## EXERCISE 19.28

**Evaluate the following integrals:**

**1.** $\int \sqrt{3+2x-x^2}\,dx$

**Solution:**

Let, I = $\int \sqrt{3+2x-x^2}\,dx$

$\therefore$ I = $\int \sqrt{3-(x^2-2(1)x)}dx = \int \sqrt{3-(x^2-2(1)x+1)+1}\,dx$

Using $a^2 - 2ab + b^2 = (a - b)^2$

We have:

I = $\int \sqrt{4-(x-1)^2}\,dx = \int \sqrt{2^2-(x-1)^2}dx$

As I match with the form:

$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

By using above form and simplifying we get

$\therefore$ I = $\frac{x-1}{2}\sqrt{4-(x-1)^2} + \frac{4}{2}\sin^{-1}(\frac{x-1}{2}) + C$

$\Rightarrow$ I = $\frac{1}{2}(x-1)\sqrt{3+2x-x^2} + 2\sin^{-1}\left(\frac{x-1}{2}\right) + C$

**2.** $\int \sqrt{x^2+x+1}\,dx$

**Solution:**

Let, I = $\int \sqrt{(x^2+x+1)}\,dx$

$\therefore$ I = $\int \sqrt{x^2+2\left(\frac{1}{2}\right)x+\left(\frac{1}{2}\right)^2+1-\left(\frac{1}{2}\right)^2}\,dx$

Using $a^2 + 2ab + b^2 = (a + b)^2$

We have:

I = $\int\sqrt{\left(x+\frac{1}{2}\right)^2+1-\frac{1}{4}}\,dx = \int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx$

As I match with the form:

$$\int\sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$$

∴ I = $\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}+\frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right|+C$

⇒ I = $\frac{1}{4}(2x+1)\sqrt{x^2+x+1}+\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right|+C$

⇒ I = $\frac{1}{4}(2x+1)\sqrt{x^2+x+1}+\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|+C$

**3.** $\int\sqrt{x-x^2}\,dx$



**Solution:**

Let, I = $\int\sqrt{x-x^2}\,dx$

∴ I = $\int\sqrt{-\left(x^2-2\left(\frac{1}{2}\right)x\right)}\,dx = \int\sqrt{\frac{1}{4}-\left(x^2-2\left(\frac{1}{2}\right)x+\left(\frac{1}{2}\right)^2\right)}\,dx$

Using $a^2-2ab+b^2=(a-b)^2$

We have:

I = $\int\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}\,dx = \int\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}\,dx$

As I match with the form: $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+C$

∴ I = $\frac{x-\frac{1}{2}}{2}\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}+\frac{\frac{1}{4}}{2}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{1}{2}}\right)+C$

$\Rightarrow I = \frac{1}{4}(2x-1)\sqrt{x-x^2} + \frac{1}{8}\sin^{-1}(2x-1) + C$

**4.** $\int \sqrt{1+x-2x^2}\, dx$

**Solution:**

Let, $I = \int \sqrt{1+x-2x^2}\, dx$

$\therefore I = \int \sqrt{1-2\left(x^2-2\left(\frac{1}{4}\right)x\right)}\, dx = \int \sqrt{1-2\left(x^2-2\left(\frac{1}{4}\right)x+\left(\frac{1}{4}\right)^2\right)+2\left(\frac{1}{4}\right)^2}\, dx$

Using $a^2 - 2ab + b^2 = (a - b)^2$

We have:

$I = \int \sqrt{\frac{9}{8}-2\left(x-\frac{1}{4}\right)^2}\, dx = \int \sqrt{2}\sqrt{\left(\frac{3}{4}\right)^2-\left(x-\frac{1}{4}\right)^2}\, dx$

As I match with the form: $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

$\therefore I = \sqrt{2}\left\{\frac{x-\frac{1}{4}}{2}\sqrt{\left(\frac{3}{4}\right)^2-\left(x-\frac{1}{4}\right)^2} + \frac{\frac{9}{16}}{2}\sin^{-1}\left(\frac{x-\frac{1}{4}}{\frac{3}{4}}\right)\right\} + C$

$\Rightarrow I = \frac{1}{8}(4x-1)\sqrt{2\left\{\left(\frac{3}{4}\right)^2-\left(x-\frac{1}{4}\right)^2\right\}} + \frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x-1}{3}\right) + C$

$\Rightarrow I = \frac{1}{8}(4x-1)\sqrt{1+x-2x^2} + \frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x-1}{3}\right) + C$

**5.** $\int \cos x\sqrt{4-\sin^2 x}\, dx$

**Solution:**

Let, $I = \int \cos x\sqrt{4-\sin^2 x}\, dx$

Let, $\sin x = t$

Differentiating both sides:

$\Rightarrow$ cos x dx = dt

Substituting sin x with t, we have:

$\therefore$ I = $\int \sqrt{4 - t^2}dt = \int \sqrt{2^2 - t^2}dt$

As I match with the form: $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

$\therefore$ I = $\frac{t}{2}\sqrt{4 - (t)^2} + \frac{4}{2}\sin^{-1}(\frac{t}{2}) + C$

Putting the value of t i.e. t = sin x

$\Rightarrow$ I = $\frac{1}{2}\sin x\sqrt{4 - \sin^2 x} + 2\sin^{-1}\left(\frac{\sin x}{2}\right) + C$