

EXERCISE 23.3

Find the equation of a line making an angle of 150° with the x-axis and cutting off an intercept 2 from y-axis.

Solution:

Given: A line which makes an angle of 150° with the x-axis and cutting off an intercept at 2

By using the formula,

The equation of a line is $y = mx + c$

We know that angle, $\theta = 150^\circ$

The slope of the line, $m = \tan \theta$

Where, $m = \tan 150^\circ$
 $= -1/\sqrt{3}$

Coordinate of y-intercept is (0, 2)

The required equation of the line is $y = mx + c$

Now substitute the values, we get

$$y = -x/\sqrt{3} + 2$$

$$\sqrt{3}y - 2\sqrt{3} + x = 0$$

$$x + \sqrt{3}y = 2\sqrt{3}$$

\therefore The equation of line is $x + \sqrt{3}y = 2\sqrt{3}$

1. Find the equation of a straight line:

(i) with slope 2 and y – intercept 3;

(ii) with slope $-1/3$ and y – intercept -4 .

(iii) with slope -2 and intersecting the x-axis at a distance of 3 units to the left of origin.

Solution:

(i) With slope 2 and y – intercept 3

The slope is 2 and the coordinates are (0, 3)

Now, the required equation of line is

$$y = mx + c$$

Substitute the values, we get

$$y = 2x + 3$$

(ii) With slope $-1/3$ and y – intercept -4

The slope is $-1/3$ and the coordinates are (0, -4)

Now, the required equation of line is

$$y = mx + c$$

Substitute the values, we get

$$y = -1/3x - 4$$
$$3y + x = -12$$

(iii) With slope -2 and intersecting the x -axis at a distance of 3 units to the left of origin
The slope is -2 and the coordinates are $(-3, 0)$

Now, the required equation of line is $y - y_1 = m(x - x_1)$

Substitute the values, we get

$$y - 0 = -2(x + 3)$$
$$y = -2x - 6$$
$$2x + y + 6 = 0$$

2. Find the equations of the bisectors of the angles between the coordinate axes.

Solution:

There are two bisectors of the coordinate axes.

Their inclinations with the positive x -axis are 45° and 135°

The slope of the bisector is $m = \tan 45^\circ$ or $m = \tan 135^\circ$

i.e., $m = 1$ or $m = -1$, $c = 0$

By using the formula, $y = mx + c$

Now, substitute the values of m and c , we get

$$y = x + 0$$
$$x - y = 0 \text{ or } y = -x + 0$$
$$x + y = 0$$

\therefore The equation of the bisector is $x \pm y = 0$

3. Find the equation of a line which makes an angle of $\tan^{-1}(3)$ with the x -axis and cuts off an intercept of 4 units on the negative direction of y -axis.

Solution:

Given:

The equation which makes an angle of $\tan^{-1}(3)$ with the x -axis and cuts off an intercept of 4 units on the negative direction of y -axis

By using the formula,

The equation of the line is $y = mx + c$

Here, angle $\theta = \tan^{-1}(3)$

So, $\tan \theta = 3$

The slope of the line is, $m = 3$

And, Intercept in the negative direction of y -axis is $(0, -4)$

The required equation of the line is $y = mx + c$

Now, substitute the values, we get

$$y = 3x - 4$$

∴ The equation of the line is $y = 3x - 4$.

4. Find the equation of a line that has y – intercept – 4 and is parallel to the line joining (2, –5) and (1, 2).

Solution:

Given:

A line segment joining (2, –5) and (1, 2) if it cuts off an intercept – 4 from y–axis

By using the formula,

The equation of line is $y = mx + C$

It is given that, $c = -4$

Slope of line joining $(x_1 - x_2)$ and $(y_1 - y_2)$,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

So, Slope of line joining (2, –5) and (1, 2),

$$m = \frac{2 - (-5)}{1 - 2} = \frac{7}{-1}$$

$$m = -7$$

The equation of line is $y = mx + c$

Now, substitute the values, we get

$$y = -7x - 4$$

$$y + 7x + 4 = 0$$

∴ The equation of line is $y + 7x + 4 = 0$.

