EXERCISE 19.4

1. $\int \frac{x^2 + 5x + 2}{x + 2}\, dx$

Solution:

Given

$$\int \frac{x^2 + 5x + 2}{x + 2}\, dx$$

By performing long division of the given equation we get

Quotient = x + 3

Remainder = –4

∴ We can write the above equation as

$\Rightarrow x + 3 - \frac{4}{x+2}$

∴ The above equation becomes

$\Rightarrow \int x + 3 - \frac{4}{x+2}\, dx$

By splitting

$\Rightarrow \int x\, dx + 3 \int dx - 4 \int \frac{1}{x+2} dx$

We know $\int x\, dx = \frac{x^n}{n+1}; \int \frac{1}{x} dx = \ln x$

$\Rightarrow \frac{x^2}{2} + 3x - 4\ln(x + 2)$ +c. (Where c is some arbitrary constant)

$= \frac{x^2}{2} + 3x - 4 \log |x + 2| + c$

2. $\int \frac{x^3}{x - 2}\, dx$

Solution:

Given

$$\int \frac{x^3}{x-2}\,dx$$

By performing long division of the given equation we get

Quotient = x^2+2x+4

Remainder = 8

∴ We can write the above equation as

$$\Rightarrow x^2+2x+4+\frac{8}{x-2}$$

∴ The above equation becomes

$$\Rightarrow \int x^2+2x+4+\frac{8}{x-2}\,dx$$

$$\Rightarrow \int x^2\,dx+2\int x\,dx+4\int dx+8\int \frac{1}{x-2}dx$$

We know $\int x\,dx=\frac{x^n}{n+1}; \int \frac{1}{x}dx=\ln x$

$$\Rightarrow \frac{x^3}{3}+2\frac{x^2}{2}+4x+8\ln(x-2)+c$$

$$\Rightarrow \frac{x^3}{3}+x^2+4x+8\ln(x-2)+c$$. (Where c is some arbitrary constant)

$$=\frac{x^3}{3}+x^2+4x+8\log|x-2|+c$$

3. $$\int \frac{x^2+x+5}{3x+2}\,dx$$

Solution:

Given

$$\int \frac{x^2+x+5}{3x+2}\,dx$$

By doing long division of the given equation we get

Quotient = $\frac{x}{3}+\frac{1}{9}$

Remainder = $\frac{43}{9}$

∴ We can write the above equation as

$\Rightarrow \frac{x}{3}+\frac{1}{9}+\frac{43}{9}\left(\frac{1}{3x+2}\right)$

∴ The above equation becomes

$\Rightarrow \int \frac{x}{3}+\frac{1}{9}+\frac{43}{9}\left(\frac{1}{3x+2}\right)dx$

$\Rightarrow \frac{1}{3}\int xdx+\frac{1}{9}\int dx+\frac{43}{9}\int \frac{1}{3x+2}dx$

We know $\int x^n dx=\frac{x^{n+1}}{n+1}; \int \frac{1}{x}dx=\ln x$

$\Rightarrow \frac{1}{3}\times\frac{x^2}{2}+\frac{1}{9}\times x+\frac{43}{9}\times\frac{1}{3}\ln(3x+2)+c$

$=\frac{x^2}{6}+\frac{1}{9}x+\frac{43}{27}\log|3x+2|+c$ (Where c is some arbitrary constant)