EXERCISE 26.1

Q1.i

Solution:

We know that,

$$[\hat{i}\ \hat{j}\ \hat{k}]+[\hat{j}\ \hat{k}\ \hat{i}]+[\hat{k}\ \hat{i}\ \hat{j}]$$

Now let us simplify the above expression, we get

$$\begin{aligned}[\hat{i}\ \hat{j}\ \hat{k}]+[\hat{j}\ \hat{k}\ \hat{i}]+[\hat{k}\ \hat{i}\ \hat{j}] &= (\hat{i}\times\hat{j})\cdot\hat{k}+(\hat{j}\times\hat{k})\cdot\hat{i}+(\hat{k}\times\hat{i})\cdot\hat{j}\\ &= \hat{k}\cdot\hat{k}+\hat{i}\cdot\hat{i}+\hat{j}\cdot\hat{j}\\ &= 1+1+1\\ &= 3\end{aligned}$$

Hence,

$$[\hat{i}\ \hat{j}\ \hat{k}]+[\hat{j}\ \hat{k}\ \hat{i}]+[\hat{k}\ \hat{i}\ \hat{j}]=3$$

ii.

Solution:

We know that,

$$[2\hat{i}\ \hat{j}\ \hat{k}]+[\hat{i}\ \hat{k}\ \hat{j}]+[\hat{k}\ \hat{j}\ 2\hat{i}]$$

Now let us simplify the above expression, we get

$$\begin{aligned}[2\hat{i}\ \hat{j}\ \hat{k}]+[\hat{i}\ \hat{k}\ \hat{j}]+[\hat{k}\ \hat{j}\ 2\hat{i}] &= (2\hat{i}\times\hat{j})\cdot\hat{k}+(\hat{i}\times\hat{k})\cdot\hat{j}+(\hat{k}\times\hat{j})\cdot 2\hat{i}\\ &= 2\hat{k}\cdot\hat{k}+(-\hat{j})\cdot\hat{j}+(-\hat{i})\cdot 2\hat{i}\\ &= 2-1-2\\ &= -1\end{aligned}$$

Hence,

$$[2\hat{i}\ \hat{j}\ \hat{k}]+[\hat{i}\ \hat{k}\ \hat{j}]+[\hat{k}\ \hat{j}\ 2\hat{i}]=-1$$

Q2.i

Solution:

We know that,

$$[\vec{a}\ \vec{b}\ \vec{c}]=\begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}$$

Now let us simplify we get,

$$\begin{aligned}[\vec{a}\ \vec{b}\ \vec{c}] &= \begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}\\ &= 2(-1-0)+3(-1+3)\\ &= -2+6\\ &= 4\end{aligned}$$

Hence,

$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=4$

ii.

Solution:

We know that,

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & -1\\ 0 & 1 & 1\end{vmatrix}$$

Now let us simplify we get,

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & -1\\ 0 & 1 & 1\end{vmatrix}$$

$$=1(1+1)+2(2+0)+3(2-0)$$
$$=2+4+6$$
$$=12$$

Hence,

$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=12$

Q3.i

Solution:

We know that the volume of a parallelepiped whose three adjacent edges is $\vec{a},\vec{b},\vec{c}=\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|$.

So we now have,

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}2 & 3 & 4\\ 1 & 2 & -1\\ 3 & -1 & 2\end{vmatrix}$$

Let us simplify we get,

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}2 & 3 & 4\\ 1 & 2 & -1\\ 3 & -1 & 2\end{vmatrix}$$

$$=2(4-1)-3(2+3)+4(-1-6)$$
$$=6-15-28$$
$$=-9-28$$
$$=-37$$

Hence,

The volume of the parallelepiped is $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=|-37|=37$ cubic unit.

ii.

Solution:

Let us consider,

$\because \vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges is $\vec{a}, \vec{b}, \vec{c} = \left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

So we now have,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$$

Let us simplify we get,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$$
$$= 2(-4-1) + 3(-2+3) + 4(-1-6)$$
$$= -10 + 3 - 28$$
$$= -10 - 25$$
$$= -35$$

Hence,

The volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right| = |-35| = 35$ cubic unit.

iii.

Solution:

Let us consider,

$\vec{a} = 11\hat{i}, \vec{b} = 2\hat{j}, \vec{c} = 13\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges is $\vec{a}, \vec{b}, \vec{c} = \left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

So we now have,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 11 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{vmatrix}$$

Let us simplify we get,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 11 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{vmatrix}$$
$$= 11(26-0) + 0 + 0$$
$$= 286$$

Hence,

The volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right| = |286| = 286$ cubic unit.

iv.

Solution:

Let us consider,

$\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}-\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges is

$\vec{a},\vec{b},\vec{c} = \left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

So we now have,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 2 & -1\end{vmatrix}$$

Let us simplify we get,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 2 & -1\end{vmatrix}$$

$$=1(1-2)-1(-1-1)+1(2+1)$$
$$=-1+2+3$$
$$=4$$

Hence,

The volume of the parallelepiped is $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=|4|=4$ cubic unit.

Q4.i

Solution:

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar if their scalar triple product is zero. i.e., $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

So we now have,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & 2 & -1\\ 3 & 2 & 7\\ 5 & 6 & 5\end{vmatrix}$$

Let us simplify we get,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & 2 & -1\\ 3 & 2 & 7\\ 5 & 6 & 5\end{vmatrix}$$

$$=1(10-42)-2(15-35)-1(18-10)$$
$$=-32+40-8$$
$$=0$$

∴ The given vectors are coplanar.

ii.

Solution:

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar if their scalar triple product is zero. i.e.,

$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0.$

So we now have,

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}-4 & -6 & -2\\ -1 & 4 & 3\\ -8 & -1 & 3\end{vmatrix}$$

Let us simplify we get,

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}-4 & -6 & -2\\ -1 & 4 & 3\\ -8 & -1 & 3\end{vmatrix}$$
$$=-4(12+3)+6(-3+24)-2(1+32)$$
$$=-60+126-66$$
$$=0$$

∴ The given vectors are coplanar.

iii.

Solution:

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar if their scalar triple product is zero. i.e.,

$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0.$

So we now have,

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}1 & -2 & 3\\ -2 & 3 & -4\\ 1 & -3 & 5\end{vmatrix}$$

Let us simplify we get,

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}1 & -2 & 3\\ -2 & 3 & -4\\ 1 & -3 & 5\end{vmatrix}$$
$$=1(15-12)+2(-10+4)+3(6-3)$$
$$=3-12+9$$
$$=0$$

∴ The given vectors are coplanar.

Q5.i

Solution:

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar if their scalar triple product is zero. i.e.,

$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0.$

So we now have,

$$\begin{vmatrix}1 & -1 & 1\\ 2 & 1 & -1\\ \lambda & -1 & \lambda\end{vmatrix}=0$$

Let us simplify we get,

$= 1(\lambda - 1) + 1(2\lambda + \lambda) + 1(-2 - \lambda)$
$= \lambda - 1 + 3\lambda - 2 - \lambda$
$3 = 3\lambda$
$1 = \lambda$

∴ The value of $\lambda = 1$.

ii.
Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e. $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

So we now have,

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ \lambda & \lambda & 5 \end{vmatrix} = 0$$

Let us simplify we get,

$= 2(10 + 3\lambda) + 1(5 + 3\lambda) + 1(\lambda - 2\lambda)$
$= 20 + 6\lambda + 5 + 3\lambda - \lambda$
$-25 = 8\lambda$
$\lambda = -\frac{25}{8}$

∴ The value of $\lambda = -25/8$.

iii.
Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e., $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

So we now have,

$$\begin{vmatrix} 1 & 2 & -3 \\ 3 & \lambda & 1 \\ 1 & 2 & 2 \end{vmatrix} = 0$$

Let us simplify we get,

$= 1(2\lambda - 2) - 2(6 - 1) - 3(6 - \lambda)$
$= 2\lambda - 2 - 12 + 2 - 18 + 3\lambda$
$= 5\lambda - 30$
$30 = 5\lambda$
$\lambda = 6$

∴ The value of $\lambda = 6$.

iv.
Solution:

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero. i.e., $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

So we now have,

$$\begin{vmatrix} 1 & 3 & 0 \\ 0 & 0 & 5 \\ \lambda & -1 & 0 \end{vmatrix} = 0$$

Let us simplify we get,

$$= 1(0+5) - 3(0-5\lambda) + 0$$
$$= 5 + 15\lambda$$
$$-5 = 15\lambda$$
$$\lambda = -\frac{1}{3}$$

∴ The value of λ = -1/3.