

EXERCISE 19.16

Evaluate the following integrals:

$$1. \int \frac{\sec^2 x}{1 - \tan^2 x} dx$$

Solution:

$$\text{Let } I = \int \frac{\sec^2 x}{1 - \tan^2 x} dx$$

$$\text{Let } \tan x = t \dots (i)$$

$$\Rightarrow \sec^2 x dx = dt$$

So, substituting these values in given equation we get

$$I = \int \frac{dt}{(1)^2 - t^2}$$

$$I = \frac{1}{2 \times 1} \log \left| \frac{1+t}{1-t} \right| + c \quad \left[\text{since, } \int \frac{1}{a^2 - (x)^2} dx = \frac{1}{2 \times a} \log \left| \frac{a+x}{a-x} \right| + c \right]$$

$$I = \frac{1}{2} \log \left| \frac{1+\tan x}{1-\tan x} \right| + c \quad [\text{Using (i)}]$$

$$2. \int \frac{e^x}{1 + e^{2x}} dx$$

Solution:

$$\text{Let } I = \int \frac{e^x}{1 + e^{2x}} dx$$

$$\text{Let } e^x = t \dots (i)$$

$$\Rightarrow e^x dx = dt$$

So, substituting these values in given equation we get

$$I = \int \frac{dt}{(1)^2 + t^2}$$

$$I = \tan^{-1} t + c$$

$$[\text{since, } \int \frac{1}{1+(x)^2} dx = \tan^{-1} x + c]$$

$$I = \tan^{-1}(e^x) + c \text{ [Using (i)]}$$

$$3. \int \frac{\cos x}{\sin^2 x + 4 \sin x + 5} dx$$

Solution:

$$\text{Let } I = \int \frac{\cos x}{\sin^2 x + 4 \sin x + 5} dx$$

$$\text{Let } \sin x = t \dots (i)$$

$$\Rightarrow \cos x dx = dt$$

$$\text{So, } I = \int \frac{dt}{t^2 + 4t + 5}$$

Adding and subtracting 2^2 to the denominator we get

$$= \int \frac{dt}{t^2 + (2t)(2) + 2^2 - 2^2 + 5}$$

Above equation can be written as

$$\int \frac{dt}{(t+2)^2 + 1}$$

$$\text{Again, let } t + 2 = u \dots (ii)$$

$$\Rightarrow dt = du$$

$$I = \int \frac{du}{u^2 + 1}$$

$$= \tan^{-1} u + c$$

$$[\text{since, } \int \frac{1}{1+(x)^2} dx = \tan^{-1} x + c]$$

$$= \tan^{-1}(\sin x + 2) + c \text{ [Using (i), (ii)]}$$

$$4. \int \frac{e^x}{e^{2x} + 5e^x + 6} dx$$

Solution:

$$\text{Let } I = \int \frac{e^x}{e^{2x} + 5e^x + 6} dx$$

$$\text{Let } e^x = t \dots (i)$$

$$\Rightarrow e^x dx = dt$$

$$= \int \frac{1}{t^2 + 5t + 6} dt$$

$$= \int \frac{1}{t^2 + 2t \times \frac{5}{2} + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6} dt$$

$$= \int \frac{1}{\left(t + \frac{5}{2}\right)^2 - \frac{1}{4}} dt$$

$$\text{Let } t + \frac{5}{2} = u \dots (ii)$$

$$\Rightarrow dt = du$$

So, substituting these values we get

$$I = \int \frac{1}{u^2 - \left(\frac{1}{2}\right)^2} du$$

$$I = \frac{1}{2 \times \frac{1}{2}} \log \left| \frac{u - \frac{1}{2}}{u + \frac{1}{2}} \right| + c$$

$$[\text{since, } \int \frac{1}{x^2 - (a)^2} dx = \frac{1}{2 \times a} \log \left| \frac{x - a}{x + a} \right| + c]$$

$$I = \log \left| \frac{2u - 1}{2u + 1} \right| + c$$

$$I = \log \left| \frac{2\left(t + \frac{5}{2}\right) - 1}{2\left(t + \frac{5}{2}\right) + 1} \right| + c \text{ [Using (ii)]}$$

$$I = \log \left| \frac{e^{3x} + 2}{e^{3x} + 3} \right| + c \text{ [Using (i)]}$$

$$5. \int \frac{e^{3x}}{4e^{6x} - 9} dx$$

Solution:

$$\text{Let } I = \int \frac{e^{3x}}{4e^{6x} - 9} dx$$

$$\text{Let } e^{3x} = t \dots \text{ (i)}$$

$$\Rightarrow 3e^{3x} dx = dt$$

$$I = \frac{1}{3} \int \frac{1}{4t^2 - 9} dt$$

Taking $(\frac{1}{4})$ as common we get

$$= \frac{1}{12} \int \frac{1}{t^2 - \frac{9}{4}} dt$$

The above equation can be written as

$$I = \frac{1}{12} \int \frac{1}{t^2 - \left(\frac{3}{2}\right)^2} dt$$

$$I = \frac{1}{36} \log \left| \frac{t - \frac{3}{2}}{t + \frac{3}{2}} \right| + c$$

$$\text{[since, } \int \frac{1}{x^2 - (a)^2} dx = \frac{1}{2 \times a} \log \left| \frac{x - a}{x + a} \right| + c]$$

$$I = \frac{1}{36} \log \left| \frac{2t-3}{2t+3} \right| + c$$

$$I = \frac{1}{36} \log \left| \frac{2e^{3x}-3}{2e^{3x}+3} \right| + c \quad [\text{Using (i)}]$$



Myclass24
Your Class. Your Pace.