$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \frac{\dfrac{\tan\frac{\pi}{4} + \tan x}{1-\tan\frac{\pi}{4}.\tan x}}{\dfrac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}.\tan x}}$$

$$\Rightarrow \frac{\dfrac{1+\tan x}{1-1.\tan x}}{\dfrac{1-\tan x}{1+1.\tan x}} = \frac{1+\tan x}{1-\tan x}.\frac{1+\tan x}{1-\tan x}$$

$$\Rightarrow \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Hence, Proved.

Q. 7. Prove that

(i) $\sin 75^0 = \frac{(\sqrt{6}+\sqrt{2})}{4}$

(ii) $\frac{\cos 135^0 - \cos 120^0}{\cos 135^0 + \cos 120^0} = (3-2\sqrt{2})$

(iii) $\tan 15^0 + \cot 15^0 = 4$

Answer : (i) sin75° = sin(90° - 15°) ……..(using sin(A - B) = sinAcosB - cosAsinB)

= sin90°cos15° - cos90°sin15°

= 1.cos15° - 0.sin15°

= cos15°

Cos15° = cos(45° - 30°) …………(using cos(A - B) = cosAcosB + sinAsinB)

= cos45°.cos30° + sin45°.sin30°

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}.1 \Rightarrow \frac{\sqrt{3}+1}{2\sqrt{2}}.\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin 75° = \cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$$

(ii) $\frac{\cos 135° - \cos 120°}{\cos 135° + \cos 120°} = \frac{\cos(180° - 45°) - \cos(180° - 60°)}{\cos(180° - 45°) + \cos(180° - 60°)}$ (using sin(180° - x) = sinx)

(using cos(180° - x) = - cosx)

$$= \frac{-\cos 45° - (-\cos 60°)}{-\cos 45° + (-\cos 60°)}$$

$$= \frac{\cos 60° - \cos 45°}{-(\cos 60° + \cos 45°)}$$

$$= -\frac{\frac{1}{2} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + \frac{1}{2}} \Rightarrow -\frac{\frac{1-\sqrt{2}}{2}}{\frac{\sqrt{2}+1}{2}} = -\frac{1-\sqrt{2}}{\sqrt{2}+1}.\frac{(-\sqrt{2}+1)}{(-\sqrt{2}+1)}$$

$$= -\frac{-\sqrt{2}+1+2-\sqrt{2}}{-2+\sqrt{2}-\sqrt{2}+1} \Rightarrow -\frac{-2\sqrt{2}+3}{-1} = 3-2\sqrt{2}$$

(iii) tan15° + cot15° =

First, we will calculate tan15°,

$$\tan 15^\circ = \frac{\sin 15^\circ}{\cos 15^\circ} \quad \text{.......................(1)}$$

$$[\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}, \sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ . \cos 30^\circ - \cos 45^\circ . \sin 30^\circ = \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} . \frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\tan 15^\circ = \frac{\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} \Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} \text{ and } \cot 15^\circ = \frac{1}{\tan 15^\circ} \frac{1}{\frac{\sqrt{3}-1}{\sqrt{3}+1}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Putting in eq(1),

$$\tan 15^\circ + \cot 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{3-1} \frac{3+1-2\sqrt{3}+3+1+2\sqrt{3}}{2}$$

$$= \frac{8}{2} = 4$$

Q. 8. Prove that

(i) $\cos 15^0 - \sin 15^0 = \frac{1}{\sqrt{2}}$

(ii) $\cot 105^0 - \tan 105^0 = 2\sqrt{3}$

(iii) $\frac{\tan 69^0 + \tan 66^0}{1 - \tan 69^0 \tan 66^0} = -1$

Answer :

(i) $\cos 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\text{Sin} 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\text{Cos} 15^0 - \sin 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{2}{2\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

(ii) cot105° - tan105° = cot(180° - 75°) - tan(180° - 75°)

(II quadrant tanx is negative and cotx as well)

= - cot75° - (- tan75°)

= tan75° - cot75°

Tan75° = $\frac{\sin 75°}{\cos 75°} \Rightarrow \frac{\sin(90° - 15°)}{\cos(90° - 15°)} = \frac{-\cos 15°}{\sin 15°}$

(using sin(90° - x) = - cosx and cos(90° - x) = sinx)

$$= -\frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} \Rightarrow \frac{-\sqrt{3}-1}{\sqrt{3}-1}$$

Cot75° = $\frac{1}{\tan 75°} \Rightarrow \frac{\sqrt{3}-1}{-\sqrt{3}-1}$

Cot105° - tan105°

= $\frac{\sqrt{3}-1}{-\sqrt{3}-1} - \frac{-\sqrt{3}-1}{\sqrt{3}-1} \Rightarrow \frac{(\sqrt{3}-1)-(-\sqrt{3}-1)}{(-\sqrt{3}-1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}-(3+1+2\sqrt{3})}{(-3+1-\sqrt{3}+\sqrt{3})}$

$$= \frac{-4\sqrt{3}}{-2} \Rightarrow 2\sqrt{3}$$

(iii) $\frac{\tan 69° + \tan 66°}{1 - \tan 69°.\tan 66°} = \tan(69° + 66°) \Rightarrow \tan 135° = \tan(180° - 45°)$

(II quadrant tanx negative)

$\Rightarrow$ - tan45° = - 1

Q. 9. Prove that $\frac{\cos 9^0 + \sin 9^0}{\cos 9^0 - \sin 9^0} = \tan 54^0$

Answer : First we will take out cos9°common from both numerator and denominator,

$$\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \frac{\cos 9(1 + \tan 9°)}{\cos 9°(1 - \tan 9°)} \Rightarrow \frac{\tan 45° + \tan 9°}{1 - \tan 45°.\tan 9°} = \tan(45° + 9°) \Rightarrow \tan 54$$

$$\left(\text{using} \tan\left(x + y\right) = \frac{\tan x + \tan y}{1 - \tan x . \tan y} \text{and} \tan 45° = 1\right)$$

Q. 10. Prove that $\dfrac{\cos 8^0 - \sin 8^0}{\cos 8^0 + \sin 8^0} = \tan 37^0$

Answer : First we will take out cos8° common from both numerator and denominator,

$$\frac{\cos 8° - \sin 8°}{\cos 8° + \sin 8°} = \frac{\cos 8°\left(1 - \tan 8°\right)}{\cos 8°\left(1 + \tan 8°\right)} \Rightarrow \frac{\tan 45° - \tan 8°}{1 + \tan 45° . \tan 8°} = \tan\left(45° - 8°\right) \Rightarrow \tan 37°$$

$$[\text{using} \tan\left(x - y\right) = \frac{\tan x - \tan y}{1 + \tan x . \tan y} \text{and} \tan 45° = 1$$

Q. 11. Prove that $\dfrac{\cos(\pi + \theta)\cos(-\theta)}{\cos(\pi - \theta)\cos\left(\dfrac{\pi}{2} + \theta\right)} = -\cot\theta$

Answer :

$$\frac{\cos\left(\pi + \theta\right) . \cos\left(-\theta\right)}{\cos\left(\pi - \theta\right) . \cos\left(\frac{\pi}{2} + \theta\right)} = \frac{-\cos\theta . \cos\theta}{-\cos\theta . -\sin\theta}$$

$$\Rightarrow \frac{\cos\theta}{-\sin\theta} = -\cot\theta$$

$$\left(\text{Using} \cos\left(\pi - \theta\right) = -\cos\theta \text{and} \cos\left(\frac{\pi}{2} - \theta\right) = -\sin\theta, \cos\left(-\theta\right) = -\cos\theta\right)$$

$$\left(\text{In III quadrant} \cos x \text{ is negative}, \cos\left(\pi + \theta\right) = -\cos\theta\right)$$

Q. 12. Prove that

$$\frac{\cos\theta}{\sin(90^0+\theta)}+\frac{\sin(-\theta)}{\sin(180^0+\theta)}-\frac{\tan(90^0+\theta)}{\cot\theta}=3$$

Answer : Using sin(90° + θ) = cosθ and sin(- θ) = sinθ,tan(90° + θ) = - cotθ

Sin(180° + θ) = - sinθ(III quadrant sinx is negative)

$$\frac{\cos\theta}{\sin(90^\circ+\theta)}+\frac{\sin(-\theta)}{\sin(180^\circ+\theta)}-\frac{\tan(90^\circ+\theta)}{\cot\theta}=\frac{\cos\theta}{\cos\theta}+\frac{-\sin\theta}{-\sin\theta}-\frac{-\cot\theta}{\cot\theta}$$

$$=1+(1)-(-1)\Rightarrow 1+1+1=3$$

Q. 13. Prove that

$$\frac{\sin(180^0+\theta)\cos(90^0+\theta)\tan(270^0-\theta)\cot(360^0-\theta)}{\sin(360^0-\theta)\cos(360^0+\theta)\operatorname{cosec}(-\theta)\sin(270^0+\theta)}=1$$

Answer : Using cos(90° + θ) = - sinθ(I quadrant cosx is positive

cosec(- θ) = - cosecθ

tan(270° - θ) = tan(180° + 90° - θ) = tan(90° - θ) = cotθ

(III quadrant tanx is positive)

Similarly sin(270° + θ) = - cosθ (IV quadrant sinx is negative

cot(360° - θ) = cotθ(IV quadrant cotx is negative)

$$=\frac{\sin(180^\circ+\theta).\cos(90^\circ+\theta).\tan(270^\circ-\theta).\cot(360^\circ-\theta)}{\sin(360^\circ-\theta).\cos(360^\circ-\theta).\operatorname{cosec}(-\theta).\sin(270^\circ+\theta)}$$

$$=\frac{-\sin\theta.-\sin\theta.\cot\theta.-\cot\theta}{-\sin\theta.\cos\theta.-\operatorname{cosec}\theta.-\cos\theta}$$

$$=\cot\theta.\tan\theta.\cot\theta.\tan\theta\Rightarrow 1$$

Q. 14. If θ and Φ lie in the first quadrant such that $\sin\theta = \frac{8}{17} \text{ and } \cos\phi = \frac{12}{13}$**, find the values of**

(i) sin (θ - Φ)
(ii) cos (θ - Φ)
(iii) tan (θ - Φ)

Answer : $\text{Given } \sin\theta = \frac{8}{17} \text{ and } \cos\phi = \frac{12}{13}$

$$\cos\theta = \sqrt{\left(1-\sin^2\theta\right)} \Rightarrow \sqrt{\left(1-\left(\frac{8}{17}\right)^2\right.} = \sqrt{\left(\frac{289-84}{289}\right)} \Rightarrow \sqrt{\left(\frac{225}{289}\right)} = \frac{15}{17}$$

$$\sin\phi = \sqrt{\left(1-\left(\frac{12}{13}\right)^2\right)} \Rightarrow \sqrt{\left(\frac{169-144}{169}\right)} = \sqrt{\left(\frac{25}{169}\right)} \Rightarrow \frac{5}{13}$$

(i) sin(θ - Φ) = sinθcosΦ + cosθsinΦ

$$= \frac{8}{17}\cdot\frac{12}{13} + \frac{15}{17}\cdot\frac{5}{13} \Rightarrow \frac{96+75}{221} = \frac{171}{221}$$

(ii) cos(θ - Φ) = cosθ.cosΦ + sinθ.sinΦ

$$= \frac{15}{17}\cdot\frac{12}{13} + \frac{8}{17}\cdot\frac{5}{13} \Rightarrow \frac{180+40}{221} = \frac{220}{221}$$

(iii) We will first find out the Values of tanθ and tanΦ,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{8/17}{15/17} = \frac{8}{15} \text{antan}\phi = \frac{\sin\phi}{\cos\phi} \Rightarrow \frac{5/13}{12/13} = \frac{5}{12}$$

$$\tan(\theta - \Phi) = \tan(\theta-\phi) = \frac{\tan\theta - \tan\phi}{1+\tan\theta.\tan\phi} \Rightarrow \frac{\frac{8}{15}-\frac{5}{12}}{1+\frac{8}{15}\cdot\frac{5}{12}}$$

Q. 15. If x and y are acute such that $\sin x = \frac{1}{\sqrt{5}}$ and $\sin y = \frac{1}{\sqrt{10}}$**, prove that** $(x + y) = \frac{\pi}{4}$

Answer : $\text{Given} \sin x = \frac{1}{\sqrt{5}} \text{ and} \sin y = \frac{1}{\sqrt{10}}$,

Now we will calculate value of cos x and cosy

$$\cos x = \sqrt{(1 - \sin^2 x)} \Rightarrow \sqrt{\left(1 - \left(\frac{1}{\sqrt{5}}\right)^2\right)} = \sqrt{\left(\frac{5-1}{5}\right)} \Rightarrow \sqrt{\left(\frac{4}{5}\right)} = \frac{2}{\sqrt{5}}$$

$$\cos y = \sqrt{(1 - \sin x^2)} \Rightarrow \sqrt{\left(1 - \left(\frac{1}{\sqrt{10}}\right)^2\right)} = \sqrt{\left(\frac{10-1}{10}\right)} \Rightarrow \sqrt{\left(\frac{9}{10}\right)} = \frac{3}{\sqrt{10}}$$

Sin(x + y) = sinx.cosy + cosx.siny

$$= \frac{1}{\sqrt{5}} \cdot \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{10}} \Rightarrow \frac{3+2}{\sqrt{50}} = \frac{5}{5\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin(x + y) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x + y = \frac{\pi}{4}$$

Q. 16. If x and y are acute angles such that $\cos x = \frac{13}{14}$ and $\cos y = \frac{1}{7}$**, prove that** $(x - y) = -\frac{\pi}{3}$.

Answer : $\text{Given } \cos x = \frac{13}{14} \text{ and } \cos y = \frac{1}{7}$

Now we will calculate value of sinx and siny

$$\sin x = \sqrt{\left(1-\cos^2 x\right)} \Rightarrow \sqrt{\left(1-\left(\frac{13}{14}\right)^2\right)} = \sqrt{\left(\frac{196-169}{196}\right)} \Rightarrow \sqrt{\left(\frac{27}{196}\right)} = \frac{3\sqrt{3}}{14}$$

$$\sin y = \sqrt{\left(1-\cos^2 y\right)} \Rightarrow \sqrt{\left(1-\left(\frac{1}{7}\right)^2\right)} = \sqrt{\left(\frac{49-1}{49}\right)} \Rightarrow \sqrt{\left(\frac{48}{49}\right)} = \frac{4\sqrt{3}}{7}$$

Hence,

Cos(x - y) = cosx.cosy + sinx.siny

$$= \frac{13}{14}.\frac{1}{7} + \frac{3\sqrt{3}}{14}.\frac{4\sqrt{3}}{7} \Rightarrow \frac{13+36}{98} = \frac{49}{98}$$

$$\cos(x-y) = \frac{1}{2}$$

$$x - y = \frac{\pi}{3}$$

Q. 17. If $\sin x = \frac{12}{3}$ and $\sin y = \frac{4}{5}$**, where**

$\frac{\pi}{2} < x < \pi$ and $0 < y < \frac{\pi}{2}$ **, find the values of**
(i) sin (x + y)
(ii) cos (x + y)
(iii) tan (x – y)

Answer : Given $\sin x = \frac{12}{13}$ and $\sin y = \frac{4}{5}$,

Here we will find values of cosx and cosy

$$\cos x = \sqrt{\left(1-\sin^2 x\right)} \Rightarrow \sqrt{\left(1-\left(\frac{12}{13}\right)^2\right)} = \sqrt{\left(\frac{169-144}{169}\right)} \Rightarrow \sqrt{\left(\frac{25}{169}\right)} = \frac{5}{13}$$

$$\cos y = \sqrt{(1-\sin^2 y)} \Rightarrow \sqrt{\left(1-\left(\frac{4}{5}\right)^2\right)} = \sqrt{\left(\frac{25-16}{25}\right)} \Rightarrow \sqrt{\left(\frac{9}{25}\right)} = \frac{3}{5}$$

(i) sin(x + y) = sinx.cosy + cosx.siny

$$\Rightarrow \frac{12}{13}.\frac{3}{5} + \frac{5}{13}.\frac{4}{5} \Rightarrow \frac{36+20}{65} = \frac{56}{65}$$

(ii) cos(x + y) = cosx.cosy + sinx.siny

$$= \frac{5}{13}.\frac{3}{5} + \frac{12}{13}.\frac{4}{5} \Rightarrow \frac{15+48}{65} = \frac{63}{65}$$

(iii) Here first we will calculate value of tanx and tany,

$$\tan x = \frac{\sin x}{\cos x} \Rightarrow \frac{12/13}{5/13} = \frac{5}{12} \text{ and } \tan y = \frac{\sin y}{\cos y} \Rightarrow \frac{4/5}{3/5} = \frac{4}{3}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x.\tan y} \Rightarrow \frac{\frac{5}{12}-\frac{4}{3}}{1+\frac{5}{12.}.\frac{4}{3}} = \frac{\frac{5-16}{12}}{\frac{36+20}{36}} \Rightarrow \frac{\frac{-11}{12}}{\frac{56}{36}} = \frac{-33}{56}$$

Q. 18.

If $\cos x = \frac{3}{5}$ and $\cos y = \frac{-24}{25}$**, where** $\frac{3\pi}{2} < x < 2\pi$ and $\pi < y < \frac{3\pi}{2}$**, find the values of**

(i) sin (x + y)
(ii) cos (x – y)
(iii) tan (x + y)

Answer : Given $\cos x = \frac{3}{5}$ and $\cos y = \frac{-24}{25}$

We will first find out value of sinx and siny,

$$\sin x = \sqrt{\left(1-\cos^2 x\right)} \Rightarrow \sqrt{\left(1-\left(\frac{3}{5}\right)^2\right)} = \sqrt{\left(\frac{25-9}{25}\right)} \Rightarrow \sqrt{\left(\frac{16}{25}\right)} = \frac{4}{5}$$

$$\sin y = \sqrt{\left(1-\cos^2 y\right)} \Rightarrow \sqrt{\left(1-\left(\frac{-24}{25}\right)^2\right)} = \sqrt{\left(\frac{625-576}{625}\right)} \Rightarrow \sqrt{\left(\frac{49}{625}\right)} = \frac{7}{25}$$

(i) sin(x + y) = sinx.cosy + cosx.siny

$$= \frac{4}{5}.\frac{-24}{25} + \frac{3}{5}.\frac{7}{25} \Rightarrow \frac{-96+21}{125} = \frac{-75}{125}$$

$$= \frac{-3}{5}$$

(ii) cos(x - y) = cosx.cosy + sinx.siny

$$= \frac{3}{5}.\frac{-24}{25} + \frac{4}{5}.\frac{7}{25} \Rightarrow \frac{-72+28}{125} = \frac{-44}{125}$$

(iii) Here first we will calculate value of tanx and tany,

$$\tan x = \frac{\sin x}{\cos x} \Rightarrow \frac{4/5}{3/5} = \frac{4}{3} \text{ and } \tan y = \frac{\sin y}{\cos y} \Rightarrow \frac{7/25}{-24/25} = \frac{7}{-24}$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x.\tan y} \Rightarrow \frac{\frac{4}{3}+\frac{-7}{24}}{1+\frac{4}{3}.\frac{-7}{24}} = \frac{\frac{32-7}{24}}{\frac{72-28}{72}} \Rightarrow \frac{\frac{25}{24}}{\frac{44}{72}} = \frac{75}{44}$$

Q. 19. Prove that

(i) $\cos\left(\frac{\pi}{3}+x\right) = \frac{1}{2}(\cos x - \sqrt{3}\sin x)$

(ii) $\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cos x$

(iii) $\frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{4}+x\right)=\frac{1}{2}(\cos x-\sin x)$

(iv) $\cos x+\cos\left(\frac{2\pi}{3}+x\right)+\cos\left(\frac{2\pi}{3}-x\right)=0$

Answer : (i) $\cos(\left(\frac{\pi}{3}+x\right) = \cos\frac{\pi}{3}.\cos x-\sin\frac{\pi}{3}.\sin x$

$\Rightarrow \frac{1}{2}.\cos x-\frac{\sqrt{3}}{2}.\sin x=\frac{1}{2}\left(\cos x-\sqrt{3}\sin x\right)$

(ii) $\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}-x\right)$

$= \sin\frac{\pi}{4}.\cos x+\cos\frac{\pi}{4}.\sin x+\sin\frac{\pi}{4}.\cos x-\cos\frac{\pi}{4}.\sin x$

$= 2.\sin\frac{\pi}{4}.\cos x \Rightarrow 2.\frac{1}{\sqrt{2}}.\cos x=\sqrt{2}.\cos x$

(iii) $\frac{1}{\sqrt{2}}.\cos\left(\frac{\pi}{4}+x\right)=\frac{1}{\sqrt{2}}.\left(\cos\frac{\pi}{4}.\cos x-\sin\frac{\pi}{4}.\sin x\right)$

$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}.\cos x-\frac{1}{\sqrt{2}}.\sin x\right)=\frac{1}{2}(\cos x-\sin x)$

(iv) $\cos x+\cos\left(\frac{2\pi}{3}+x\right)+\cos\left(\frac{2\pi}{3}-x\right)$

$$= \cos x + \cos\frac{2\pi}{3}.\cos x - \sin\frac{2\pi}{3}.\sin x + \cos\frac{2\pi}{3}.\cos x + \sin\frac{2\pi}{3}.\sin x$$

$$= \cos x + 2.\cos\left(\pi - \frac{\pi}{3}\right).\cos x$$

$$= \cos x + 2.\left(-\frac{1}{2}\right).\cos x$$

$$= \cos x - \cos x \Rightarrow 0$$

Q. 20. Prove that

(i) $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12} = \frac{1}{2}$

(ii) $2\cos\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}$

(iii) $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{(2+\sqrt{3})}{2}$

Answer : (i) $2\sin\frac{5\pi}{12}.\sin\frac{\pi}{12} = -\left(\cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)\right)$

.........[Using –2sinx.siny = cos(x + y)–cos (x–y)]

$$= -\left(\cos\frac{6\pi}{12} - \cos\frac{4\pi}{12}\right)$$

$$= -\left(\cos\frac{\pi}{2} - \cos\frac{\pi}{3}\right) \Rightarrow -\left(0 - \frac{1}{2}\right) = \frac{1}{2}$$

(ii) $2\cos\frac{5\pi}{12}.\cos\frac{\pi}{12} = \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) + \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$

...........[using 2cosx.cosy = cos(x + y) + cos(x–y)]

$$= \cos\frac{6\pi}{12} + \cos\frac{4\pi}{12} \Rightarrow \cos\frac{\pi}{2} + \cos\frac{\pi}{3} = 0 + \frac{1}{2}$$

$$= \frac{1}{2}$$

(iii) $$2\sin\frac{5\pi}{12}.\cos\frac{\pi}{12} = \sin\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) + \sin\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$$

...[Using2sinx.cosy = sin(x + y) + sin(x–y)]

$$= \sin\frac{6\pi}{12} + \sin\frac{4\pi}{12} \Rightarrow \sin\frac{\pi}{2} + \sin\frac{\pi}{3}$$

$$= 1 + \frac{\sqrt{3}}{2} \Rightarrow \frac{2 + \sqrt{3}}{2}$$

Exercise 15C

Q. 1. Prove that

$\sin(150^0 + x) + \sin(150^0 - x) = \cos x$

Answer : In this question the following formula will be used:

Sin(A +B)= sinA cos B + cosA sinB

Sin(A - B)= sinA cos B - cosA sinB

$= \sin 150^\circ \cos x + \cos 150^\circ \sin x + \sin 150^\circ \cos x - \cos 150^\circ \sin x$

$= 2\sin 150^\circ \cos x$

$= 2\sin(90^\circ + 60^\circ)\cos x$

$= 2\cos 60^\circ \cos x$

$= 2 \times \frac{1}{2} \cos x$

= cosx

Q. 2. Prove that

$\cos x + \cos(120^0 - x) + \cos(120^0 + x) = 0$

Answer : In this question the following formulas will be used:

cos (A + B) = cosAcosB – sinAsinB

cos (A - B) = cosAcosB+ sinAsinB

$= \cos x + \cos 120^0 \cos x - \sin 120 \sin x + \cos 120^0 \cos x + \sin 120 \sin x$

= cosx + 2cos120 cosx

= cosx + 2cos (90 + 30) cosx

= cosx + 2 (-sin30) cosx

$= \cos x - 2 \times \frac{1}{2} \cos x$

$= \cos x - \cos x$

= 0.

Q. 3. Prove that

$$\sin\left(x - \frac{\pi}{6}\right) + \cos\left(x - \frac{\pi}{3}\right) = \sqrt{3}\sin x$$

Answer : In this question the following formulas will be used:

sin (A - B) = sinA cos B - cosA sinB

cos (A - B) = cosAcosB+ sinAsinB

$= \sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6} + \cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3}$

$= \sin x \times \frac{\sqrt{3}}{2} - \cos x \times \frac{1}{2} + \cos x \times \frac{1}{2} + \sin x \times \frac{\sqrt{3}}{2}$

$= \sin x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{\sqrt{3}}{2}$

$= (\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}) \sin x$

$= \sqrt{3} \sin x$.

Q. 4. Prove that

$$\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$$

Answer : In this question the following formulas will be used:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$\tan(\frac{\pi}{4} + x) = \frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \tan x}$

$= \frac{1 + \tan x}{1 - \tan x} \because \tan\frac{\pi}{4} = 1$

Q. 5. Prove that

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan x}{1 + \tan x}$$

Answer : In this question the following formulas will be used:

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$\tan(\frac{\pi}{4} - x) = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \tan x}$

$= \frac{1-\tan x}{1+\tan x} \because \tan\frac{\pi}{4} = 1$

Q. 6. Express each of the following as a product.

1. sin 10x + sin 6x
2. sin 7x – sin 3x
3. cos 7x + cos 5x
4. cos2x – cos 4x

Answer :

$1. \sin 10x + \sin 6x = 2\sin\frac{10x+6x}{2}\cos\frac{10x-sinx}{2}$

$=2\sin\frac{18x}{2}\cos\frac{4x}{2}$

$=2\sin 9x \cos 2x$

Using,

sin(A +B)= sinA cos B + cosA sinB

$2. \sin 7x - \sin 3x = 2\cos\frac{7x+3x}{2}\sin\frac{7x-3x}{2}$

$= 2\cos\frac{10x}{2}\sin\frac{4x}{2}$

= 2cos5x sin2x

Using,

sin(A - B)= sinA cos B - cosA sinB

$3. \cos 7x + \cos 5x = 2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}$

$= 2\cos\frac{12x}{2}\cos\frac{2x}{2}$

$= 2\cos 6x \cos x$

Using,

cos (A + B) = cosAcosB – sinAsinB

$4. \cos 2x - \cos 4x = -2\sin\frac{2x+4x}{2}\sin\frac{2x-4x}{2}$

$= -2\sin\frac{6x}{2}\sin\frac{-2x}{2}$

= 2sin3x sinx

Using,

cos (A - B) = cosAcosB+ sinAsinB

Q. 7. Express each of the following as an algebraic sum of sines or cosines :

(i) 2sin 6x cos 4x
(ii) 2cos 5x din 3x
(iii) 2cos 7x cos 3x
(iv) 2sin 8x sin 2x

Answer : (i) 2sin 6x cos 4x = sin (6x+4x) + sin (6x-4x)

= sin 10x + sin 2x

Using,

2sinAcosB = sin (A+ B) + sin (A - B)

(ii) 2cos 5x sin3x = sin (5x + 3x) – sin (5x – 3x)

= sin8x – sin2x

Using,

2cosAsinB = sin(A + B) – sin (A - B)

(iii) 2cos7xcos3x = cos (7x+3x) + cos (7x – 3x)

= cos10x + cos 4x

Using,

2cosAcosB = cos (A+ B) + cos (A - B)

(iv) 2sin8 x sin2 x = cos (8x - 2x) – cos (8x + 2x)

= cos6x – cos10x

Using,

2sinAsinB = cos (A - B) – cos (A+ B)

Q. 8. Prove that

$$\frac{\sin x + \sin 3x}{\cos x - \cos 3x} = \cot x$$

Answer :

$$\frac{\sin x + \sin 3x}{\cos x - \cos 3x}$$

$=\frac{2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}}{-2\sin\frac{x+3x}{2}\sin\frac{x-3x}{2}}$

$=\frac{2\sin\frac{4x}{2}\cos\frac{2x}{2}}{2\sin\frac{4x}{2}\sin\frac{2x}{2}}$

$=\frac{\cos x}{\sin x}$

= cotx

Using the formula,

sinA + sinB = 2sin$\frac{A+B}{2}$cos$\frac{A-B}{2}$

cosA - cosB = -2sin$\frac{A+B}{2}$sin$\frac{A-B}{2}$

Q. 9. Prove that

$$\frac{\sin 7x - \sin 5x}{\cos 7x + \cos 5x} = \tan x$$

Answer :

$$\frac{\sin 7x - \sin 5x}{\cos 7x + \cos 5x}$$

$= \frac{2\cos\frac{7x+5x}{2}\sin\frac{7x-5x}{2}}{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}}$

$= \frac{2\cos 6x \sin x}{2\cos 6x \cos x}$

$= \frac{\sin x}{\cos x}$

= tanx

Using the formula,

sinA - sinB = $2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

cosA + cosB = $2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 10. Prove that

$$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

Answer :

$$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$=\frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$$

$$=\frac{2\sin 4x\cos x}{2\cos 4x\cos x}$$

= tan4x

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 11. Prove that

$$\frac{\cos 9x - \cos 5x}{\cos 17x - \sin 3x} = \frac{-\sin 2x}{\cos 10x}$$

Answer :

$= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$= \frac{-2\sin\frac{9x+5x}{2}\sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}$

$= \frac{-2\sin 7x \sin 2x}{2\cos 10x \sin 7x}$

$= \frac{-\sin 2x}{\cos 10x}$

Using the formula,

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 12. Prove that

$$\frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \tan 3x$$

Answer :

$=\frac{\sin x+\sin 3x+\sin 5x}{\cos x+\cos 3x+\cos 5x}$

$=\frac{(\sin 5x+\sin x)+\sin 3x}{(\cos 5x+\cos x)+\cos 3x}$

$=\frac{2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2}+\sin 3x}{2\cos\frac{5x+x}{2}\cos\frac{5x-x}{2}+\cos 3x}$

$=\frac{2\sin 3x\cos x+\sin 3x}{2\cos 3x\cos x+\cos 3x}$

$=\frac{\sin 3x(2\cos x+1)}{\cos 3x(2\cos x+1)}$

= tan3x.

Using the formula,

$\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 13. Prove that

$$\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}=\tan 6x$$

Answer :

$$=\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$$

$$=\frac{2\sin\frac{7x+5x}{2}\cos\frac{7x-5x}{2}+2\sin\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}+2\cos\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}$$

$$=\frac{2\sin 6x\cos x+2\sin 6x\cos 3x}{2\cos 6x\cos x+2\cos 6x\cos 3x}$$

$$=\frac{2\sin 6x(\cos x+\cos 3x)}{2\cos 6x(\cos x+\cos 3x)}$$

$$=\frac{\sin 6x}{\cos 6x}$$

$$=\tan 6x$$

Using the formula,

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

Q. 14. Prove that

cot 4x (sin 5x + sin 3x) = cot x (sin 5x – sin 3x)

Answer : L.H.S

cot 4x (sin 5x + sin3x)

$= \cot 4x\ (2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2})$

= cot 4x (2 sin4x cosx)

$= \frac{\cos 4x}{\sin 4x}$(2 sin4x cosx)

= 2cos4xcosx

R.H.S

cot x (sin 5x - sin3x)

$= \cot x\ (2\cos\frac{5x+3x}{2}\sin\frac{5x-3x}{2})$

$= \cot x\ (2 \cos 4x \sin x)$

$= \frac{\cos x}{\sin x}(2 \cos 4x \sin x)$

$= 2\cos 4x \cos x$

L.H.S=R.H.S

Hence, proved.

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 15. Prove that

(sin 3x + sin x) sin x + (cos 3x – cos x) cos x = 0

Answer : = (sin 3x + sin x) sin x + (cos 3x – cos x) cos x

$= (2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}) \sin x + (-2\sin \frac{3x+x}{2}\sin\frac{3x-x}{2}) \cos x$

= (2sin2x cosx) sinx-(2sin2x sinx) cosx

= 0.

Using the formula,

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Q. 16. Prove that

$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$

Answer : = $(\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$$=\left(-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}\right)^2 + \left(2\cos\frac{x+y}{2}\sin\frac{x-y}{2}\right)^2$$

$$=4\sin^2\left(\frac{x-y}{2}\right)\left(\sin^2\left(\frac{x-y}{2}\right)+\cos^2\left(\frac{x-y}{2}\right)\right)$$

$$=4\sin^2\left(\frac{x-y}{2}\right)$$

Using the formula,

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

Q. 17. Prove that

$$\frac{\sin 2x - \sin 2y}{\cos 2y - \cos 2x} = \cot(x+y)$$

Answer :

$$=\frac{\sin 2x-\sin 2y}{\cos 2y-\cos 2x}$$

$$=\frac{2\cos\frac{2x+2y}{2}\sin\frac{2x-2y}{2}}{-2\sin\frac{2x+2y}{2}\sin\frac{2y-2x}{2}}$$

$$=\frac{\cos(x+y)\sin(x-y)}{\sin(x+y)\sin(x-y)}$$

$$=\frac{\cos(x+y)}{\sin(x+y)}$$

$$=\cot(x+y)$$

Using the formula,

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 18. Prove that

$$\frac{\cos x+\cos y}{\cos y-\cos x}=\cot\left(\frac{x+y}{2}\right)\cot\left(\frac{x-y}{2}\right)$$

Answer :

$=\frac{\cos x-\cos y}{\cos y-\cos x}$

$=\frac{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{-2\sin\frac{x+y}{2}\sin\frac{y-x}{2}}$

$=\frac{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{2\sin\frac{x+y}{2}\sin\frac{x-y}{2}}$

$=\frac{\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{\sin\frac{x+y}{2}\sin\frac{x-y}{2}}$

$=\cot\frac{x+y}{2}\cot\frac{x-y}{2}$

Using the formula,

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 19. Prove that

$$\frac{\sin x+\sin y}{\sin x-\sin y}=\tan\left(\frac{x+y}{2}\right)\cot\left(\frac{x-y}{2}\right)$$

Answer :

$=\frac{\sin x+\sin y}{\sin x-\sin y}$

$=\frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}$

$=\tan\frac{x+y}{2}\cot\frac{x-y}{2}$

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 20. Prove that

$$\sin 3x + \sin 2x - \sin x = 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$$

Answer : =Sin3x+sin2x-sinx

= (sin3x- sinx)+sin2x

$= \left(2\cos\frac{3x+x}{2}\sin\frac{3x-2x}{2}\right) + \sin 2x$

= 2cos2xsinx +sin2x

= 2cos2xsinx + 2sinxcosx

= 2sinx (cos2x + cosx)

$= 2\sin x\ (2\cos\frac{2x+x}{2}\cos\frac{2x-x}{2})$

$= 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Using the formula,

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 21. Prove that

$$\frac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x \sin x + \cos 6x \cos x} = \tan 2x$$

Answer :

$$= \frac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x \sin x + \cos 6x \cos x}$$

$$= \frac{2\cos 4x \sin 3x - 2\cos 2x \sin x}{2\sin 4x \sin x + 2\cos 6x \cos x}$$

$$= \frac{\sin(4x+3x) - \sin(4x-3x) - \{\sin(2x+x) - \sin(2x-x)\}}{\cos(4x-x) - \cos(4x+x) + \cos(6x+x) + \cos(6x-x)}$$

$$= \frac{\sin 7x + \sin x - \sin 3x + \sin x}{\cos 3x - \cos 5x + \cos 7x + \cos 5x}$$

$$= \frac{\sin 7x - \sin 3x}{\cos 3x + \cos 7x}$$

$$= \frac{2\cos\frac{7x+3x}{2}\sin\frac{7x-3x}{2}}{2\cos\frac{7x+3x}{2}\cos\frac{7x-3x}{2}}$$

Using the formulas,

2cosAsinB = sin (A + B) – sin (A - B)

2cosAcosB = cos (A + B) + cos (A - B)

2sinAsinB = cos (A - B) – cos (A + B)

Q. 22. Prove that

$$\frac{\cos 2x \sin x + \cos 6x \sin 3x}{\sin 2x \sin x + \sin 6x \sin 3x} = \cot 5x$$

Answer :

$$= \frac{\cos 2x \sin x + \cos 6x \sin 3x}{\sin 2x \sin x + \sin 6x \sin 3x}$$

$$= \frac{2\cos 2x \sin x + 2\cos 6x \sin 3x}{2\sin 2x \sin x + 2\sin 6x \sin 3x}$$

$$= \frac{\sin(2x+x) - \sin(2x-x) + \{\sin(6x+3x) - \sin(6x-3x)\}}{\cos(2x-x) - \cos(2x+x) + \cos(6x-3x) - \cos(6x+3x)}$$

$$= \frac{\sin 3x - \sin x + \sin 9x - \sin 3x}{\cos x - \cos 3x + \cos 3x - \cos 9x}$$

$$= \frac{\sin 9x - \sin x}{\cos x - \cos 9x}$$

$$= \frac{2\cos\frac{9x+x}{2}\sin\frac{9x-x}{2}}{-2\sin\frac{x+9x}{2}\sin\frac{x-9x}{2}}$$

$$=\frac{2\cos\frac{9x+x}{2}\sin\frac{9x-x}{2}}{2\sin\frac{x+9x}{2}\sin\frac{9x-x}{2}}$$

$$=\frac{\cos 5x \sin 4x}{\sin 5x \cos 4x}$$

$$=\cot 5x$$

Using the formulas,

2cosAsinB = sin (A + B) – sin (A - B)

2sinAsinB = cos (A - B) – cos (A + B)

Q. 23. Prove that

$$\sin 10^0 \sin 30^0 \sin 50^0 \sin 70^0 = \frac{1}{16}$$

Answer : L.H.S