NCERT Solutions for Class-XII Maths

Chapter-5.5

NCERT Math Class 12

Differentiate the functions given in Exercises 1 to 11 w. r. t. x.

1. cos x . cos 2 x . cos 3x

1. Let $y = \cos x \cdot \cos 2x \cdot \cos 3x$, taking log on both the sides

$\log y = \log\cos x + \log\cos 2x + \log\cos 3x$

Therefore,

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}\cdot\frac{d}{dx}\cos x + \frac{1}{\cos 2x}\cdot\frac{d}{dx}\cos 2x + \frac{1}{\cos 3x}\cdot\frac{d}{dx}\cos 3x$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{\cos x}\cdot(-\sin x) + \frac{1}{\cos 2x}\cdot(-\sin 2x)\cdot 2 + \frac{1}{\cos 3x}\cdot(-\sin 3x)\cdot 3\right]$$

$$\Rightarrow \frac{dy}{dx} = \cos x\cdot\cos 2x\cdot\cos 3x\left[-\tan x - 2\tan 2x - 3\tan 3x\right]$$

2. $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

2. Given: $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Let $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} = \left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]^{\frac{1}{2}}$

Taking log on both sides, we get

$$\log y = \log\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]^{\frac{1}{2}}$$

$$\Rightarrow \log y = \frac{1}{2}\log\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]$$

$$\Rightarrow \log y = \frac{1}{2}[\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)]$$

Now, differentiate both sides with respect to x

$$\Rightarrow \frac{d}{dx}(\log y) = \frac{1}{2}\left[\frac{d}{dx}\log(x-1) + \frac{d}{dx}\log(x-2) - \frac{d}{dx}\log(x-3) - \frac{d}{dx}\log(x-4) - \frac{d}{dx}\log(x-5)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}$$

$$\left[\frac{1}{x-1}.\frac{d}{dx}(x-1) + \frac{1}{x-2}.\frac{d}{dx}(x-2) - \frac{1}{x-3}.\frac{d}{dx}(x-3) - \frac{1}{x-4}.\frac{d}{dx}(x-4) - \frac{1}{x-5}.\frac{d}{dx}(x-5)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}.\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}.\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

3. $(\log x)^{\cos x}$

3. Let $y = (\log x)^{\cos x}$, taking log on both the sides

$\log y = \log(\log x)^{\cos x} = \cos x \cdot \log\log x$

Therefore,

$$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \frac{d}{dx}\log\log x + \log\log x \cdot \frac{d}{dx}\cos x$$

$$\Rightarrow \frac{dy}{dx} = y\left[\cos x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \log\log x \cdot (-\sin x)\right]$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x - \sin x \log\log x}{x\log x}\right]$$

4. $x^x - 2^{\sin x}$

4. Given: $x^x - 2^{\sin x}$

Lety= $x^x - 2^{\sin x}$

Lety = u - v

$\Rightarrow u = x^x$ and $v = 2^{\sin x}$

For, $u = x^x$

Taking log on both sides, we get

$\log u = \log x^x$

$\Rightarrow \log u = x.\log(x)$

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log u) = \frac{d}{dx}[x.\log(x)]$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = x.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(x)$$

$$\Rightarrow \frac{du}{dx} = u\left[x.\frac{1}{x} + \log x.(1)\right]$$

$$\Rightarrow \frac{du}{dx} = x^x(1+\log x)$$

For, v = $2^{\sin x}$

Taking log on both sides, we get

log v = log $2^{\sin x}$

⇒log v = sin x. log (2)

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log \text{v}) = \frac{d}{dx}\left[\sin \text{x}.\log(2)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \log 2.\frac{d}{dx}(\sin x)$$

$$\Rightarrow \frac{dv}{dx} = v\left[\log 2.(\cos x)\right]$$

$$\Rightarrow \frac{dv}{dx} = 2^{\sin x}.\cos x.\log 2$$

Because, y = u - v

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = x^x(1+\log x) + 2^{\sin x}.\cos x\ .\log 2$$

5. $(x + 3)^2. (x + 4)^3. (x + 5)^4$

5. Let $y = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$, taking log on both the sides

Therefore,

$$\frac{1}{y}\frac{dy}{dx} = 2\cdot\frac{1}{(x+3)} + 3\cdot\frac{1}{(x+4)} + 4\cdot\frac{1}{(x+5)}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{2(x+4)(x+5)+3(x+3)(x+5)+4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{2\left(x^2+9x+20\right)+3\left(x^2+8x+15\right)+4\left(x^2+7x+12\right)}{(x+3)(x+4)(x+5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)^2\cdot(x+4)^3\cdot(x+5)^4\left[\frac{9x^2+70x+133}{(x+3)(x+4)(x+5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)\cdot(x+4)^2\cdot(x+5)^3\left(9x^2+70x+133\right)$$

6. $\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

6. Given: $\left(x+\frac{1}{x}\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$

Lety= $\left(x+\frac{1}{x}\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$

Also,Lety = u + v

$\Rightarrow u=\left(x+\frac{1}{x}\right)^{x}$ *and* $v=x^{\left(1+\frac{1}{x}\right)}$

$for,\ u=\left(x+\frac{1}{x}\right)^{x}$

Taking log on both sides, we get

$\log u=\log\left(x+\frac{1}{x}\right)^{x}$

$\Rightarrow \log u=x.\log\left(x+\frac{1}{x}\right)$

Now, differentiate both sides with respect to x

$\frac{d}{dx}(\log \mathrm{u})=\frac{d}{dx}\left[x.\log\left(x+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{1}{u}\frac{du}{dx}=x.\frac{d}{dx}\left(\log\left(x+\frac{1}{x}\right)\right)+\log\left(x+\frac{1}{x}\right).\frac{d}{dx}(x)$

$\Rightarrow \frac{du}{dx}=u\left[x.\frac{1}{\left(x+\frac{1}{x}\right)}.\frac{d}{dx}\left(x+\frac{1}{x}\right)+\log\left(x+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{du}{dx}=u\left[x.\frac{1}{\left(x+\frac{1}{x}\right)}.\left(\frac{dx}{dx}+\frac{d}{dx}\left(\frac{1}{x}\right)\right)+\log\left(x+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{du}{dx}=u\left[\frac{x}{\left(x+\frac{1}{x}\right)}.\left(1-\frac{1}{x^{2}}\right)+\log\left(x+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{du}{dx}=u\left[\frac{x}{\left(x+\frac{1}{x}\right)}.\left(\frac{x^{2}-1}{x^{2}}\right)+\log\left(x+\frac{1}{x}\right)\right]$

$$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x \left[\left(\frac{x^2-1}{x^2+1}\right) + \log\left(x+\frac{1}{x}\right)\right]$$

For, $v = x^{\left(1+\frac{1}{x}\right)}$

Taking log on both sides, we get

$$\log v = \log x^{\left(1+\frac{1}{x}\right)}$$

$$\Rightarrow \log v = \left(1+\frac{1}{x}\right).\log x$$

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log v) = \frac{d}{dx}\left[\left(1+\frac{1}{x}\right).\log x\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \log x.\frac{d}{dx}\left(1+\frac{1}{x}\right) + \left(1+\frac{1}{x}\right).\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{dv}{dx} = v\left[\log x.\left(0-\frac{1}{x^2}\right) + \left(1+\frac{1}{x}\right).\frac{1}{x}\right]$$

$$\Rightarrow \frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left[-\frac{\log x}{x^2} + \left(\frac{1}{x}+\frac{1}{x^2}\right)\right]$$

$$\Rightarrow \frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left[\frac{-\log x + x + 1}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left[\frac{x+1-\log x}{x^2}\right]$$

Because, y = u + v

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\left(\frac{x^2-1}{x^2+1}\right) + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)}\left[\frac{x+1-\log x}{x^2}\right]$$

7. $(\log x)^x + x^{\log x}$

7. Let $u = (\log x)^x$ and $v = x^{\log x}$, therefore, $y = u + v$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

Here, $u = (\log x)^x$, taking log on both the sides

$\log u = x \log \log x$, therefore,

$$\frac{1}{u}\frac{du}{dx}=x\cdot\frac{d}{dx}\log\log x+\log\log x\cdot\frac{d}{dx}x$$

$$=x\cdot\frac{1}{\log x}\cdot\frac{1}{x}+\log\log x\cdot 1=\frac{1}{\log x}+\log\log x$$

$$\frac{du}{dx}=(\log x)^{x}\left[\frac{1+\log x\cdot\log\log x}{\log x}\right]$$

$$=(\log x)^{x-1}\left(1+\log x\cdot\log\log x\right)$$

and, $v=x^{\log x}$, taking log on both the sides

$\log v=\log x\log x$, therefore,

$$\frac{1}{v}\frac{dv}{dx}=\log x\cdot\frac{d}{dx}\log x+\log x\cdot\frac{d}{dx}\log x$$

$$=\log x\cdot\frac{1}{x}+\log x\cdot\frac{1}{x}$$

$$\frac{dv}{dx}=v\left[\frac{2\log x}{x}\right]=x^{\log x}\left[\frac{2\log x}{x}\right]=x^{\log x-1}\left(2\log x\right)\ldots(3)$$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get

$$\frac{dy}{dx}=(\log x)^{x-1}\left(1+\log x\cdot\log\log x\right)+x^{\log x-1}\left(2\log x\right)$$

8. $(\sin x)^x+\sin^{-1}\sqrt{x}$

8. Given: $(\sin x)^x+sin^{-1}\sqrt{x}$

Lety= $(\sin x)^x+sin^{-1}\sqrt{x}$

Lety = u + v

$\Rightarrow u=(\sin x)^x\ and\ v=sin^{-1}\sqrt{x}$

$for,\ u=(\sin x)^x$

Taking log on both sides, we get

$\log u=\log(\sin x)^x$

$\Rightarrow \log u=\text{x}.\log\sin x$

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log \text{u})=\frac{d}{dx}[\text{x}.\log(\sin x)]$$

$$\Rightarrow\frac{1}{u}\frac{du}{dx}=x.\frac{d}{dx}\log(\sin x))+\log(\sin x).\frac{d}{dx}(x)$$

$$\Rightarrow\frac{du}{dx}=u\left[x.\frac{1}{\sin x}\frac{d}{dx}(\sin x)+\log(\sin x).(1)\right]$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x \left[\frac{x}{\sin x}.\cos x + \log(\sin x).(1) \right]$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x \left[x.\cot x + \log \sin x \right]$$

$for,\ v = sin^{-1}\sqrt{x}$

Now, differentiate both sides with respect to x

$$\frac{dv}{dx} = \frac{d}{dx}\left[sin^{-1}\sqrt{x} \right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}.\frac{d}{dx}\left(\sqrt{x}\right)$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}}.\frac{1}{2(\sqrt{x})}$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{x-x^2}}$$

Because, y = u + v

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^x \left[x.\cot x + \log \sin x \right] + \frac{1}{2\sqrt{x-x^2}}$$

9. $x^{\sin x} + (\sin x)^{\cos x}$

9. Let $u = x^{\sin x}$ and $v = (\sin x)^{\cos x}$ therefore, $y = u + v$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

Here, $u = x^{\sin x}$, taking log on both the sides

$\log u = \sin x \log x$, therefore,

$$\frac{1}{u}\frac{du}{dx} = \sin x \cdot \frac{d}{dx}\log x + \log x \cdot \frac{d}{dx}\sin x = \sin x \cdot \frac{1}{x} + \log x \cdot \cos x = \frac{\sin x}{x} + \log x \cos x$$

$$\frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x \cos x \right] = x^{\sin x - 1}\left(\sin x + x \log x \cos x \right) \ldots (2)$$

and , $v = (\sin x)^{\cos x}$, taking log on both the sides $\log v = \cos x \log \sin x$,

Therefore,

$$\frac{1}{v}\frac{dv}{dx} = \cos x \cdot \frac{d}{dx}\log \sin x + \log \sin x \cdot \frac{d}{dx}\cos x = \cos x \cdot \frac{1}{\sin x}\cos x + \log \sin x(-\sin x)$$

$$\frac{dv}{dx} = v\left[\cos x \cot x - \sin x \log \sin x \right] = (\sin x)^{\cos x}\left(\cos x \cot x - \sin x \log \sin x \right) \ldots (3)$$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get

$$\frac{dy}{dx}=x^{\sin x-1}\left(\sin x+x\log x\cos x\right)+(\sin x)^{\cos x}\left(\cos x\cot x-\sin x\log\sin x\right)$$

10. $x^{x\cos x}+\frac{x^2+1}{x^2-1}$

10. Given: $x^{x\cos x}+\frac{x^2+1}{x^2-1}$

Lety= $x^{x\cos x}+\frac{x^2+1}{x^2-1}$

Lety = u + v

$\Rightarrow u=x^{x\cos x} and\ v=\frac{x^2+1}{x^2-1}$

For, $u=x^{x\cos x}$

Taking log on both sides, we get

$\log u=\log x^{x\cos x}$

$\Rightarrow$log u = x.cos x.log x

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log u)=\frac{d}{dx}[x.\cos x.\log x]$$

$$\Rightarrow\frac{1}{u}\frac{du}{dx}=\cos x.\log x.\frac{d}{dx}(x)+x.\log x.\frac{d}{dx}(\cos x)+x.\cos x.\frac{d}{dx}\log x)$$

$$\Rightarrow\frac{du}{dx}=u\left[\cos x.\log x+x.\log x(-\sin x)+x.\cos x.\left(\frac{1}{x}\right)\right]$$

$$\Rightarrow\frac{du}{dx}=x^{x\cos x}\left[\cos x.\log x-x.\log x.\sin x+\cos x\right]$$

$$\Rightarrow\frac{du}{dx}=x^{x\cos x}\left[\cos\ x(1+\log x)-x.\log x.\sin x\right]$$

For, $v=\frac{x^2+1}{x^2-1}$

Taking log on both sides, we get

$$\log v=\log\left(\frac{x^2+1}{x^2-1}\right)$$

$\Rightarrow$ log v = log (x^2 + 1) – log (x^2 – 1)

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log v)=\frac{d}{dx}\left[\log\left(x^2+1\right)-\log(x^2-1)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \frac{1}{x^2+1}.\frac{d}{dx}\left(x^2\right) - \frac{1}{x^2-1}.\frac{d}{dx}\left(x^2\right)$$

$$\Rightarrow \frac{dv}{dx} = v.\left[\frac{1}{x^2+1}.(2x) - \frac{1}{x^2-1}.(2x)\right]$$

$$\Rightarrow \frac{dv}{dx} = \left(\frac{x^2+1}{x^2-1}\right).\left[\frac{2x\left(x^2-1\right)-2x\left(x^2+1\right)}{(x^2+1)\left(x^2-1\right)}\right]$$

$$\Rightarrow \frac{dv}{dx} = \left(\frac{x^2+1}{x^2-1}\right).\left[\frac{-4x}{(x^2+1)\left(x^2-1\right)}\right]$$

$$\Rightarrow \frac{dv}{dx} = \left[\frac{-4x}{(x^2-1)^2}\right]$$

Because, y = u + v

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = x^{x\cos x}\left[\cos\ x(1+\log x) - x.\log x.\sin x\right] - \left[\frac{4x}{(x^2-1)^2}\right]$$

11. $\left(x\ cos\ x\right)^x + \left(xsin\ x\right)^{\frac{1}{x}}$

11. Let $u = (x\cos x)^x$ and $v = (x\sin x)^{\frac{1}{x}}$, therefore, $y = u + v$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

Here, $u = (x\cos x)^x$, Taking log on both the sides

$\log u = x\log\left(x\cos x\right)$, therefore,

$$\frac{1}{u}\frac{du}{dx} = x\cdot\frac{d}{dx}\log\left(x\cos x\right) + \log\left(x\cos x\right)\cdot\frac{d}{dx}x$$

$$= x\cdot\frac{1}{\left(x\cos x\right)}\left(-x\sin x + \cos x\right) + \log\left(x\cos x\right)\cdot 1 = -x\tan x + 1 + \log\left(x\cos x\right)$$

$$\frac{du}{dx} = (x\cos x)^x\left[1 - x\tan x + \log\left(x\cos x\right)\right]$$

$$= (x\cos x)^x\left[1 - x\tan x + \log\left(x\cos x\right)\right]\ldots(2)$$

and, $v = (x\sin x)^{\frac{1}{x}}$, Taking log on both the sides

$\log v = \frac{1}{x}\log\left(x\sin x\right)$, therefore,

$$\frac{1}{v}\frac{dv}{dx}=\frac{1}{x}\cdot\frac{d}{dx}\log(x\sin x)+\log(x\sin x)\cdot\frac{d}{dx}\frac{1}{x}$$

$$=\frac{1}{x}\cdot\frac{1}{x\sin x}(x\cos x+\sin x)+\log(x\sin x)\left(-\frac{1}{x^2}\right)$$

$$\frac{dv}{dx}=v\left[\frac{x\cot x+1-\log(x\sin x)}{x^2}\right]$$

$$=(x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x+1-\log(x\sin x)}{x^2}\right]$$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get

$$\frac{dy}{dx}=(x\cos x)^x\left[1-x\tan x+\log(x\cos x)\right]+(x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x+1-\log(x\sin x)}{x^2}\right]$$

Find $\frac{dy}{dx}$ of the functions given in exercises 12 to 15.

12. $x^y + y^x = 1$

12. Given: $x^y + y^x = 1$

Lety= $x^y + y^x = 1$

Let $u = x^y$ and $v = y^x$

Then, $\Rightarrow u + v = 1$

$$\Rightarrow \frac{du}{dx}+\frac{dv}{dx}=0$$

For, $u = x^y$

Taking log on both sides, we get

Log u =log x^y

$\Rightarrow$log u = y.log(x)

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log u)=\frac{d}{dx}\left[y.\log(x)\right]$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx}=\left\{y.\frac{d}{dx}(\log x)+\log x.\frac{d}{dx}(y)\right\}$$

$$\Rightarrow \frac{du}{dx}=u\left[y.\frac{1}{x}+\log x.\left(\frac{dy}{dx}\right)\right]$$

$$\Rightarrow \frac{du}{dx}=x^y\left[\frac{y}{x}+\log x.\left(\frac{dy}{dx}\right)\right]$$

For, $v = y^x$

Taking log on both sides, we get

Log v =log y^x

$\Rightarrow \log v = x.\log(y)$

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log v) = \frac{d}{dx}\left[x.\log(y)\right]$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left\{x.\frac{d}{dx}(\log y) + \log y.\frac{d}{dx}x\right\}$$

$$\Rightarrow \frac{dv}{dx} = v\left[x.\frac{1}{y}.\frac{dy}{dx} + \log y.\left(\frac{dx}{dx}\right)\right]$$

$$\Rightarrow \frac{dv}{dx} = y^x\left[\frac{x}{y}.\frac{dy}{dx} + \log y\right]$$

because, $\frac{du}{dx} + \frac{dv}{dx} = 0$

so, $x^y\left[\frac{y}{x} + \log x.\left(\frac{dy}{dx}\right)\right] + y^x\left[\frac{x}{y}.\frac{dy}{dx} + \log y\right] = 0$

$$\Rightarrow \left(x^y \log x + xy^{x-1}\right).\frac{dy}{dx} + \left(yx^{y-1} + y^x \log y\right) = 0$$

$$\Rightarrow \left(x^y \log x + xy^{x-1}\right).\frac{dy}{dx} = -\left(yx^{y-1} + y^x \log y\right)$$

$$\frac{dy}{dx} = -\frac{\left(yx^{y-1} + y^x \log y\right)}{\left(x^y \log x + xy^{x-1}\right)}$$

13. $y^x = x^y$

13. $y^x = x^y$

Taking log on both the sides, $x\log y = y\log x$, therefore,

$$x \cdot \frac{d}{dx}\log y + \log y \cdot \frac{d}{dx}x = y \cdot \frac{d}{dx}\log x + \log x \cdot \frac{d}{dx}y$$

$$\Rightarrow x \cdot \frac{1}{y}\frac{dy}{dx} + \log y \cdot 1 = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x}{y} - \log x\right) = \frac{y}{x} - \log y$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x - y\log x}{y}\right) = \frac{y - x\log y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\left(y - x\log y\right)}{x\left(x - y\log x\right)}$$

14. $(\cos x)^y = (\cos y)^x$

14. Given: $(\cos x)^y = (\cos y)^x$

Taking log on both sides, we get

log (cos x)^y =log (cos y)^x

⇒y log (cos x) = x log (cos y)

Now, differentiate both sides with respect to x

$$y.\frac{d}{dx}\log(\cos x)+\log(\cos x).\frac{d}{dx}y=x.\frac{d}{dx}\log(\cos y)+\log(\cos y).\frac{d}{dx}x$$

$$y.\frac{1}{\cos x}.\frac{d}{dx}(\cos x)+\log(\cos x).\frac{dy}{dx}=x.\frac{1}{\cos y}.\frac{d}{dx}(\cos y)+\log(\cos y).\frac{dx}{dx}$$

$$\frac{y}{\cos x}.(-\sin x)+\log(\cos x).\frac{dy}{dx}=\frac{x}{\cos y}.(-\sin y).\frac{dy}{dx}+\log(\cos y).(1)$$

$$\frac{dy}{dx}\left(\frac{x.\sin y}{\cos y}+\log(\cos x)\right)=y.\frac{\sin x}{\cos x}+\log(\cos y)$$

$$\frac{dy}{dx}(x.\tan x+\log(\cos x))=y.\tan x+\log(\cos y)$$

$$\frac{dy}{dx}=\left(\frac{y.\tan x+\log(\cos y)}{x.\tan x+\log(\cos x)}\right)$$

15. xy = e$^{(x-y)}$

15. $xy=e^{(x-y)}$

Taking log on both the sides, $\log x+\log y=(x-y)\log e\Rightarrow \log x+\log y=(x-y)$, therefore,

$$\frac{1}{x}+\frac{1}{y}\cdot\frac{dy}{dx}=1-\frac{dy}{dx}$$

$$\Rightarrow\frac{dy}{dx}\left(\frac{1}{y}+1\right)=1-\frac{1}{x}\Rightarrow\frac{dy}{dx}\left(\frac{1+y}{y}\right)=\frac{x-1}{x}\Rightarrow\frac{dy}{dx}=\frac{y(x-1)}{x(y+1)}$$

16. Find the derivative of the function given by f (x) = (1 + x) (1 + x^2) (1 + x^4) (1 + x^8) and hence find f ′(1).

16. Given: f (x) = (1 + x) (1 + x^2) (1 + x^4) (1 + x^8)

Taking log on both sides, we get

log f (x) =log (1 + x) + log (1 + x^2) + log (1 + x^4) + log (1 + x^8)

Now, differentiate both sides with respect to x

$$\frac{d}{dx}\log f(x)=\frac{d}{dx}\log(1+x)+\frac{d}{dx}\log(1+x^2)+\frac{d}{dx}\log(1+x^4)+\frac{d}{dx}\log(1+x^8)$$

$$\Rightarrow\frac{1}{f(x)}.\frac{d}{dx}[\mathrm{f}(\mathrm{x})]=\frac{1}{1+\mathrm{x}}.\frac{d}{dx}(1+x)+\frac{1}{1+x^2}.\frac{d}{dx}(1+x^2)+\frac{1}{1+x^4}.\frac{d}{dx}(1+x^4)+\frac{1}{1+x^8}.\frac{d}{dx}(1+x^8)$$

$$\Rightarrow\mathrm{f}'(\mathrm{x})=f(x)\left[\frac{1}{1+\mathrm{x}}+\frac{1}{1+x^2}.(2x)+\frac{1}{1+x^4}.(4x^3)+\frac{1}{1+x^8}.(8x^7)\right]$$

$$\Rightarrow f'(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}\right]$$

$$\Rightarrow f'(1)=(1+1)(1+1^2)(1+1^4)(1+1^8)\left[\frac{1}{1+1}+\frac{2(1)}{1+1}+\frac{4(1)^3}{1+(1)^4}+\frac{8(1)^7}{1+(1)^8}\right]$$

$$\Rightarrow f'(1)=(2)(2)(2)(2)\left[\frac{1}{2}+\frac{2}{2}+\frac{4}{2}+\frac{8}{2}\right]$$

$$\Rightarrow f'(1)=16\left(\frac{1+2+4+8}{2}\right)$$

$$\Rightarrow f'(1)=16\left(\frac{15}{2}\right)$$

$$\Rightarrow f'(1)=120$$

17. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below:
(i) by using product rule
(ii) by expanding the product to obtain a single polynomial.
(iii) by logarithmic differentiation.
Do they all give the same answer?

17. Let $y=(x^2-5x+8)(x^3+7x+9$

(i) Differentiating using product rule

$$\frac{dy}{dx}=(x^2-5x+8)\frac{d}{dx}(x^3+7x+9)+(x^3+7x+9)\frac{d}{dx}(x^2-5x+8)$$

$$=(x^2-5x+8)(3x^2+7)+(x^3+7x+9)(2x-5)$$

$$=(3x^4+7x^2-15x^3-35x+24x^2+56)+2x^4-5x^3+14x^2-35x+18x-45$$

$$=5x^4-20x^3+45x^2-52x+11$$

(ii) Differentiating by expanding the product to obtain a single polynomial

$$y=(x^2-5x+8)(x^3+7x+9)$$

$$=x^5+7x^3+9x^2-5x^4-35x^2-45x+8x^3+56x+72$$

$$=x^5-5x^4+15x^3-26x^2+11x+72$$

$$\frac{dy}{dx}=\frac{d}{dx}x^5-5\frac{d}{dx}x^4+15\frac{d}{dx}x^3-26\frac{d}{dx}x^2+11\frac{d}{dx}x+\frac{d}{dx}72$$

$$=5x^4-20x^3+45x^2-52x+11$$

(iii) Logarithmic differentiation

$$y=(x^2-5x+8)(x^3+7x+9)$$

Taking log on both sides, $\log y=\log(x^2-5x+8)+\log(x^3+7x+9)$

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{\left(x^2-5x+8\right)}\cdot\frac{d}{dx}\left(x^2-5x+8\right)+\frac{1}{\left(x^3+7x+9\right)}\cdot\frac{d}{dx}\left(x^3+7x+9\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{x^2-5x+8}\cdot\left(2x-5\right)+\frac{1}{x^3+7x+9}\cdot\left(3x^2+7\right)$$

$$\frac{dy}{dx}=y\left[\frac{\left(2x-5\right)\left(x^3+7x+9\right)+\left(3x^2+7\right)\left(x^2-5x+8\right)}{\left(x^2-5x+8\right)\left(x^3+7x+9\right)}\right]$$

$$=\text{y}\left[\frac{2x^4+14x^2+18x-5x^3-35x-45+3x^4-15x^3+24x^2+7x^2-35x+56}{\left(x^2-5x+8\right)\left(x^3+7x+9\right)}\right]$$

$$\Rightarrow\frac{dy}{dx}=\left(x^2-5x+8\right)\left(x^3+7x+9\right)\left[\frac{5x^5-20x^3+45x^2-52x+11}{\left(x^2-5x+8\right)\left(x^3+7x+9\right)}\right]$$

$$\Rightarrow\frac{dy}{dx}==5x^4-20x^3+45x^2-52x+11$$

Hence, all the three answers are same.

18. If u, v and w are functions of x, then show that

$$\frac{d}{dx}\left(u.v.w\right)=\frac{du}{dx}v.w+u.\frac{dv}{dx}.w+u.v.\frac{dw}{dx}$$

in two ways – first by repeated application of product rule, second by logarithmic differentiation.

18. **Toprove:** $\frac{d}{dx}\left(u.v.w\right)=\frac{du}{dx}v.w+u.\frac{dv}{dx}.w+u.v.\frac{dw}{dx}$

Lety=u.v.w=u.(v.w)

by applying product rule differentiate both sides with respect to x

$$\frac{dy}{dx}=\left(v.w\right).\frac{du}{dx}+u.\frac{d}{dx}\left(v.w\right)$$

$$\Rightarrow\frac{dy}{dx}=\left(v.w\right).\frac{du}{dx}+u.\left[v.\frac{d}{dx}\left(w\right)+w.\frac{d}{dx}\left(v\right)\right]$$

$$\Rightarrow\frac{dy}{dx}=\left(v.w\right).\frac{du}{dx}+\left(u.v\right).\frac{dw}{dx}+\left(u.w\right).\frac{dv}{dx}$$

Taking log on both sides, we get

as, y=u.v.w

log y = log (u.v.w)

log y = log u + log v + log w

Now, differentiate both sides with respect to x

$$\Rightarrow\frac{d}{dx}\left(\log y\right)=\frac{d}{dx}\log u+\frac{d}{dx}\log v+\frac{d}{dx}\log w$$

$$\Rightarrow \frac{1}{y}.\frac{d}{dx}(y)=\frac{1}{u}.\frac{d}{dx}(u)+\frac{1}{v}.\frac{d}{dx}(v)+\frac{1}{w}.\frac{d}{dx}(w)$$

$$\Rightarrow \frac{d}{dx}(y)=y\left[\frac{1}{u}.\frac{du}{dx}+\frac{1}{v}.\frac{dv}{dx}+\frac{1}{w}.\frac{dw}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx}=u.v.w\left[\frac{1}{u}.\frac{du}{dx}+\frac{1}{v}.\frac{dv}{dx}+\frac{1}{w}.\frac{dw}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx}=v.w.\frac{du}{dx}+u.w.\frac{dv}{dx}+u.v.\frac{dw}{dx}$$