27. Straight Line in Space

Exercise 27A

1. Question

A line passes through the point (3, 4, 5) and is parallel to the vector $\left(2\hat{i}+2\hat{j}-3\hat{k}\right)$. Find the equations of the line in the vector as well as Cartesian forms.

Answer

Given: line passes through point (3, 4, 5) and is parallel to $2\hat{i}+2\hat{j}-3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}-3\hat{k}$

Therefore,

Vector form:

$$\vec{r}=3\hat{i}+4\hat{j}+5\hat{k}+\lambda\left(2\hat{i}+2\hat{j}-3\hat{k}\right)$$

Cartesian form:

$$\frac{x-3}{2}=\frac{y-4}{2}=\frac{z-5}{-3}$$

2. Question

A line passes through the point (2, 1, -3) and is parallel to the vector $\left(\hat{i}-2\hat{j}+3\hat{k}\right)$. Find the equations of the line in vector and Cartesian forms.

Answer

Given: line passes through (2, 1, -3) and is parallel to $\hat{i}-2\hat{j}+3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

Cartesian form:

$$\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+3}{3}$$

3. Question

Find the vector equation of the line passing through the point with position vector $\left(2\hat{i} + \hat{j} - 5\hat{k}\right)$ and parallel to the vector $\left(\hat{i} + 3\hat{j} - \hat{k}\right)$. Deduce the Cartesian equations of the line.

Answer

Given: line passes through $2\hat{i} + \hat{j} - 5\hat{k}$ and is parallel to $\hat{i} + 3\hat{j} - \hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a} = 2\hat{i} + \hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} + \hat{j} - 5\hat{k} + \lambda(\hat{i} + 3\hat{j} - \hat{k})$$

Cartesian form:

$$\frac{x-2}{1} = \frac{y-1}{3} = \frac{z+5}{-1}$$

4. Question

A line is drawn in the direction of $\left(\hat{i} + \hat{j} - 2\hat{k}\right)$ and it passes through a point with position vector $\left(2\hat{i} - \hat{j} - 4\hat{k}\right)$. Find the equations of the line in the vector as well as Cartesian forms.

Answer

Given: line passes through $2\hat{i} - \hat{j} - 4\hat{k}$ and is drawn in the direction of $\hat{i} + \hat{j} - 2\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since line is drawn in the direction of $(\hat{i} + \hat{j} - \widehat{2k})$, it is parallel to $(\hat{i} + \hat{j} - 2\hat{k})$

Here, $\vec{a} = 2\hat{i} - \hat{j} - 4\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} - \hat{j} - 4\hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$$

Cartesian form:

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z+4}{-2}$$

5. Question

The Cartesian equations of a line are $\frac{x-3}{2} = \frac{y+2}{-5} = \frac{z-6}{4}$. Find the vector equation of the line.

Answer

Given: Cartesian equation of line

$$\frac{x-3}{2} = \frac{y+2}{-5} = \frac{z-6}{4}$$

To find: equation of line in vector form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

From the Cartesian equation of the line, we can find $\vec{a}$ and $\vec{b}$

Here, $\vec{a} = 3\hat{\imath} - \widehat{2}\jmath + 6\hat{k}$ and $\vec{b} = \widehat{2}\imath - 5\hat{\jmath} + 4\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k} + \lambda(2\hat{\imath} - 5\hat{\jmath} + 4\hat{k})$$

6. Question

The Cartesian equations of a line are 3x + 1 = 6y − 2 = 1 − z. Find the fixed point through which it passes, its direction ratios and also its vector equation.

Answer

Given: Cartesian equation of line are 3x + 1 = 6y − 2 = 1 - z

To find: fixed point through which the line passes through, its direction ratios and the vector equation.

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ is a vector parallel to the line and also its direction ratio.

Explanation:

The Cartesian form of the line can be rewritten as:

$$\frac{x+\frac{1}{3}}{\frac{1}{3}}=\frac{y-\frac{1}{3}}{\frac{1}{6}}=\frac{z-1}{-1}=\lambda$$

$$\Rightarrow \frac{x+\frac{1}{3}}{2}=\frac{y-\frac{1}{3}}{1}=\frac{z-1}{-6}=\lambda$$

Therefore, $\vec{a}=\frac{-1}{3}\hat{\imath}+\frac{1}{3}\hat{\jmath}+\hat{k}$ and $\vec{b}=2\hat{\imath}+\hat{\jmath}-6\hat{k}$

So, the line passes through $\left(\frac{-1}{3},\frac{1}{3},1\right)$ and direction ratios of the line are (2, 1, -6) and vector form is:

$$\vec{r}=\frac{-1}{3}\hat{\imath}+\frac{1}{3}\hat{\jmath}+\hat{k}+\lambda(2\hat{\imath}+\hat{\jmath}-6\hat{k})$$

7. Question

Find the Cartesian equations of the line which passes through the point (1, 3, -2) and is parallel to the line given by $\frac{x+1}{3}=\frac{y-4}{5}=\frac{z+3}{-6}$. Also, find the vector form of the equations so obtained.

Answer

Given: line passes through (1, 3, -2) and is parallel to the line

$$\frac{x+1}{3}=\frac{y-4}{5}=\frac{z+3}{-6}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say L_1) is parallel to another line (say L_2), L_1 has the same direction ratios as that of L_2

Here, $\vec{a}=\hat{\imath}+3\hat{\jmath}-2\hat{k}$

Since the equation of L_2 is

$$\frac{x+1}{3}=\frac{y-4}{5}=\frac{z+3}{-6}$$

$$\vec{b}=3\hat{\imath}+5\hat{\jmath}-6\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 3\hat{j} - 2\hat{k} + \lambda(3\hat{i} + 5\hat{j} - 6\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{3} = \frac{y-3}{5} = \frac{z+2}{-6}$$

8. Question

Find the equations of the line passing through the point (1, -2, 3) and parallel to the line $\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$. Also find the vector form of this equation so obtained.

Answer

Given: line passes through (1, -2, 3) and is parallel to the line

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say L_1) is parallel to another line (say L_2), L_1 has the same direction ratios as that of L_2

Here, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

Since the equation of L_2 is

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

$$\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} - 4\hat{j} + 5\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z-3}{5}$$

9. Question

Find the Cartesian and vector equations of a line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$.

Answer

Given: line passes through (1, 2, 3) and is parallel to the line

$$\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say L_1) is parallel to another line (say L_2), L_1 has the same direction ratios as that of L_2

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Equation of L_2 can be rewritten as:

$$\frac{x+2}{-1} = \frac{y+3}{7} = \frac{z-3}{\frac{3}{2}}$$

$$\Rightarrow \frac{x+2}{-2} = \frac{y+3}{14} = \frac{z-3}{3}$$

$$\vec{b} = -2\hat{i} + 14\hat{j} + 3\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-2\hat{i} + 14\hat{j} + 3\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{-2}=\frac{y-2}{14}=\frac{z-3}{3}$$

10. Question

Find the equations of the line passing through the point (-1, 3, -2) and perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$.

Answer

Given: line passes through (-1, 3, -2) and is perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

Here, $\vec{a}=-\hat{i}+3\hat{j}-2\hat{k}$

Let the direction ratios of the line be $b_1:b_2:b_3$

Direction ratios of the other two lines are 1 : 2 : 3 and -3 : 2 : 5

Since the other two line are perpendicular to the given line, we have

$b_1 + 2b_2 + 3b_3 = 0$

$-3b_1 + 2b_2 + 5b_3 = 0$

Solving,

$$\frac{b_1}{\begin{vmatrix}2 & 3\\ 2 & 5\end{vmatrix}}=\frac{-b_2}{\begin{vmatrix}1 & 3\\ -3 & 5\end{vmatrix}}=\frac{b_3}{\begin{vmatrix}1 & 2\\ -3 & 2\end{vmatrix}}$$

$$\Rightarrow\frac{b_1}{4}=\frac{b_2}{-14}=\frac{b_3}{8}$$

$$\Rightarrow\frac{b_1}{2}=\frac{b_2}{-7}=\frac{b_3}{4}$$

$\vec{b} = 2\hat{i} - 7\hat{j} + 4\hat{k}$

Therefore,

Vector form of the line is:

$\vec{r} = -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$

Cartesian form of the line is:

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$$

11. Question

Find the Cartesian and vector equations of the line passing through the point (1, 2, -4) and perpendicular to each of the lines $\frac{x-8}{8} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$.

Answer

Given: line passes through (1, 2, -4) and is perpendicular to each of the lines $\frac{x-8}{8} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

Here, $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

Let the direction ratios of the line be $b_1:b_2:b_3$

Direction ratios of other two lines are 8 : -16 : 7 and 3 : 8 : -5

Since the other two line are perpendicular to the given line, we have

$8b_1 - 16b_2 + 7b_3 = 0$

$3b_1 + 8b_2 - 5b_3 = 0$

Solving,

$$\frac{b_1}{\begin{vmatrix} -16 & 7 \\ 8 & -5 \end{vmatrix}} = \frac{-b_2}{\begin{vmatrix} 8 & 7 \\ 3 & -5 \end{vmatrix}} = \frac{b_3}{\begin{vmatrix} 8 & -16 \\ 3 & 8 \end{vmatrix}}$$

$$\Rightarrow \frac{b_1}{24} = \frac{b_2}{61} = \frac{b_3}{112}$$

$$\vec{b} = 24\hat{i} + 61\hat{j} + 112\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(24\hat{i} + 61\hat{j} + 112\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{24} = \frac{y-2}{61} = \frac{z+4}{112}$$

12. Question

Prove that the lines $\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect each other and find the point of their intersection.

Answer

Given: The equations of the two lines are

$$\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7} \text{ and } \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7} = \lambda_1$$

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda_2$$

So a point on the first line is $(\lambda_1 + 4, 4\lambda_1 - 3, 7\lambda_1 - 1)$

A point on the second line is $(2\lambda_2 + 1, -3\lambda_2 - 1, 8\lambda_2 - 10)$

If they intersect they should have a common point.

$\lambda_1 + 4 = 2\lambda_2 + 1 \Rightarrow \lambda_1 - 2\lambda_2 = -3$ ... (1)

$4\lambda_1 - 3 = -3\lambda_2 - 1 \Rightarrow 4\lambda_1 + 3\lambda_2 = 2$ ... (2)

Solving (1) and (2),

$11\lambda_2 = 14$

$\lambda_2 = \frac{14}{11}$

Therefore, $\lambda_1 = \frac{-5}{11}$

Substituting for the z coordinate, we get

$7\lambda_1 - 1 = \frac{-46}{11}$ and $8\lambda_2 - 10 = \frac{2}{11}$

So, the lines do not intersect.

13. Question

Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect each other. Also, find the point of their intersection.

Answer

Given: The equations of the two lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-4}{5} = \frac{y-1}{2} = z$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda_1$$

$$\frac{x-4}{5}=\frac{y-1}{2}=z=\lambda_2$$

So a point on the first line is $(2\lambda_1 + 1, 3\lambda_1 + 2, 4\lambda_1 + 3)$

A point on the second line is $(5\lambda_2 + 4, 2\lambda_2 + 1, \lambda_2)$

If they intersect they should have a common point.

$2\lambda_1 + 1 = 5\lambda_2 + 4 \Rightarrow 2\lambda_1 - 5\lambda_2 = 3$ ... (1)

$3\lambda_1 + 2 = 2\lambda_2 + 1 \Rightarrow 3\lambda_1 - 2\lambda_2 = -1$ ... (2)

Solving (1) and (2),

$-11\lambda_2 = 11$

$\lambda_2 = -1$

Therefore, $\lambda_1 = -1$

Substituting for the z coordinate, we get

$4\lambda_1 + 3 = -1$ and $\lambda_2 = -1$

So, the lines intersect and their point of intersection is (-1, -1, -1)

14. Question

Show that the lines $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}, z=2$ do not intersect each other.

Answer

Given: The equations of the two lines are

$$\frac{x-1}{2}=\frac{y+1}{3}=z \text{ and } \frac{x+1}{5}=\frac{y-2}{1}, z=2$$

To Prove: the lines do not intersect each other.

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2}=\frac{y+1}{3}=z=\lambda_1$$

$\frac{x+1}{5} = \frac{y-2}{1} = \lambda_2, z = 2$

So a point on the first line is $(2\lambda_1 + 1, 3\lambda_1 - 1, \lambda_1)$

A point on the second line is $(5\lambda_2 - 1, \lambda_2 + 1, 2)$

If they intersect they should have a common point.

$2\lambda_1 + 1 = 5\lambda_2 - 1 \Rightarrow 2\lambda_1 - 5\lambda_2 = -2$ ... (1)

$3\lambda_1 - 1 = \lambda_2 + 1 \Rightarrow 3\lambda_1 - \lambda_2 = 2$ ... (2)

Solving (1) and (2),

$-13\lambda_2 = -10$

$\lambda_2 = \frac{10}{13}$

Therefore, $\lambda_1 = \frac{33}{65}$

Substituting for the z coordinate, we get

$\lambda_1 = \frac{33}{65}$ and $z = 2$

So, the lines do not intersect.

15. Question

Find the coordinates of the foot of the perpendicular drawn from the point (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, find the length of the perpendicular from the given point to the line.

Answer

Given: Equation of line is $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

To find: coordinates of foot of the perpendicular from (1, 2, 3) to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

2. Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Explanation:

Let

$$\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}=\lambda$$

So the foot of the perpendicular is (3λ + 6, 2λ + 7, -2λ + 7)

Direction ratio of the line is 3 : 2 : -2

Direction ratio of the perpendicular is

⇒ (3λ + 6 - 1) : (2λ + 7 - 2) : (-2λ + 7 - 3)

⇒ (3λ + 5) : (2λ + 5) : (-2λ + 4)

Since this is perpendicular to the line,

3(3λ + 5) + 2(2λ + 5) – 2(-2λ + 4) = 0

⇒ 9λ + 15 + 4λ + 10 + 4λ – 8 = 0

⇒ 17λ = -17

⇒ λ = -1

So the foot of the perpendicular is (3, 5, 9)

Distance $= \sqrt{(3-1)^2+(5-2)^2+(9-3)^2}$

$= \sqrt{4+9+36}$

= 7 units

Therefore, the foot of the perpendicular is (3, 5, 9) and length of perpendicular is 7 units.

16. Question

Find the length and the foot of the perpendicular drawn from the point (2, -1, 5) to the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.

Answer

Given: Equation of line is $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.

To find: coordinates of foot of the perpendicular from (2, -1, 5) to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

2. Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Explanation:

Let

$$\frac{x - 11}{10} = \frac{y + 2}{-4} = \frac{z + 8}{-11} = \lambda$$

So the foot of the perpendicular is (10λ + 11, -4λ - 2, -11λ - 8)

Direction ratio of the line is 10 : -4 : -11

Direction ratio of the perpendicular is

⇒ (10λ + 11 - 2) : (-4λ - 2 + 1) : (-11λ - 8 - 5)

⇒ (10λ + 9) : (-4λ - 1) : (-11λ - 13)

Since this is perpendicular to the line,

10(10λ + 9) - 4(-4λ - 1) - 11(-11λ - 13) = 0

⇒ 100λ + 90 + 16λ + 4 + 121λ + 143 = 0

⇒ 237λ = -237

⇒ λ = -1

So the foot of the perpendicular is (1, 2, 3)

Distance $= \sqrt{(1 - 2)^2 + (2 + 1)^2 + (3 - 5)^2}$

$= \sqrt{1 + 9 + 4}$

= √14 units

Therefore, the foot of the perpendicular is (1, 2, 3) and length of perpendicular is √14 units.

17. Question

Find the vector and Cartesian equations of the line passing through the points A(3, 4, -6) and B(5, -2, 7).

Answer

Given: line passes through the points (3, 4, -6) and (5, -2, 7)

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = 3\hat{i} + 4\hat{j} - 6\hat{k}$

The direction ratios of the line are (3 - 5) : (4 + 2) : (-6 - 7)

⇒ -2 : 6 : -13

⇒ 2 : -6 : 13

So, $\vec{b} = 2\hat{i} - 6\hat{j} + 13\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 3\hat{i} + 4\hat{j} - 6\hat{k} + \lambda(2\hat{i} - 6\hat{j} + 13\hat{k})$$

Cartesian form:

$$\frac{x-3}{2} = \frac{y-4}{-6} = \frac{z+6}{13}$$

18. Question

Find the vector and Cartesian equations of the line passing through the points A(2, -3, 0) and B(-2, 4, 3).

Answer

Given: line passes through the points (2, -3, 0) and (-2, 4, 3)

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = 2\hat{i} - 3\hat{j}$

The direction ratios of the line are (2 + 2) : (-3 - 4) : (0 - 3)

⇒ 4 : -7 : -3

⇒ -4 : 7 : 3

So, $\vec{b} = -4\hat{i} + 7\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} - 3\hat{j} + \lambda(-4\hat{i} + 7\hat{j} + 3\hat{k})$$

Cartesian form:

$$\frac{x-2}{-4} = \frac{y+3}{7} = \frac{z}{3}$$

19. Question

Find the vector and Cartesian equations of the line joining the points whose position vectors are $(\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 3\hat{j} - 2\hat{k})$.

Answer

Given: line passes through the points whose position vectors are $(\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 3\hat{j} - 2\hat{k})$.

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

The direction ratios of the line are (1 - 1) : (-2 - 3) : (1 + 2)

⇒ 0 : -5 : 3

⇒ 0 : 5 : -3

So, $\vec{b} = -5\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = \hat{i} - 2\hat{j} + \hat{k} + \lambda(5\hat{j} - 3\hat{k})$$

Cartesian form:

$$\frac{x-1}{0}=\frac{y+2}{5}=\frac{z-1}{-3}$$

20. Question

Find the vector equation of a line passing through the point A(3, -2, 1) and parallel to the line joining the points B(-2, 4, 2) and C(2, 3, 3). Also, find the Cartesian equations of the line.

Answer

Given: line passes through the point (3, -2, 1) and is parallel to the line joining points B(-2, 4, 2) and C(2, 3, 3).

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$

The direction ratios of the line are (-2 - 2) : (4 - 3) : (2 - 3)

⇒ -4 : 1 : -1

⇒ 4 : -1 : 1

So, $\vec{b}=4\hat{i}-\hat{j}+\hat{k}$

Therefore,

Vector form:

$$\vec{r}=3\hat{i}-2\hat{j}+\hat{k}+\lambda\left(4\hat{i}-\hat{j}+\hat{k}\right)$$

Cartesian form:

$$\frac{x-3}{4}=\frac{y+2}{-1}=\frac{z-1}{1}$$

21. Question

Find the vector equation of a line passing through the point having the position vector $\left(\hat{i}+2\hat{j}-3\hat{k}\right)$ and parallel to the line joining the points with position vectors $\left(\hat{i}-\hat{j}+5\hat{k}\right)$ and $\left(2\hat{i}+3\hat{j}-4\hat{k}\right)$.

Also, find the Cartesian equivalents of this equation.

Answer

Given: line passes through the point with position vector $\hat{i} + 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 5\hat{k}$ and $2\hat{i} + 3\hat{j} - 4\hat{k}$.

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

The direction ratios of the line are (1 - 2) : (-1 - 3) : (5 + 4)

⇒ -1 : -4 : 9

⇒ 1 : 4 : -9

So, $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$

Therefore,

Vector form:

$$\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \lambda(\hat{i} + 4\hat{j} - 9\hat{k})$$

Cartesian form:

$$\frac{x-1}{1} = \frac{y-2}{4} = \frac{z+3}{-9}$$

22. Question

Find the coordinates of the foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining the points B(1, 4, 6) and C(5, 4, 4).

Answer

Given: perpendicular drawn from point A (1, 2, 1) to line joining points B (1, 4, 6) and C (5, 4, 4)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (1, 4, 6) is a point on the line.

Therefore, $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

Also direction ratios of the line are (1 - 5) : (4 – 4) : (6 – 4)

⇒ -4 : 0 : 2

⇒ -2 : 0 : 1

So, equation of the line in Cartesian form is

$$\frac{x-1}{-2} = \frac{y-4}{0} = \frac{z-6}{1} = \lambda$$

Any point on the line will be of the form (-2λ + 1, 4, λ + 6)

So the foot of the perpendicular is of the form (-2λ + 1, 4, λ + 6)

The direction ratios of the perpendicular is

(-2λ + 1 – 1) : (4 - 2) : (λ + 6 - 1)

⇒ (-2λ) : 2 : (λ + 5)

From the direction ratio of the line and the direction ratio of its perpendicular, we have

-2(-2λ) + 0 + λ + 5 = 0

⇒ 4λ + λ = -5

⇒ λ = -1

So, the foot of the perpendicular is (3, 4, 5)

23. Question

Find the coordinates of the foot of the perpendicular drawn from the point A(1, 8, 4) to the line joining the points B(0, -1, 3) and C(2, -3, -1).

Answer

Given: perpendicular drawn from point A (1, 8, 4) to line joining points B (0, -1, 3) and C (2, -3, -1)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (0, -1, 3) is a point on the line.

Therefore, $\vec{a}=-\hat{j}+3\hat{k}$

Also direction ratios of the line are (0 - 2) : (-1 + 3) : (3 + 1)

⇒ -2 : 2 : 4

⇒ -1 : 1 : 2

So, equation of the line in Cartesian form is

$$\frac{x}{-1}=\frac{y+1}{1}=\frac{z-3}{2}=\lambda$$

Any point on the line will be of the form (-λ, λ - 1, 2λ + 3)

So the foot of the perpendicular is of the form (-λ, λ - 1, 2λ + 3)

The direction ratios of the perpendicular is

(-λ - 1) : (λ − 1 - 8) : (2λ + 3 - 4)

⇒ (-λ - 1) : (λ − 9) : (2λ − 1)

From the direction ratio of the line and the direction ratio of its perpendicular, we have

-1(-λ - 1) + λ − 9 + 2(2λ − 1) = 0

⇒ λ + 1 + λ − 9 + 4λ − 2 = 0

⇒ 6λ = 10

$$\Rightarrow \lambda=\frac{5}{3}$$

So, the foot of the perpendicular is $\left(\frac{-5}{3},\frac{2}{3},\frac{19}{3}\right)$

24. Question

Find the image of the point (0, 2, 3) in the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.

Answer

Given: Equation of line is $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.

To find: image of point (0, 2, 3)

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

Explanation:

Let

$$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda$$

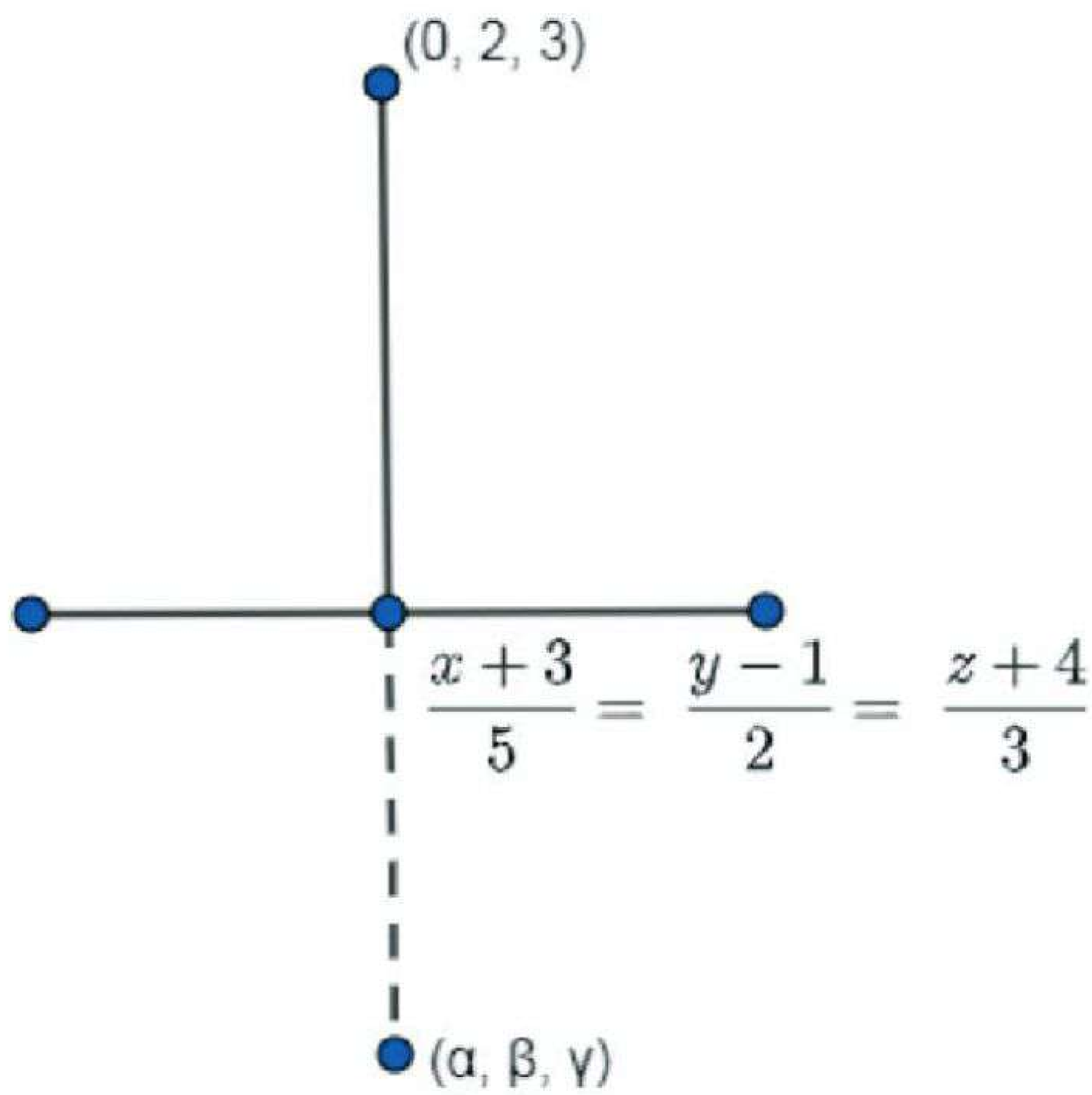
So the foot of the perpendicular is $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

The direction ratios of the perpendicular is

$(5\lambda - 3 - 0) : (2\lambda + 1 - 2) : (3\lambda - 4 - 3)$

$\Rightarrow (5\lambda - 3) : (2\lambda - 1) : (3\lambda - 7)$

Direction ratio of the line is 5 : 2 : 3



From the direction ratio of the line and the direction ratio of its perpendicular, we have

$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$\Rightarrow 38\lambda = 38$

$\Rightarrow \lambda = 1$

So, the foot of the perpendicular is (2, 3, -1)

The foot of the perpendicular is the mid-point of the line joining (0, 2, 3) and (α, β, γ)

So, we have

$$\frac{\alpha + 0}{2} = 2 \Rightarrow \alpha = 4$$

$$\frac{\beta + 2}{2} = 3 \Rightarrow \beta = 4$$

$$\frac{\gamma + 3}{2} = -1 \Rightarrow \gamma = -5$$

So, the image is (4, 4, -5)

25. Question

Find the image of the point (5, 9, 3) in the line $\frac{x-1}{2} = \frac{y=2}{3} = \frac{z-3}{4}$.

Answer

Given: Equation of line is $\frac{x-1}{2}=\frac{y=2}{3}=\frac{z-3}{4}$.

To find: image of point (5, 9, 3)

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

Explanation:

Let

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$$

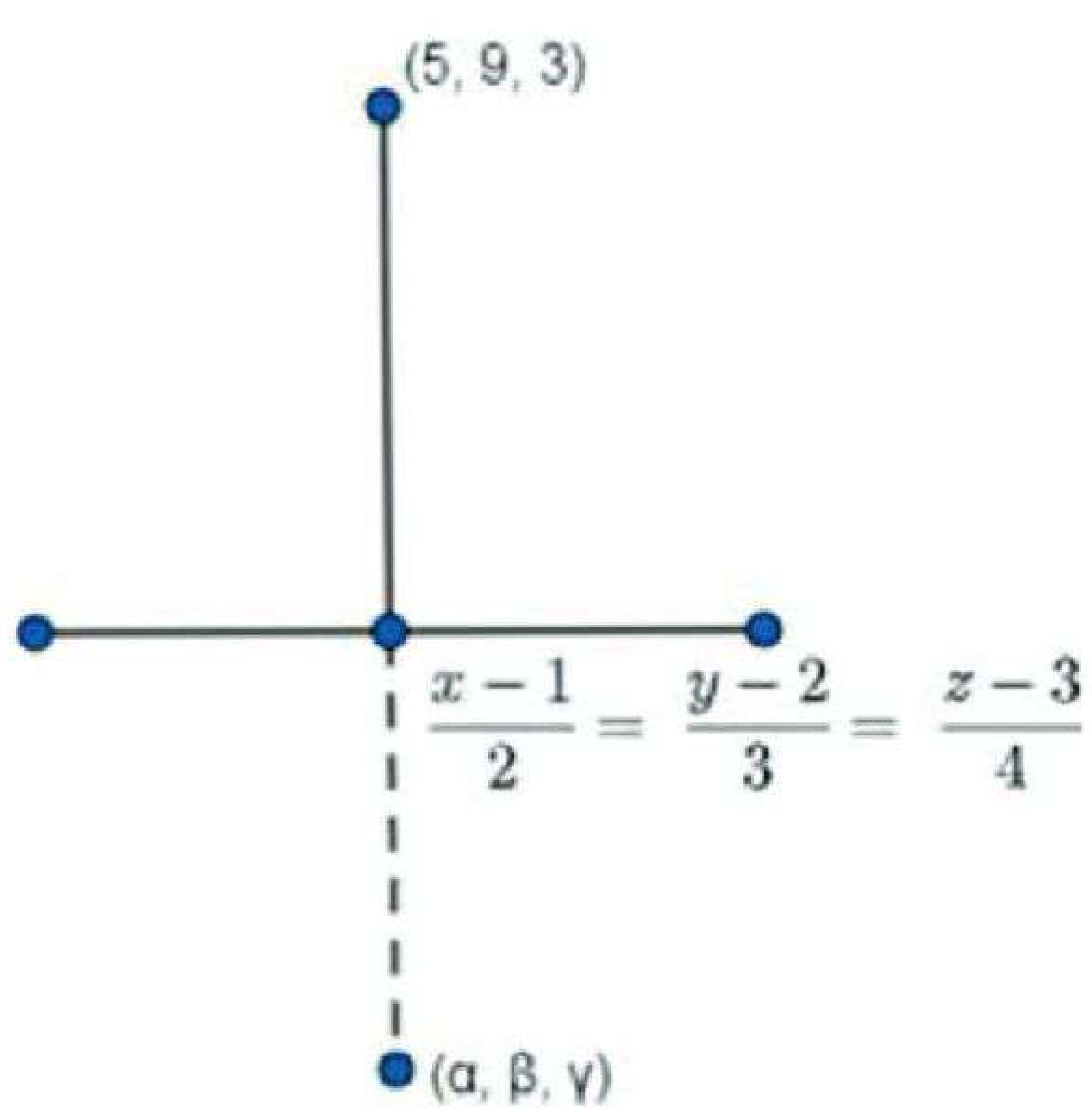
So the foot of the perpendicular is (2λ + 1, 3λ + 2, 4λ + 3)

The direction ratios of the perpendicular is

(2λ + 1 - 5) : (3λ + 2 - 9) : (4λ + 3 - 3)

⇒ (2λ – 4) : (3λ – 7) : (4λ)

Direction ratio of the line is 2 : 3 : 4



From the direction ratio of the line and the direction ratio of its perpendicular, we have

$2(2\lambda - 4) + 3(3\lambda - 7) + 4(4\lambda) = 0$

$\Rightarrow 4\lambda - 8 + 9\lambda - 21 + 16\lambda = 0$

$\Rightarrow 29\lambda = 29$

$\Rightarrow \lambda = 1$

So, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and (α, β, γ)

So, we have

$$\frac{\alpha + 5}{2} = 3 \Rightarrow \alpha = 1$$

$$\frac{\beta + 9}{2} = 5 \Rightarrow \beta = 1$$

$$\frac{\gamma + 3}{2} = 7 \Rightarrow \gamma = 11$$

So, the image is (1, 1, 11)

26. Question

Find the image of the point (2, -1, 5) in the line

$$\vec{r} = \left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$$

Answer

Given: Point (2, -1, 5)

Equation of line $= \left(11\,\hat{i} - 2\,\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\,\hat{j} - 11\hat{k}\right)$

The equation of line can be re-arranged as $\frac{x-11}{10} = \frac{x+2}{-4} = \frac{x+8}{-11} = r$

The general point on this line is

(10r + 11, -4r – 2, -11r – 8)

Let N be the foot of the perpendicular drawn from the point P(2, 1, -5) on the given line.

Then, this point is N(10r + 11, -4r – 2, -11r – 8) for some fixed value of r.

D.r.'s of PN are (10r + 9, -4r - 3, -11r - 3)

D.r.'s of the given line is 10, -4, -11.

Since, PN is perpendicular to the given line, we have,

10(10r + 9) – 4(-4r – 3) – 11(-11r – 3) = 0

100r + 90 + 16r + 12 + 121r + 33 = 0

237r = 135

$r = \frac{135}{237}$

Then, the image of the point is

$$\frac{\alpha - 11}{-11} = 0, \frac{\beta + 2}{7} = 1, \frac{\gamma + 8}{9} = 1$$

Therefore, the image is (0, 5, 1).

Exercise 27B

1. Question

Show that the points A(2, 1, 3), B(5, 0, 5) and C(-4, 3, -1) are collinear.

Answer

Given -

A = (2,1,3)

B = (5,0,5)

C = (-4,3,-1)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

((5-2),(0-1),(5-3))

=(3,-1,-2)

Similarly, the direction ratios of the line BC can be given by

((-4-5),(3-0),(-1-5))

=(-9,3,-6)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(3,-1,-2)

=(-1/3)X(-9,3,-6)

=(-1/3)Xd.r. of BC

Hence, **A, B and C are collinear**

2. Question

Show that the points A(2, 3, -4), B(1, -2, 3) and C(3, 8, -11) are collinear.

Answer

Given -

A = (2,3,-4)

B = (1,-2,3)

C = (3,8,-11)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1-2),(-2-3),(3+4))

=(-1,-5,7)

Similarly, the direction ratios of the line BC can be given by

((3-1),(8+2),(-11-3))

=(2,10,-14)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-5,7)

=(-1/2)X(2,10,-14)

=(-1/2)Xd.r. of BC

Hence, **A, B and C are collinear**

3. Question

Find the value of λ for which the points A(2, 5, 1), B(1, 2, -1) and C(3, λ, 3) are collinear.

Answer

Given -

A = (2,5,1)

B = (1,2,-1)

C = (3,λ,3)

To find – The value of λ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1-2),(2-5),(-1-1))

=(-1,-3,-2)

Similarly, the direction ratios of the line BC can be given by

((3-1),(λ-2),(3+1))

=(2,λ-2,4)

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-3,-2)

=(-1/2)X(2,λ-2,4)

=(-1/2)Xd.r. of BC

Since, A, B and C are collinear,

$\therefore -\frac{1}{2}(\lambda - 2) = -3$

$\Rightarrow \lambda - 2 = 6$

$\Rightarrow \boldsymbol{\lambda = 8}$

4. Question

Find the values of λ and μ so that the points A(3, 2, -4), B(9, 8, -10) and C(λ, μ -6) are collinear.

Answer

Given -

A = (3,2,-4)

B = (9,8,-10)

C = (λ,μ,-6)

To find – The value of λ and μ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((9-3),(8-2),(-10+4))

=(6,6,-6)

Similarly, the direction ratios of the line BC can be given by

((λ-9),(μ-8),(-6+10))

=(λ-9,μ-8,4)

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(6,6,-6)

=(-6/4)X(-4,-4,4)

=(-3/2)Xd.r. of BC

Since, A, B and C are collinear,

$$\therefore -\frac{3}{2}(\lambda - 9) = 6$$

$$\Rightarrow \lambda - 9 = -4$$

$$\Rightarrow \boldsymbol{\lambda = 5}$$

And,

$$\therefore -\frac{3}{2}(\mu - 8) = 6$$

$$\Rightarrow \mu - 8 = -4$$

$$\Rightarrow \boldsymbol{\lambda = 4}$$

5. Question

Find the values of λ and μ so that the points A(-1, 4, -2), B(λ, μ 1) and C(0, 2, -1) are collinear.

Answer

Given -

A = (-1,4,-2)

B = (λ,μ,1)

C = (0,2,-1)

To find – The value of λ and μ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((λ+1),(μ-4),(1+2))

=(λ+1,μ-4,3)

Similarly, the direction ratios of the line BC can be given by

$((0-\lambda),(2-\mu),(-1-1))$

$=(-\lambda,2-\mu,-2)$

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

$=(\lambda+1,\mu-4,3)$

Say, α be an arbitrary constant such that d.r. of AB = α X d.r. of BC

So, 3 = α X (-2)

i.e. α = -3/2

Since, A, B and C are collinear,

$\therefore -\frac{3}{2}(-\lambda) = \lambda + 1$

$\Rightarrow 3\lambda = 2\lambda + 2$

$\Rightarrow \boldsymbol{\lambda = 2}$

And,

$\therefore -\frac{3}{2}(2-\mu) = \mu - 4$

$\Rightarrow -6 + 3\mu = 2\mu - 8$

$\Rightarrow \boldsymbol{\mu = -2}$

6. Question

The position vectors of three points A, B and C are $\left(-4\hat{i}+2\hat{j}-3\hat{k}\right),\left(\hat{i}+3\hat{j}-2\hat{k}\right)$ and $\left(-9\hat{i}+\hat{j}-4\hat{k}\right)$ respectively. show that the points A, B and C are collinear.

Answer

Given -

$\vec{A} = -4\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{C} = -9\hat{i} + \hat{j} - 4\hat{k}$

It can thus be written as:

A = (-4,2,-3)

B = (1,3,-2)

C = (-9,1,-4)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

((1+4),(3-2),(-2+3))

=(5,1,1)

Similarly, the direction ratios of the line BC can be given by

((-9-1),(1-3),(-4+2))

=(-10,-2,-2)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(5,1,1)

=(-1/2)X(-10,-2,-2)

=(-1/2)Xd.r. of BC

Hence, **A, B and C are collinear**

Exercise 27C

1. Question

Find the angle between each of the following pairs of lines:

$\vec{r}=\left(3\hat{i}+\hat{j}-2\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-2\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}-\hat{j}-5\hat{k}\right)+\mu\left(3\hat{i}-5\hat{j}-4\hat{k}\right)$

Answer

Given – $\overrightarrow{L_1}=(3\hat{i}+\hat{j}-2\hat{k})+\lambda(\hat{i}-\hat{j}-2\hat{k})$

& $\overrightarrow{L_2}=(2\hat{i}-\hat{j}-5\hat{k})+\mu(3\hat{i}-5\hat{j}-4\hat{k})$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,-1,-2)

Direction ratios of L_2 = (3,-5,-4)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3+(-1)\times(-5)+(-2)\times(-4)}{\sqrt{1^2+1^2+2^2}\sqrt{3^2+5^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{3+5+8}{\sqrt{6}\sqrt{50}}\right)$$

$$= \cos^{-1}\left(\frac{16}{5\sqrt{6}\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$$

2. Question

Find the angle between each of the following pairs of lines:

$\vec{r} = \left(3\hat{i}-4\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+3\hat{k}\right)$ and $\vec{r} = 5\hat{i}+\mu\left(-\hat{i}+\hat{j}+\hat{k}\right)$

Answer

Given – $\overrightarrow{L_1} = (3\hat{\imath}-4\hat{\jmath}+2\hat{k})+\lambda(\hat{\imath}+3\hat{k})$

& $\overrightarrow{L_2} = (5\hat{\imath})+\mu(-\hat{\imath}+\hat{\jmath}+\hat{k})$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,0,3)

Direction ratios of L_2 = (-1,1,1)

Tip – If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times(-1)+0\times 1+3\times 1}{\sqrt{1^2+0^2+3^2}\sqrt{1^2+1^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{-1+3}{\sqrt{10}\sqrt{3}}\right)$$

$= \cos^{-1}\left(\frac{2}{\sqrt{30}}\right)$

$= \cos^{-1}\left(\frac{\sqrt{30}}{15}\right)$

3. Question

Find the angle between each of the following pairs of lines:

$\vec{r} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = 3\hat{k} + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

Answer

Given – $\overrightarrow{L_1} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$

& $\overrightarrow{L_2} = (3\hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (2,-2,1)

Direction ratios of L_2 = (1,2,-2)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2} \times \sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{2 \times 1 + (-2) \times 2 + 1 \times (-2)}{\sqrt{2^2 + 2^2 + 1^2}\sqrt{1^2 + 2^2 + 2^2}}\right)$

$= \cos^{-1}\left(\frac{2 - 4 - 2}{3 \times 3}\right)$

$= \cos^{-1}\left(-\frac{4}{9}\right)$

4. Question

Find the angle between each of the following pairs of lines:

$\frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$ and $\frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$

Answer

Given – $\overrightarrow{L_1} = \frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

& $\overrightarrow{L_2} = \frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,1,2)

Direction ratios of L_2 = (3,5,4)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3 + 1\times 5 + 2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{3^2+5^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{3+5+8}{\sqrt{6}\times\sqrt{50}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)}$$

5. Question

Find the angle between each of the following pairs of lines:

$$\frac{x-4}{4} = \frac{y+1}{4} = \frac{z-6}{5} \text{ and } \frac{x-5}{1} = \frac{2y+5}{-2} = \frac{z-3}{1}$$

Answer

Given – $\overrightarrow{L_1} = \frac{x-4}{4} = \frac{y+1}{3} = \frac{z-6}{5}$

& $\overrightarrow{L_2} = \frac{x-5}{1} = \frac{y+5/2}{-1} = \frac{z-3}{1}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (4,3,5)

Direction ratios of L_2 = (1,-1,1)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{4 \times 1 + 3 \times (-1) + 5 \times 1}{\sqrt{4^2 + 3^2 + 5^2}\sqrt{1^2 + 1^2 + 1^2}}\right)$$

$$= \cos^{-1}\left(\frac{4 - 3 + 5}{5\sqrt{2} \times \sqrt{3}}\right)$$

$$= \cos^{-1}\left(\frac{6}{5\sqrt{6}}\right)$$

$$= \cos^{-1}\left(\frac{2\sqrt{6}}{15}\right)$$

6. Question

Find the angle between each of the following pairs of lines:

$$\frac{3-x}{-2} = \frac{y+5}{1} = \frac{1-z}{3} \text{ and } \frac{x}{3} = \frac{1-y}{-2} = \frac{z+2}{-1}$$

Answer

Given – $\overrightarrow{L_1} = \frac{x-3}{2} = \frac{y+5}{1} = \frac{z-1}{-3}$

& $\overrightarrow{L_2} = \frac{x}{3} = \frac{y-1}{2} = \frac{z+2}{-1}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (2,1,-3)

Direction ratios of L_2 = (3,2,-1)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2+b^2+c^2} \times \sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2 \times 3 + 1 \times 2 + (-3) \times (-1)}{\sqrt{2^2 + 1^2 + 3^2}\sqrt{3^2 + 2^2 + 1^2}}\right)$$

$$= \cos^{-1}\left(\frac{6 + 2 + 3}{\sqrt{14} \times \sqrt{14}}\right)$$

$$= \cos^{-1}\left(\frac{11}{14}\right)$$

7. Question

Find the angle between each of the following pairs of lines: