.3.4+..........+k(k+1)(k+2)+(k+1)(k+1+1)(k+2+1)$$

$$\Rightarrow\left[1.2.3+2.3.4+..........+k(k+1)(k+2)\right]+(k+1)(k+2)(k+3)$$

$$\Rightarrow\left[\frac{k(k+1)(k+2)(k+3)}{4}\right]+(k+1)(k+2)(k+3) \text{ [From equation (1)]}$$

$$\Rightarrow(k+1)(k+2)(k+3)\left[\frac{k}{4}+1\right]$$

$$\Rightarrow\frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

$$\Rightarrow\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)}{2}$$

Which is equal to the Right hand side for n = k + 1.We proved that P(k + 1) is true.

Hence by principle of mathematical induction it is true for all n ϵ N.

**5.** Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1.3+2.3^2+3.3^3+\ldots+n.3^n=\frac{(2n-1)3^{n+1}+3}{4}$$

**5.** Let the given statement be $P(n)$, i.e.,

$$P(n): 1.3 + 2.3^2 + 3.3^3 + \ldots + n3^n = \frac{(2n-1)3^{n+1}+3}{4}$$

For $n=1$, we have

$P(1): 1.3 = 3 = \frac{(2.1-1)3^{1+1}+3}{4} = \frac{3^2+3}{4} = \frac{12}{4} = 3$, which is true.

Let $P(k)$ be true for some positive integer $k$, i.e.,

$$1.3 + 2.3^2 + 3.3^3 + \ldots + k3^k = \frac{(2k-1)3^{k+1}+3}{4}$$

We shall now prove that $P(k+1)$ is true.

Consider

$$1.3 + 2\cdot 3^2 + 3\cdot 3^3 + \ldots + k\cdot 3^k + (k+1)\cdot 3^{k+1}$$

$$= \left(1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \ldots + k\cdot 3^k\right) + (k+1)\cdot 3^{k+1}$$

$$= \frac{(2k-1)3^{k+1}+3}{4} + (k+1)3^{k+1}$$

$$= \frac{(2k-1)3^{k+1}+3+4(k+1)3^{k+1}}{4}$$

$$= \frac{3^{k+1}\{2k-1+4(k+1)\}+3}{4}$$

$$= \frac{3^{k+1}\{6k+3\}+3}{4}$$

$$= \frac{3^{k+1}\cdot 3\{2k+1\}+3}{4}$$

$$= \frac{3^{(k+1)+1}\{2k+1\}+3}{4}$$

$$= \frac{\{2(k+1)-1\}3^{(k+1)+1}+3}{4}$$

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., $N$.

**6.** Prove the following by using the principle of mathematical induction for all $n \in N$:

$$= \left[\frac{n(n+1)(n+2)}{3}\right]$$

**6.** Let the given statement be P(n), as

$P(n): 1.2+2.3+.........+n(n+1)=\frac{n(n+1)(n+2)}{3}$

First, we check if it is true for n = 1,

$P(1): 1.2=2=\frac{1(2)(3)}{3}=2;$

∴ It is true for n = 1.

Now we assume that it is true for some positive integer k, such that

$P(k): 1.2+2.3+.........+k(k+1)=\frac{k(k+1)(k+2)}{3}$ ..............(1)

We shall prove that P(k + 1)is true,

Solving the left hand side with n = k + 1

$1.2+2.3+.........+k(k+1)+(k+1)(k+1+1)$

$\Rightarrow[1.2+2.3+.........+k(k+1)]+(k+1)(k+2)$

$\Rightarrow\left[\frac{k(k+1)(k+2)}{3}\right]+(k+1)(k+2)$ [From equation (1)]

$\Rightarrow(k+1)(k+2)\left[\frac{k}{3}+1\right]$

$\Rightarrow\frac{(k+1)(k+2)(k+3)}{3}$

$\Rightarrow\frac{(k+1)(k+1+1)(k+1+2)}{2}$

Which is equal to the Right hand side for n = k + 1.We proved that P(k + 1) is true.

Hence by principle of mathematical induction it is true for all n ϵ N.

**7.** Prove the following by using the principle of mathematical induction for all $n\in N$:

$1.3+3.5+5.7+\ldots+(2n-1)(2n+1)=\frac{n(4n^2+6n-1)}{3}$

**7.** Let the given statement be $P(n)$, i.e.,

$P(n): 1.3+3.5+5.7+\ldots+(2n-1)(2n+1)=\frac{n(4n^2+6n-1)}{3}$

For $n=1$, we have

$P(1): 1.3=3=\frac{1(4\cdot 1^2+6.1-1)}{3}=\frac{4+6-1}{3}=\frac{9}{3}=3$, which is true.

Let $P(k)$ be true for some positive integer $k$, i.e.,

$$1.3+3.5+5.7+\ldots\ldots+(2k-1)(2k+1)=\frac{k\left(4k^2+6k-1\right)}{3}$$

We shall now prove that $P(k+1)$ is true.

Consider

$$(1.3+3.5+5.7+\ldots+(2k-1)(2k+1)+\{2(k+1)-1\}\{2(k+1)+1\}$$

$$=\frac{k\left(4k^2+6k-1\right)}{3}+(2k+2-1)(2k+2+1)$$

$$=\frac{k\left(4k^2+6k-1\right)}{3}+(2k+1)(2k+3)$$

$$=\frac{k\left(4k^2+6k-1\right)}{3}+\left(4k^2+8k+3\right)$$

$$=\frac{k\left(4k^2+6k-1\right)+3\left(4k^2+8k+3\right)}{3}$$

$$=\frac{4k^3+6k^2-k+12k^2+24k+9}{3}$$

$$=\frac{4k^3+18k^2+23k+9}{3}$$

$$=\frac{4k^3+14k^2+9k+4k^2+14k+9}{3}$$

$$=\frac{(k+1)\left\{4\left(k^2+2k+1\right)+6(k+1)-1\right\}}{3}$$

$$=\frac{(k+1)\left\{4k^2+8k+4+6k+6-1\right\}}{3}$$

$$=\frac{(k+1)\left\{4(k+1)^2+6(k+1)-1\right\}}{3}$$

$$=\frac{\left(kk^2+14k+9\right)}{3}$$

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., $N$.

**8.** Prove the following by using the principle of mathematical induction for all $n \in N$:

**8.** Let the given statement be P(n), as

$P(n): 1.2 + 2.2^2 + \ldots\ldots\ldots + n.2^n = (n-1)2^{n+1} + 2$

First, we check if it is true for n = 1,

$P(1): 1.2 = 2 = (1-1)2^{1+1} + 2 = 0 + 2 = 2;$

∴ It is true for n = 1.

Now we assume that it is true for some positive integer k, such that

$P(k): 1.2 + 2.2^2 + \ldots\ldots\ldots + k.2^k = (k-1)2^{k+1} + 2 \ldots\ldots\ldots\ldots(1)$

We shall prove that P(k + 1)is true,

Solving the left hand side with n = k + 1

$1.2 + 2.2^2 + \ldots\ldots\ldots + k.2^k + (k+1)2^{k+1}$

$\Rightarrow \left[1.3 + 2.2^2 + \ldots\ldots\ldots + k.2^k\right] + (k+1)2^{k+1}$

[From equation (1)]

$\Rightarrow (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$

$\Rightarrow (2k)2^{k+1} + 2$

$\Rightarrow (k)2^{k+2} + 2$

$\Rightarrow ((k+1)-1)2^{(k+1)+1} + 2$

Which is equal to the Right hand side for n = k + 1.We proved that P(k + 1) is true. Hence by principle of mathematical induction it is true for all n ϵ N.

**9.** Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

**9.** Let the given statement be $P(n)$, i.e.,

$$P(n): \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

For $n = 1$, we have

$P(1): \frac{1}{2} = 1 - \frac{1}{2^1} = \frac{1}{2}$, which is true.

Let $P(k)$ be true for some positive integer $k$, i.e.,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots. + \frac{1}{2^k} = 1 - \frac{1}{2^k}$$

We shall now prove that $P(k+1)$ is true.

Consider

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^k}\right) + \frac{1}{2^{k+1}}$$

$$=\left(1-\frac{1}{2^k}\right)+\frac{1}{2^{k+1}}$$

$$=1-\frac{1}{2^k}+\frac{1}{2\cdot 2^k}$$

$$=1-\frac{1}{2^k}\left(1-\frac{1}{2}\right)$$

$$=1-\frac{1}{2^k}\left(\frac{1}{2}\right)$$

$$=1-\frac{1}{2^{k+1}}$$

[ Using (i) ]

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers i.e., $N$.

**10.** Prove the following by using the principle of mathematical induction for all n N:

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+....+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$$

**10.** Let the given statement be P(n), as

$$P(n): \frac{1}{2.5}+\frac{1}{5.8}+....+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$$

First, we check if it is true for n = 1,

$$P(1): \frac{1}{2.5}=\frac{1}{10}=\frac{1}{(6+4)}=\frac{1}{10};$$

∴ It is true for n = 1.

Now we assume that it is true for some positive integer k, such that

$$P(k): \frac{1}{2.5}+\frac{1}{5.8}+....+\frac{1}{(3k-1)(3k+2)}=\frac{k}{(6k+4)} \quad ..............(1)$$

We shall prove that P(k + 1)is true,

Solving the left hand side with n = k + 1

$$\frac{1}{2.5}+\frac{1}{5.8}+....+\frac{1}{(3k-1)(3k+2)}+\frac{1}{(3k+2)(3k+5)}$$

$$\Rightarrow\left[\frac{1}{2.5}+\frac{1}{5.8}+....+\frac{1}{(3k-1)(3k+2)}\right]+\frac{1}{(3k+2)(3k+5)} \quad \text{[From equation (1)]}$$

$$\Rightarrow\frac{k}{(6k+4)}+\frac{1}{(3k+2)(3k+5)}$$

$$\Rightarrow \frac{k(3k+5)+2}{2(3k+2)(3k+5)}$$

$$\Rightarrow \frac{3k^2+5k+2}{2(3k+2)(3k+5)}$$

$$\Rightarrow \frac{3k^2+3k+2k+2}{2(3k+2)(3k+5)}$$

$$\Rightarrow \frac{(3k+2)(k+1)}{2(3k+2)(3k+5)}$$

$$\Rightarrow \frac{(k+1)}{2(3k+5)}$$

$$\Rightarrow \frac{(k+1)}{(6(k+1)+4)}$$

Which is equal to the Right-hand side for n = k + 1. We proved that P(k + 1) is true. Hence by principle of mathematical induction it is true for all n ϵ N.