

## EXERCISE 3.2

If  $f(x) = x^2 - 3x + 4$ , then find the values of  $x$  satisfying the equation  $f(x) = f(2x + 1)$ .

**Solution:**

Given:

$$f(x) = x^2 - 3x + 4.$$

Let us find  $x$  satisfying  $f(x) = f(2x + 1)$ .

We have,

$$\begin{aligned} f(2x + 1) &= (2x + 1)^2 - 3(2x + 1) + 4 \\ &= (2x)^2 + 2(2x)(1) + 1^2 - 6x - 3 + 4 \\ &= 4x^2 + 4x + 1 - 6x + 1 \\ &= 4x^2 - 2x + 2 \end{aligned}$$

Now,  $f(x) = f(2x + 1)$

$$x^2 - 3x + 4 = 4x^2 - 2x + 2$$

$$4x^2 - 2x + 2 - x^2 + 3x - 4 = 0$$

$$3x^2 + x - 2 = 0$$

$$3x^2 + 3x - 2x - 2 = 0$$

$$3x(x + 1) - 2(x + 1) = 0$$

$$(x + 1)(3x - 2) = 0$$

$$x + 1 = 0 \text{ or } 3x - 2 = 0$$

$$x = -1 \text{ or } 3x = 2$$

$$x = -1 \text{ or } 2/3$$

∴ The values of  $x$  are  $-1$  and  $2/3$ .

**1. If  $f(x) = (x - a)^2(x - b)^2$ , find  $f(a + b)$ .**

**Solution:**

Given:

$$F(x) = (x - a)^2(x - b)^2$$

Let us find  $f(a + b)$ .

We have,

$$f(a + b) = (a + b - a)^2(a + b - b)^2$$

$$f(a + b) = (b)^2(a)^2$$

$$\therefore f(a + b) = a^2b^2$$

**2. If  $y = f(x) = (ax - b) / (bx - a)$ , show that  $x = f(y)$ .**

**Solution:**

Given:

$$y = f(x) = (ax - b) / (bx - a) \Rightarrow f(y) = (ay - b) / (by - a)$$

Let us prove that  $x = f(y)$ .

We have,

$$y = (ax - b) / (bx - a)$$

By cross-multiplying,

$$y(bx - a) = ax - b$$

$$bxy - ay = ax - b$$

$$bxy - ax = ay - b$$

$$x(by - a) = ay - b$$

$$x = (ay - b) / (by - a) = f(y)$$

$$\therefore x = f(y)$$

Hence proved.

**3. If  $f(x) = 1 / (1 - x)$ , show that  $f[f\{f(x)\}] = x$ .**

**Solution:**

Given:

$$f(x) = 1 / (1 - x)$$

Let us prove that  $f[f\{f(x)\}] = x$ .

Firstly, let us solve for  $f\{f(x)\}$ .

$$f\{f(x)\} = f\{1/(1-x)\}$$

$$= 1 / [1 - (1/(1-x))]$$

$$= 1 / [(1-x-1)/(1-x)]$$

$$= 1 / (-x/(1-x))$$

$$= (1-x) / -x$$

$$= (x-1) / x$$

$$\therefore f\{f(x)\} = (x-1) / x$$

Now, we shall solve for  $f[f\{f(x)\}]$

$$f[f\{f(x)\}] = f[(x-1)/x]$$

$$= 1 / [1 - (x-1)/x]$$

$$= 1 / [(x - (x-1))/x]$$

$$= 1 / [(x - x + 1)/x]$$

$$= 1 / (1/x)$$

$$\therefore f[f\{f(x)\}] = x$$

Hence proved.

**4. If  $f(x) = (x + 1) / (x - 1)$ , show that  $f[f(x)] = x$ .**

**Solution:**

Given:

$$f(x) = (x + 1) / (x - 1)$$

Let us prove that  $f[f(x)] = x$ .

$$\begin{aligned} f[f(x)] &= f[(x+1)/(x-1)] \\ &= [(x+1)/(x-1) + 1] / [(x+1)/(x-1) - 1] \\ &= [[(x+1) + (x-1)]/(x-1)] / [[(x+1) - (x-1)]/(x-1)] \\ &= [(x+1) + (x-1)] / [(x+1) - (x-1)] \\ &= (x+1+x-1)/(x+1-x+1) \\ &= 2x/2 \\ &= x \end{aligned}$$

$$\therefore f[f(x)] = x$$

Hence proved.

### 5. If

$$f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ x, & \text{when } 0 \leq x < 1 \\ \frac{1}{x}, & \text{when } x \geq 1 \end{cases}$$

**Find:**

(i)  $f(1/2)$

(ii)  $f(-2)$

(iii)  $f(1)$

(iv)  $f(\sqrt{3})$

(v)  $f(\sqrt{-3})$

**Solution:**

(i)  $f(1/2)$

When,  $0 \leq x < 1$ ,  $f(x) = x$

$$\therefore f(1/2) = 1/2$$

(ii)  $f(-2)$

When,  $x < 0$ ,  $f(x) = x^2$

$$f(-2) = (-2)^2$$

$$= 4$$

$$\therefore f(-2) = 4$$

(iii)  $f(1)$

When,  $x \geq 1$ ,  $f(x) = 1/x$

$$f(1) = 1/1$$

$$\therefore f(1) = 1$$

(iv)  $f(\sqrt{3})$

We have  $\sqrt{3} = 1.732 > 1$

When,  $x \geq 1$ ,  $f(x) = 1/x$

$$\therefore f(\sqrt{3}) = 1/\sqrt{3}$$

(v)  $f(\sqrt{-3})$

We know  $\sqrt{-3}$  is not a real number and the function  $f(x)$  is defined only when  $x \in \mathbb{R}$ .

$\therefore f(\sqrt{-3})$  does not exist.



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