NCERT Solutions for Class-XI Maths

Chapter-7 Exercise-7.4

NCERT Math Class 11

1. If ${}^nC_8 = {}^nC_2$, find ${}^nC_2$

1. It is known that, ${}^nC_n = {}^nC_b \Rightarrow a = b$ or $n = a + b$

Therefore,

$${}^nC_8 = {}^nC_2 \Rightarrow n = 8 + 2 = 10$$

$$\therefore {}^nC_2 = {}^{10}C_2 = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10\times 9\times 8!}{2\times 1\times 8!} = 45$$

2. Determine n if

(i) ${}^{2n}C_3 : {}^nC_3 = 12 : 1$ (ii) ${}^{2n}C_3 : {}^nC_3 = 11 : 1$

2. (i)

Given: ${}^{2n}C_3 : {}^nC_3 = 12 : 1$

$$\Rightarrow \frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)\times(2n-3)!}{3!(2n-3)!}}{\frac{n\times(n-1)\times(n-2)\times(n-3)!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)}{3!}}{\frac{n\times(n-1)\times(n-2)}{3!}} = \frac{12}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times 2\times(n-1)}{n\times(n-1)\times(n-2)}=\frac{12}{1}$$

$$\Rightarrow \frac{4\times n\times(2n-1)}{n\times(n-2)}=\frac{12}{1}$$

$$\Rightarrow \frac{4\times(2n-1)}{(n-2)}=\frac{12}{1}$$

$\Rightarrow$ 4× (2n—1) = 12×(n-2)

$\Rightarrow$ 8n – 4 = 12n – 24

$\Rightarrow$ 12n – 8n = 24 – 4

$\Rightarrow$ 4n = 20

$\therefore$ n = 5

(ii)

Given: ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$

$$\Rightarrow \frac{2n_{C_3}}{n_{C_3}}=\frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}}=\frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)\times(2n-3)!}{3!(2n-3)!}}{\frac{n\times(n-1)\times(n-2)\times(n-3)!}{3!(n-3)!}}=\frac{11}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)}{3!}}{\frac{n\times(n-1)\times(n-2)}{3!}}=\frac{11}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)}=\frac{11}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times 2\times(n-1)}{n\times(n-1)\times(n-2)}=\frac{11}{1}$$

$$\Rightarrow \frac{4\times n\times(2n-1)}{n\times(n-2)}=\frac{11}{1}$$

$$\Rightarrow \frac{4\times(2n-1)}{(n-2)}=\frac{11}{1}$$

$\Rightarrow 4\times(2n—1) = 11\times(n-2)$
$\Rightarrow 8n - 4 = 11n - 22$
$\Rightarrow 11n - 8n = 22 - 4$
$\Rightarrow 3n = 18$
$\therefore n = 6$

3. How many chords can be drawn through 21 points on a circle?

3. For drawing one chord on a circle, only 2 points are required.
To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.
Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.

Thus, required number of chords $= {}^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21!}{2!19!} = \frac{21\times 20}{2} = 210$

4. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

4. Given: 5 boys and 4 girls
We can select 3 boys from 5 boys in ${}^{5}C_3$ ways
Similarly, we can select 3 boys from 54 girls in ${}^{4}C_3$ ways
$\therefore$ No. of ways a team of 3 boys and 3 girls can be selected is ${}^{5}C_3 \times {}^{4}C_3$

$\Rightarrow {}^{5}C_3 \times {}^{4}C_3 = \frac{5!}{3!(5-3)!}\times\frac{4!}{3!(4-3)!} = \frac{5!}{3!\times 2!}\times\frac{4!}{3!\times 1!}$

$\Rightarrow {}^{5}C_3 \times {}^{4}C_3 = \frac{5\times 4\times 3!}{3!\times 2!}\times\frac{4\times 3!}{3!\times 1!} = \frac{20}{2\times 1}\times\frac{4}{1} = 10\times 4 = 6$

$\therefore$ No. of ways a team of 3 boys and 3 girls can be selected is ${}^{5}C_3 \times {}^{4}C_3 = 40$ ways

5. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

5. There are a total of 6 red balls, 5 white balls, and 5 blue balls.
9 balls have to be selected in such a way that each selection consists of 3 balls of each colour.
Here,
3 balls can be selected from 6 red balls in ${}^{6}C_3$ ways.
3 balls can be selected from 5 white balls in ${}^{5}C_3$ ways.
3 balls can be selected from 5 blue balls in ${}^{5}C_3$ ways.
Thus, by multiplication principle, required number of ways of selecting 9 balls

$= {}^{6}C_3 \times {}^{5}C_3 \times {}^{5}C_3 = \frac{6!}{3!3!}\times\frac{5!}{3!2!}\times\frac{5!}{3!2!}$

$$= \frac{6\times5\times4\times3!}{3!\times3\times2}\times\frac{5\times4\times3!}{3!\times2\times1}\times\frac{5\times4\times3!}{3!\times2\times1}$$

$$= 20\times10\times10 = 2000$$

6. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

6. Given: a deck of 52 cards

There are 4 Ace cards in a deck of 52 cards.

According to question, we need to select 1 Ace card out the 4 Ace cards

∴ No. Of ways to select 1 Ace from 4 Ace cards is $^4C_1$

⇒ More 4 cards are to be selected now from 48 cards (52 cards – 4 Ace cards)

∴ No. Of ways to select 4 cards from 48 cards is $^{48}C_4$

∴ Total number of ways = $^4C_1 \times {}^{48}C_4$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4!}{1!(4-1)!}\times\frac{48!}{4!(48-4)!} = \frac{4!}{1!\times3!}\times\frac{48!}{4!\times44!}$$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4\times3!}{1!\times3!}\times\frac{48\times47\times46\times45\times44!}{4!\times44!} = \frac{4}{1}\times\frac{4669920}{24} = 4\times194580 = 778320$$

∴ Number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination 778320.

7. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

7. Out of 17 players, 5 players are bowlers.

A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers.

4 bowlers can be selected in $^5C_4$ ways and the remaining 7 players can be selected out of the 12 players in $^{12}C_7$ ways.

Thus, by multiplication principle, required number of ways of selecting cricket team

$$= {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!1!}\times\frac{12!}{7!5!} = 5\times\frac{12\times11\times10\times9\times8}{5\times4\times3\times2\times1} = 3960$$

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

8. Given: A bag contains 5 black and 6 red balls

Number of ways we can select 2 black balls from 5 black balls are $^5C_2$

Number of ways we can select 3 red balls from 6 red balls are $^6C_3$

∴ Number of ways 2 black and 3 red balls can be selected are: $^5C_2\times {}^6C_3$

$$\therefore {}^5C_2\times {}^6C_3 = \frac{5!}{2!(5-2)!}\times\frac{6!}{3!(6-3)!} = \frac{5!}{2!\times3!}\times\frac{6!}{3!\times3!}$$

$\Rightarrow {}^5C_2 \times {}^6C_3 = \frac{5\times4\times3!}{2!\times3!}\times\frac{6\times5\times4\times3!}{3!\times3!}=\frac{20}{2}\times\frac{120}{6}=10\times20=200$

∴ Number of ways in which 2 black and 3 red balls can be selected from 5 black and 6 red balls are : 200

9. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

9. There are 9 courses available out of which, 2 specific courses are compulsory for every student.

Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in ${}^7C_3$ ways.

Thus, required number of ways of choosing the programme

$= {}^7C_3 = \frac{7!}{3!4!} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!} = 35$