Activity 11

Objective

To inerpret geometrically the meaning of $i = \sqrt{-1}$ and its integral powers.

Material Required

Cardboard, chart paper, sketch pen, ruler, compasses, adhesive, nails, thread.

Method of Construction

1. Paste a chart paper on the cardboard of a convenient size.
2. Draw two mutually perpendicular lines $X'X$ and $Y'Y$ interesting at the point O (see Fig. 11).
3. Take a thread of a unit length representing the number 1 along OX . Fix one end of the thread to the nail at 0 and the other end at A as shown in the figure.
4. Set free the other end of the thread at A and rotate the thread through angles of 90º, 180º, 270º and 360º and mark the free end of the thread in different cases as A_1, A_2, A_3 and A_4 , respectively, as shown in the figure.

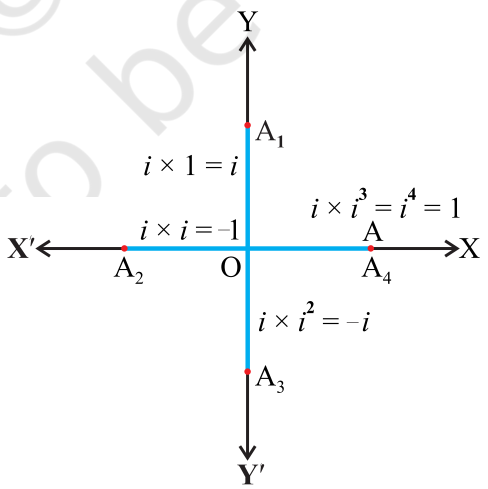

Fig. 11

DEMONSTRATION

1. In the argand plane, OA, OA_1, OA_2, OA_3, OA_4 represent, respectively, 1, i, –1, – i, 1.
2. $OA_1 = i = 1 \times i$, $OA_2 = -1 = i \times i = i^2$, $OA_3 = -i = i \times i \times i = i^3$ and so on. Each time, rotation of OA by 90º is equivalent to multiplication by i. Thus, i is referred to as the multiplying factor for a rotation of 90º.

OBSERVATION

1. On rotating OA through 90º, $OA_1 = 1 \times i =$ _________.
2. On rotating OA through an angle of 180°, $OA_2 = 1 \times$ __ $\times$ __ = ______.
3. On rotation of OA through 270º (3 right angles), $OA_3 =$

   $1 \times$ _______ $\times$ ______ $\times$ ______ = ______.
4. On rotating OA through 360º (4 right angles),

   $OA_4 = 1 \times$ _______ $\times$ ______ $\times$ ______ $\times$ ______ = __________.
5. On rotating OA through n-right angles

   $OA_n = 1 \times$ _______ $\times$ ______ $\times$ ______ $\times$ ______ $\times$ ... n times = __________.

APPLICATION

This activity may be used to evaluate any integral power of i.

Activity 12

Objective

To obtain a quadratic function with the help of linear functions graphically.

Material Required

Plywood sheet, pieces of wires.

Method of Construction

1. Take two wires of equal length.
2. Fix them at O in a plane (on the plywood sheet) at right angle to each other to represent x-axis and y-axis (see Fig.12)
3. Take a piece of wire and fix it in such a way that it meets the x-axis at a distance of a units from O in the positive direction and meets y-axis at a distance of a units below O as shown in the figure. Mark these points as B and A, respectively.

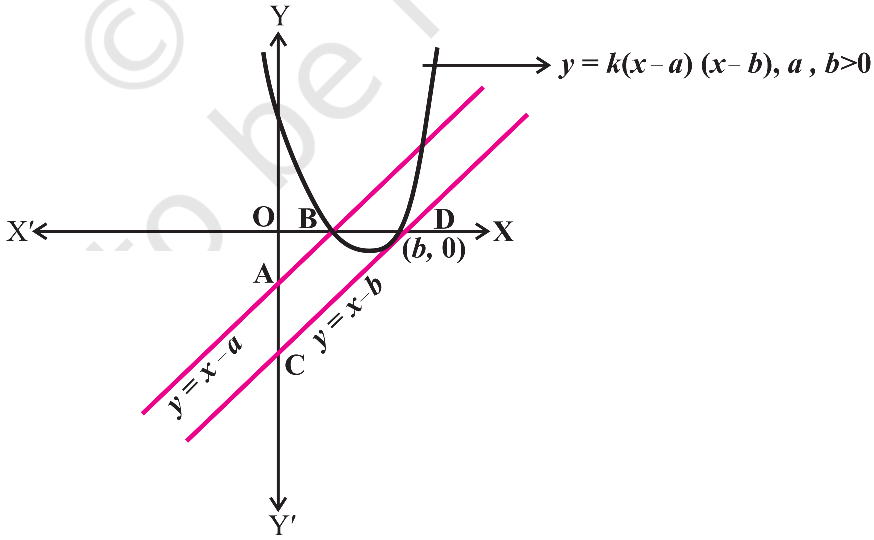

Fig. 12

4. Similarly, take another wire and fix it in such a way that it meets the x-axis at a distance of b units from O in the positive direction and meets y-axis at a distance of b units below O as shown in the Fig.12. Mark these points as D and C, respectively.
5. Take one more wire and fix it in such a way that it passes through the points where straight wires meet the x-axis and the wire takes the shape of a curve (parabola) as shown in the Fig.12.

DEMONSTRATION

1. The wire through the points A and B represents the straight line given by $y = x - a$ intersecting the x and y-axis at $(a, 0)$ and $(0, -a)$, respectively.
2. The wire through the points C and D represents the straight line given by $y = x - b$ intersecting x and y axis at $(b, 0)$ and $(0, -b)$, respectively.
3. The wire through B and D represents a curve given by the function $y = k(x - a)(x - b) = k[x^2 - (a + b)x + ab]$, where k is an arbitrary constant.

OBSERVATION

1. The line given by the linear function $y = x - a$ intersects the x-axis at the point _______ whose coordinates are __________.
2. The line given by the linear function $y = x - b$ intersects the x-axis at the point _______ whose coordinates are ________.
3. The curve passing through B and D is given by the function $y =$ ______, which is a ________ function.

APPLICATION

This activity is useful in understanding the zeroes and the shape of graph of a quadratic polynomial.

Activity 13

Objective

To verify that the graph of a given inequality, say $5x + 4y - 40 < 0$, of the form $ax + by + c < 0$, $a, b > 0$, $c < 0$ represents only one of the two half planes.

Material Required

Cardboard, thick white paper, sketch pen, ruler, adhesive.

Method of Construction

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw two perpendicular lines X′OX and Y′OY to represent x-axis and y-axis, respectively.
3. Draw the graph of the linear equation corresponding to the given linear inequality.
4. Mark the two half planes as I and II as shown in the Fig. 13.

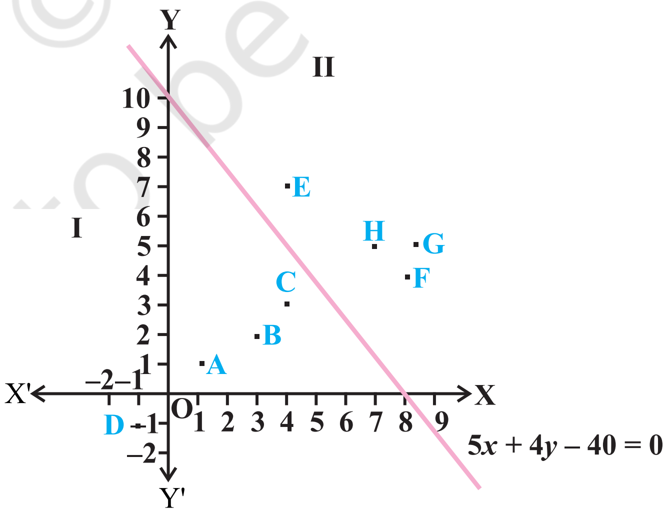

Fig. 13

DEMONSTRATION

1. Mark some points O(0, 0), A(1, 1), B(3, 2), C(4, 3), D(–1, –1) in half plane I and points E(4, 7), F(8, 4), G(9, 5), H(7, 5) in half plane II.

2. (i) Put the coordinates of O (0,0) in the left hand side of the inequality.

   Value of LHS = $5\,(0) + 4\,(0) - 40 = -40 < 0$

   So, the coordinates of O which lies in half plane I, satisfy the inequality.

   (ii) Put the coordinates of the point E (4, 7) in the left hand side of the inequality.

   Value of LHS = $5(4) + 4(7) - 40 = 8 \not< 0$ and hence the coordinates of the point E which lie in the half plane II does not satisfy the given inequality.

   (iii) Put the coordinates of the point F(8, 4) in the left hand side of the inequality. Value of LHS = $5(8) + 4(4) - 40 = 16 \not< 0$

   So, the coordinates of the point F which lies in the half plane II do not satisfy the inequality.

   (iv) Put the coordinates of the point C(4, 3) in the left hand side of the inequality.

   Value of LHS = $5(4) + 4(3) - 40 = -8 < 0$

   So, the coordinates of C which lies in the half plane I, satisfy the inequality.

   (v) Put the coordinates of the point D(–1, –1) in the left hand side of the inequality.

   Value of LHS = $5(-1) + 4\,(-) - 40 = -49 < 0$

So, the coordinates of D which lies in the half plane I, satisfy the inequality.

(iv) Similarly points A (1, 1), lies in a half plane I satisfy the inequality. The points G (9, 5) and H (7, 5) lies in half plane II do not satisfy the inequality.

Thus, all points O, A, B, C, satisfying the linear inequality $5x + 4y - 40 <$ lie only in the half plane I and all the points E, F, G, H which do not satisfy the linear inequality lie in the half plane II.

Thus, the graph of the given inequality represents only one of the two corresponding half planes.

OBSERVATION

Coordinates of the point A __________ the given inequality (satisfy/does not satisfy).

Coordinates of G __________ the given inequality.

Coordinates of H __________ the given inequality.

Coordinates of E are __________ the given inequality.

Coordinates of F __________ the given inequality and is in the half plane____.

The graph of the given inequality is only half plane _________.

APPLICATION

This activity may be used to identify the half plane which provides the solutions of a given inequality.

NOTE

The activity can also be performed for the inequality of the type $ax + by + c > 0$.

Activity 14

Objective

To find the number of ways in which three cards can be selected from given five cards.

Material Required

Cardboard sheet, white paper sheets, sketch pen, cutter.

Method of Construction

1. Take a cardboard sheet and paste white paper on it.
2. Cut out 5 identical cards of convenient size from the cardboard.
3. Mark these cards as C_1, C_2, C_3, C_4 and C_5.

Demonstration

1. Select one card from the given five cards.
2. Let the first selected card be C_1. Then other two cards from the remaining four cards can be : C_2C_3, C_2C_4, C_2C_5, C_3C_4, C_3C_5 and C_4C_5.Thus, the possible selections are : $C_1C_2C_3$, $C_1C_2C_4$, $C_1C_2C_5$,$C_1C_3C_4$, $C_1C_3C_5$, $C_1C_4C_5$. Record these on a paper sheet.
3. Let the first selected card be C_2. Then the other two cards from the remaining 4 cards can be : C_1C_3, C_1C_4, C_1C_5, C_3C_4, C_3C_5, C_4C_5.Thus, the possible selections are: $C_2C_1C_3$, $C_2C_1C_4$, $C_2C_1C_5$,$C_2C_3C_4$, $C_2C_3C_5$, $C_2C_4C_5$. Record these on the same paper sheet.
4. Let the first selected card be C_3. Then the other two cards can be : C_1C_2, C_1C_4, C_1C_5, C_2C_4, C_2C_5, C_4C_5.Thus, the possible selections are : $C_3C_1C_2$, $C_3C_1C_4$, $C_3C_1C_5$,$C_3C_2C_4$, $C_3C_2C_5$, $C_3C_4C_5$. Record them on the same paper sheet.
5. Let the first selected card be C_4. Then the other two cards can be : C_1C_2, C_1C_3, C_2C_3, C_1C_5, C_2C_5, C_3C_5 Thus, the possible selections are: $C_4C_1C_2$, $C_4C_1C_3$,$C_4C_2C_3$, $C_4C_1C_5$, $C_4C_2C_5$, $C_4C_3C_5$. Record these on the same paper sheet.

6. Let the first selected card be C_5. Then the other two cards can be: C_1C_2, C_1C_3, C_1C_4, C_2C_3, C_2C_4, C_3C_4 Thus, the possible selections are: $C_5C_1C_2$, $C_5C_1C_3$, $C_5C_1C_4$, $C_5C_2C_3$, $C_5C_2C_4$, $C_5C_3C_4$. Record these on the same paper sheet.
7. Now look at the paper sheet on which the possible selectios are listed. Here, there are in all 30 possible selections and each of the selection is repeated thrice. Therefore, the number of distinct selection $= 30 \div 3 = 10$ which is same as $5C_3$.

Observation

1. $C_1C_2C_3$, $C_2C_1C_3$ and $C_3C_1C_2$ represent the _______ selection.
2. $C_1C_2C_4$, ______________, ____________ represent the same selection.
3. Among $C_2C_1C_5$, $C_1C_2C_5$, $C_1C_2C_3$, ________ and ________ represent the same selection.
4. $C_2C_1C_5$, $C_1C_2C_3$, represent _______ selections.
5. Among $C_3C_1C_5$, $C_1C_4C_3$, $C_5C_3C_4$, $C_4C_2C_5$, $C_2C_4C_3$, $C_1C_3C_5$

   $C_3C_1C_5$, ________ represent the same selections.

   $C_3C_1C_5$, $C_1C_4C_5$, ______, _____, represent different selections.

Application

Activities of this type can be used in understanding the general formula for finding the number of possible selections when r objects are selected from given n distinct objects, i.e., ${}^nC_r = \frac{n!}{r!(n-r)!}$.

Activity 15

Objective

To construct a Pascal's Triangle and to write binomial expansion for a given positive integral exponent.

Material Required

Drawing board, white paper, matchsticks, adhesive.

Method of Construction

1. Take a drawing board and paste a white paper on it.
2. Take some matchsticks and arrange them as shown in Fig.15.

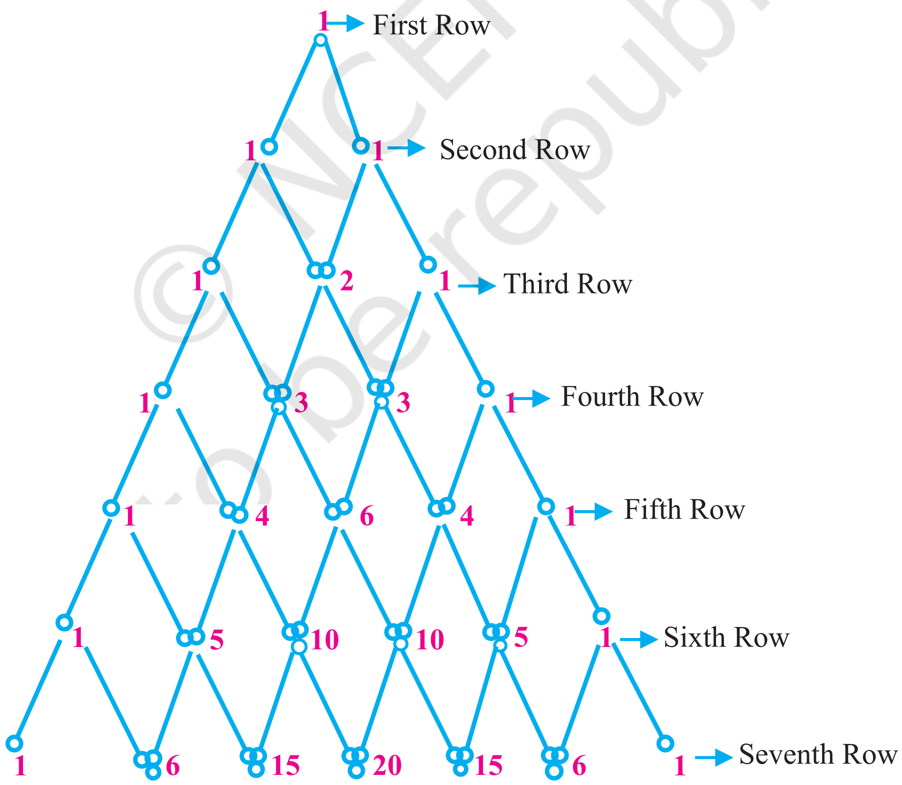

Fig. 15

3. Write the numbers as follows:

   1 (first row)
   1 1 (second row)
   1 2 1 (third row)
   1 3 3 1 (fourth row), 1 4 6 4 1 (fifth row) and so on (see Fig. 15).

4. To write binomial expansion of $(a + b)^n$, use the numbers given in the $(n + 1)^{th}$ row.

Demonstration

1. The above figure looks like a triangle and is referred to as Pascal's Triangle.
2. Numbers in the second row give the coefficients of the terms of the binomial expansion of $(a + b)^1$. Numbers in the third row give the coefficients of the terms of the binomial expansion of $(a + b)^2$, numbers in the fourth row give coefficients of the terms of binomial expansion of $(a + b)^3$. Numbers in the fifth row give coefficients of the terms of binomial expansion of $(a + b)^4$ and so on.

Observation

1. Numbers in the fifth row are ___________, which are coefficients of the binomial expansion of ___________.
2. Numbers in the seventh row are _____________, which are coefficients of the binomial expansion of _______.
3. $(a + b)^3$ = ___ a^3 + ___a^2b + ___ab^2 + ___b^3
4. $(a + b)^5$ = ___ +___+ ___+ ___ + ___+ ___.
5. $(a + b)^6$ =___a^6 +___a^5b + ___a^4b^2 + ___a^3b^3 + ___a^2b^4 + ___ab^5 + ___b^6.
6. $(a + b)^8$ = ___ +___ +___+ ___ + ___+ ___ + ___ + ___+ ___.
7. $(a + b)^{10}$ =___ + ___ + ___+ ___ + ___+ ___ + ___+___+ ___+ ___+ __.

Application

The activity can be used to write binomial expansion for $(a + b)^n$, where n is a positive integer.