EXERCISE 29.1

1. Show that $\lim_{x\to0}\frac{x}{|x|}$ does not exist.

Solution:

Firstly let us consider LHS:

$$\lim_{x\to0^-}\left(\frac{x}{|x|}\right)$$

So, let x = 0 – h, where, h = 0

$$\lim_{x\to0}\frac{x}{|x|} = \lim_{h\to0}\left(\frac{0-h}{|0-h|}\right)$$

$$= \lim_{h\to0}\left(\frac{-h}{h}\right)$$

$$= -1$$

Now, let us consider RHS:

$$\lim_{x\to0^+}\frac{(x)}{|x|}$$

So, let x = 0 + h, where, h = 0

$$\lim_{x\to0}\frac{x}{|x|} = \lim_{h\to0}\left(\frac{0+h}{|0+h|}\right)$$

$$= \lim_{h\to0}\left(\frac{h}{h}\right)$$

$$= 1$$

Since LHS ≠ RHS

∴ Limit does not exist.

2. Find k so that $\lim_{x\to2}f(x)$ may exist, where $f(x)=\begin{cases}2x+3, x\le2\\ x+k,\ x>2\end{cases}$

Solution:

Firstly let us consider LHS:

$$\lim_{x\to2^-}f(x)$$

$$\lim_{x\to2^-}f(x) = \lim_{x\to2^-}(2x+3)$$

So, let x = 2 – h, where h = 0

Substituting the value of x, we get

$$\lim_{h\to 0}[2(2-h)+3]$$

=> 2 (2 - 0) + 3 = 7

Now let us consider RHS:

$$\lim_{x\to 2^+} f(x)$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x+k)$$

So, let x = 2 + h, where, h = 0

$$\lim_{h\to 0}(2+h+k)$$

=> 2 + 0 + k = 2 + k

Since, Limit exists, LHS = RHS

7 = 2 + k

k = 7 – 2

= 5

∴ Value of k is 5.

3. Show that $\lim_{x\to 0}\frac{1}{x}$ does not exist.

Solution:

Firstly let us consider LHS:

$$\lim_{x\to 0^-}\left(\frac{1}{x}\right)$$

So, let x = 0 – h, where h = 0

$$\lim_{x\to 0^-}\left(\frac{1}{x}\right) = \lim_{h\to 0}\left(\frac{1}{0-h}\right)$$

$$= -\infty$$

Now, let us consider RHS:

$$\lim_{x\to 0^+}\left(\frac{1}{x}\right)$$

So, let x = 0 + h, where h = 0

$$\lim_{x\to 0^+}\left(\frac{1}{x}\right) = \lim_{h\to 0}\left(\frac{1}{0+h}\right)$$

$$= \infty$$

Since, LHS ≠ RHS

∴ Limit does not exist.

4. Let f(x) be a function defined by $f(x)=\begin{cases}\frac{3x}{|x|+2x}, & x\neq 0\\ 0, & x=0\end{cases}$

Show that $\lim_{x\to 0} f(x)$ **does not exist.**

Solution:

Firstly let us consider LHS:

$$\lim_{x\to 0^-}\left[\frac{3x}{|x|+2x}\right]$$

So, let x = 0 – h, where h = 0

Substituting the value of x, we get

$$\lim_{x\to 0^-}\left[\frac{3x}{|x|+2x}\right]=\lim_{h\to 0}\left[\frac{3(-h)}{|-h|+2(-h)}\right]$$

$$=\lim_{h\to 0}\left[\frac{-3h}{h-2h}\right]$$

$$=\lim_{h\to 0}\left[\frac{-3h}{-h}\right]$$

$= 3$

Now, let us consider RHS:

$$\lim_{x\to 0^+}\left(\frac{3x}{|x|+2x}\right)$$

So, let x = 0 + h, where h = 0

Substituting the value of x, we get

$$\lim_{x\to 0^+}\left(\frac{3x}{|x|+2x}\right)=\lim_{h\to 0}\left(\frac{3h}{|h|+2h}\right)$$

$$=\lim_{h\to 0}\left(\frac{3h}{h+2h}\right)$$

$= 1$

Since, LHS ≠ RHS

∴ Limit does not exist.

5. Let $f(x)=\begin{cases}x+1, \text{if } x>0\\ x-1, \text{if } x<0\end{cases}$ **. Prove that** $\lim_{x\to 0} f(x)$ **does not exist.**

Solution:

Firstly let us consider LHS:

$$\lim_{x\to 0^-} f(x)$$

So, let x = 0 – h, where h = 0

Substituting the value of x, we get

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0}(0-h-1)$$

$$= -1$$

Now, let us consider RHS

$$\lim_{x\to 0^+} f(x)$$

So, let x = 0 + h, where h = 0

Substituting the value of x, we get

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(x+1)$$

$$= \lim_{h\to 0}(0+h+1)$$

$$= 1$$

Since, LHS ≠ RHS

∴ Limit does not exist.