We have, A = set of triangles and B = set of rectangles.

Now, we can see elements of A does not belong to B since the set of rectangles does not include a set of triangles.

Q. 1 K. State in each case whether A ⊂ B or A ⊄ B.

A = {x : x is an even natural number less than 8}, B = {x : x is a natural number which divides 32}

Answer : A ⊄ B

Explanation: we have, A = {2,4,6} and B = {1,2,4,8,16,32}.

Thus , A ⊄ B , since 6 ∉ B.

Q. 2. Examine whether the following statements are true of false:

(i) {a, b} ⊄{b, c, a}

(ii) {a}∈ {a, b, c}

(iii) ϕ ⊂{a, b, c}

(iv) {a, e} ⊂{x : x is a vowel in the English alphabet}

(v) {x : x ∈ W, x + 5 = 5} =ϕ

(vi) a ∈ {{a}, b}

(vii) {a} ⊂ {{a}, b}

(viii) {b, c} ⊂{a, {b, c}}

(ix) {a, a, b, b} = {a, b}

(x) {a, b, a, b, a, b, ….} is an infinite set.

(xi) If A = set of all circles of unit radius in a plane and B = set of all circles in the same plane then A⊂B.

Answer : (i) False

Explanation: Since elements of {a,b} are also elements of {b,c,a} hence {a, b}⊂{b, c, a}.

(ii) False

{a} is not in {a,b,c}. Hence, {a} ∉ {a, b, c}.

(iii) True

Explanation: ϕ is a subset of every set.

(iv) True

Explanation: a, e are vowels of English alphabet.

(v) False

Explanation: 0+5 = 5 , 0 ϵ W

Hence, {0} ≠ ϕ

(vi) False

Explanation: a is not an element of {{a}, b}

(vii) False

As a is not an element of set {{a}, b}

(viii) False

Explanation: {b,c} is an element of {a, {b, c}} and element cannot be subset of set.

(ix) True

Explanation: In a set all the elements are taken as distinct. Repetition of elements in a set do not change a set.

(x) False

Explanation: Given set is {a,b}, which is finite set. In a set all the elements are taken as distinct.Repetition of elements in a set do not change a set.

(xi) True

Explanation: Circle in a plane with unit radius is subset of circle in a plane of any radius.

Q. 3. If A = {1} and B = {{1}, 2} then show that A ⊄ B.

Hint 1 ϵ A but 1⊄ B.

Answer : There is only one element in set A.

Now, 1 is not an element of B.

Therefore, A ⊄ B.

Q. 4. Write down all subsets of each of the following sets:

(i) A = {A}

(ii) B = {a, b}

(iii) C = {–2, 3}

(iv) D = {–1, 0, 1}

(v) E =ϕ

(vi) F = {2, {3}}

(vii) G = {3, 4, {5, 6}}

Answer : (i) The subsets of {A} are ϕ and {A}

(ii) The subsets of {a, b} are ϕ, {a}, {b}, and {a, b}.

(iii) The subsets of {–2, 3} are ϕ, {-2}, {3}, and {-2, 3}.

(iv) The subsets of {–1,0,1} are ϕ ,{-1},{0},{1},{-1,0},{0,1},{-1,1},{–1, 0, 1}

(v)ϕ has only one subset ϕ.

(vi) let x = {3}

Then, F = {2,x}

The subsets of {2,x} are ϕ , {2},{x},{2,x}

i.e ϕ , {2},{{3}},{2,{3}}

(vii) let x = {5,6}

Then, G = {3,4,x}

The subsets of {3,4,x } are ϕ ,{3},{4},{x},{3,x},{4,x},{3,4}, {3,4,x} i.e. ϕ ,{3},{4},{{5,6}},{3,{5,6}},{4,{5,6}},{3,4},{3,4,{5,6}}

Q. 5. Express each of the following sets as an interval:

(i) A = {x : x ϵR, –4 < x < 0}

(ii) B = {x : x ϵ R, 0 ≤ x < 3}

(iii) C = {x : x ϵ R, 2 < x ≤ 6}

(iv) D = {x : x ϵ R, –5 ≤ x ≤ 2}

(v) E = {x : x ϵ R, –3 ≤ x < 2}

(vi) F = {x : x ϵ R, –2 ≤ x < 0}

Answer : (i) A = (-4,0)

Explanation: All the points between -4 and 0 belong to the open interval (-4,0) but -4 ,0 themselves do not belong to this interval.

(ii) B = [0,3)

Explanation: B = {x : x ϵ R, 0 ≤ x < 3} is an open interval from 0 to 3, including 0 but excluding 3.

(iii) C = (2,6]

Explanation: C = {x : x ϵ R, 2 < x ≤ 6} is an open interval from 2 to 6, including 6 but excluding 2.

(iv) D = [-5,2]

Explanation: D = {x : x ϵ R, –5 ≤ x ≤ 2} is a closed interval from -5 to 2 and contains the end points.

(v) E = [-3,2)

Explanation: E = {x : x ϵ R, –3 ≤ x < 2} is an open interval from -3 to 2, including -3 but excluding 2.

(vi) F = [-2,0)

Explanation: F = {x : x ϵ R, –2 ≤ x < 0} is an open interval from -2 to 0, including -2 but excluding 0.

Q. 6. Write each of the following intervals in the set-builder from:

(i) A = (–2, 3)

(ii) B = [4, 10]

(iii) C = [–1, 8)

(iv) D = (4, 9]

(v) E = [–10, 0)

(vi) F = (0, 5]

Answer : (i) A = {x : x ∈R, –2 < x < 3}

(ii) B = {x : x ∈ R, 4 ≤ x ≤ 10}

(iii) C= {x : x ∈ R, –1 ≤ x < 8}

(iv) D = {x : x ∈ R, 4 < x ≤ 9}

(v) E = {x : x ∈ R, –10 ≤ x < 0}

(vi) F = {x : x ∈ R, 0 < x ≤ 5}

Q. 7. if A = {3, {4, 5}, 6} find which of the following statements are true.

(i) {4, 5} ⊄A

(ii) {4, 5} ∈A

(iii) {{4, 5}} ⊆A

(iv) 4∈A

(v) {3} ⊆A

(vi) {ϕ} ⊆A

(vii) ϕ⊆A

(viii) {3, 4, 5} ⊆A

(ix) {3, 6} ⊆A

Answer : (i) True

Explanation: we have, A = {3, {4, 5}, 6}

Let {4,5} = x

Now, A = {3, x, 6}

4,5 is not in A, {4,5} is an element of A and element cannot be subset of set,thus {4, 5} $\not\subset$A.

(ii) True

Explanation: we have, A = {3, {4, 5}, 6}

Let {4,5} = x

Now, A = {3, x, 6}

Now, x is in A.

So, x $\in$ A.

Thus, {4, 5} $\in$ A

(iii) True

Explanation: {4,5} is an element of set {{4,5}}.

Let {4,5} = x

{{4,5}} = {x}

we have, A = {3, {4, 5}, 6}

Now, A = {3, x, 6}

So, x is in {x} and x is also in A.

So , {x} is a subset of A.

Hence, {{4, 5}} $\subseteq$A

(iv) False

Explanation: 4 is not an element of A.

(v) True

Explanation: 3 is in {3} and also 3 is in A.

(vi) False

Explanation: ϕ is an element in { ϕ} but not in A.

Thus, {ϕ} ⊄ A

(vii) True

Explanation: ϕ is a subset of every set.

(viii) False

Explanation: we have, A = {3, {4, 5}, 6}

Let {4,5} = x

Now, A = {3, x, 6}

4,5 is in {3,4,5} but not in A, thus {3,4, 5} ⊄A.

(ix) True

Explanation: 3,6 is in {3,6} and also in A, thus {3, 6} ⊆A.

Q. 8. If A = {a, b, c}, find P(A) and n{P(A)}.

Answer : The collection of all subsets of a set A is called the power set of A. It is denoted by P(A).

Now, We know that ϕ is a subset of every set. So, ϕ is a subset of {a, b, c}.

Also, {a},{b},{c},{a,b},{b,c},{a,c} are also subsets of {a, b, c}

We know that every set is a subset of itself. So, {a, b, c} is a subset of {a, b, c}.

Thus, the set {a, b, c} has, in all eight subsets, viz. ϕ , {a},{b},{c},{a,b},{b,c},{a,c},{a, b, c}.

∴ P(A) = { ϕ , {a},{b},{c},{a,b},{b,c},{a,c},{a, b, c}}

Now, n{P(A)} = 2^m , where m = n(A) = 3

$\Rightarrow n\{P(A)\} = 2^3 = 8$

Q. 9. If A = {1, {2, 3}}, find P(A) and n {P(A)}.

Answer : Let {2,3} = x

Now, A = {1,x}

Subsets of A are ϕ , {1} , {x} , {1,x}

⇒ Subsets of A are ϕ , {1} , {2,3} , {1, {2,3}}

Now, $n\{P(A)\} = 2^m$, where m = n(A) = 2

$\Rightarrow n\{P(A)\} = 2^2 = 4$

Q. 10. If A = ϕ then find n{P(A)}.

Answer : We have, A = ϕ , i.e. A is a null set.

Then, n(A)= 0

$\therefore n\{P(A)\} = 2^m$, where m = n(A)

$\Rightarrow n\{P(A)\} = 2^0 = 1$.

Thus, P(A) has one element.

Q. 11. If A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 8} then find the universal set.

Answer : Elements of A+B+C = {1,3,5,2,4,6,0,8}

Thus, the universal set for A,B and C = {0,1,2,3,4,5,6,8}

Q. 12. Prove that A ⊆ B, B⊆ C and C⊆ A ⇒A = C.

Answer : We have A ⊆B , B⊆ C and C⊆ A

Now , A is a subset of B and B is a subset of C, So A is a subset of C.

Given that C⊆ A.

Hence, A = C.

Q. 13. For any set A, prove that A⊆ϕ ⇔ A =ϕ

Answer : Let $A \subseteq \phi$

A is a subset of the null set , then A is also an empty set.

$\Rightarrow A = \phi$

Now, let $A = \phi$

$\Rightarrow$ A is an empty set.

Since, every set is a subset of itself.

$\Rightarrow A \subseteq \phi$

Hence, for any set A, $A \subseteq \phi \Leftrightarrow A = \phi$

Q. 14. State whether the given statement is true false:

(i) If $A \subset B$ and $x \notin B$ than $x \notin A$.

(ii) If $A \subseteq \phi$ then $A = \phi$

(iii) If A, B and C are three sets such than $A \in B$ and $B \subset C$ then $A \subset C$.

(iv) If A, B and C are three sets such than $A \subset B$ and $B \in C$ then $A \in C$.

(v) If A, B and C are three sets such that $A \not\subset B$ and $B \not\subset C$ then $A \not\subset C$.

(vi) If A and B are sets such that x A and $A \in B$ then $x \in B$.

Answer : (i) True

Explanation: We have $A \subset B$ since A is a subset of B then all elements of A should be in B.

Let A = {1,2} and B = {1,2,3}

Let $x = 4 \notin B$

Also we observe that $4 \notin A$.

Hence, If $A \subset B$ and $x \notin B$ than $x \notin A$.

(ii) True

Explanation: We have, $A \subseteq \phi$

Now, A is a subset of null set , this implies A is also an empty set.

$\Rightarrow A = \phi$

(iii) False

Explanation: Let A = {a}, B = {{a}, b}

here , $A \in B$

Now, let C = {{a}, b, c}.

Since, {a},b is in B and also in C thus, $B \subset C$.

But, A ={a} and {a} is an element of C, since the element of a set cannot be a subset of a set.

Hence,$A \not\subset C$.

(iv) False

Explanation: Let A = {a},B = {a, b} and C = {{a, b}, c}.

Then,$A \subset B$ and $B \in C$. But, $A \notin C$ since {a} is not an element of C.

(v) False.

Explanation: Let A = {a}, B = {b, c} and C = {a, c}.

Since $a \in A$ and $a \notin B$.Then, $A \not\subset B$

Now, $b \in B$ and $b \notin C \Rightarrow B \not\subset C$.

But, $A \subset C$ since, $a \in A$ and $a \in C$.

(vi) False.

Explanation: Let A = {x}, B = {{x}, y}

Now, $x \in A$ and {x} is an element of B $\Rightarrow A \in B$

But, x is not an element of B. Thus, $x \notin B$.

Exercise 1D

Q. 1. If A = {a, b, c, d, e, f}, B = {c, e, g, h} and C = {a, e, m, n}, find:

(i) A ∪ B

(ii) B ∪ C

(iii) B ∪ C

(iv) C ∩ A

(vi) A ∩ B

Answer : Given; A = {a, b, c, d, e, f}, B = {c, e, g, h} and C = {a, e, m, n}

(i) A ∪ B = {a, b, c, d, e, f, g, h}

(ii) B ∪ C = {a, c, e, g, h, m, n}

(iii) B ∪ C = {a, c, e, g, h, m, n}

(iv) C ∩ A = {a, e}

(vi) A ∩ B = {c, e}

Q. 2. If A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8} and C = {10, 11, 12, 13, 14}, find:

(i) A ∪ B

(ii) B ∪ C

(iii) A ∪ C

(iv) B ∪ D

(v) (A ∪ B) ∪ C

(vi) (A ∪B) ∩C

(vii) (A ∩ B) ∪D

(viii) (A ∩ B) ∪ (B ∩ C)

(ix) (A ∩ C) ∩ (C ∪ D

Answer : Given; A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8} and C = {10, 11, 12, 13, 14}

(i) A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8}

(ii) B ∪ C = {4, 5, 6, 7, 8, 10, 11, 12, 13, 14}

(iii) A ∪ C = {1, 2, 3, 4, 5, 10, 11, 12, 13, 14}

(iv) B ∪ D

(v) (A ∪ B) ∪ C = {1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14}

(vi) (A ∪B) ∩C = Φ or {}

(vii) (A ∩ B) ∪D =

(viii) (A ∩ B) ∪ (B ∩ C) = Φ or {}

(ix) (A ∩ C) ∩ (C ∪ D =

Q. 3. If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13} and C = {11, 13, 15}, and D = {15, 17}, find:

(i) A ∩ B

(ii) A ∩ C

(iii) B ∩ C

(iv) B ∩ D

(v) B ∩ (C ∪ D)

(vi) A ∩ (B ∪ C)

Answer : Given; A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13} and C = {11, 13, 15}, and D = {15, 17}

(i) A ∩ B = {7, 9, 11}

(ii) A ∩ C = {11}

(iii) B ∩ C = {11, 13}

(iv) B ∩ D = Φ or {}

(v) B ∩ (C ∪ D) = {11, 13}

(vi) A ∩ (B ∪ C) = {7, 9, 11}

Q. 4. If A = {x : x ϵ N}, B = {x : x ϵ N and x is even), C = {x : x ϵ N and x is odd} and D = {x : x ϵ N and x is prime} then find:

(i) A ∩ B

(ii) A ∩ C

(iii) A ∩ D

(iv) B ∩ C

(v) B ∩ D

(vi) C ∩ D

Answer : Given; A = {x : x ϵ N}, B = {x : x ϵ N and x is even), C = {x : x ϵ N and x is odd} and D = {x : x ϵ N and x is prime}

(i) A ∩ B = {x : x ϵ N and x is even}

(ii) A ∩ C = {x : x ϵ N and x is odd}

(iii) A ∩ D = {x : x ϵ N and x is prime}

(iv) B ∩ C = Φ or {}

(v) B ∩ D = {2} [∵ 2 is the only even prime number]

(vi) C ∩ D = {x : x ϵ N and x is prime and x ≠ 2}

Q. 5. If A = {2x : x ϵ N}, 1 ≤ x < 4}, B = {x + 2) : x ϵ N and 2 ≤ x < 5} and C = {x : x ϵ N and 4 < x < 8}, find:

(i) A ∩ B

(ii) A ∪ B

(iii) (A ∪ B) ∩ C

Answer : Given; A = {2x : x ϵ N}, 1 ≤ x < 4}, B = {x + 2) : x ϵ N and 2 ≤ x < 5} and C = {x : x ϵ N and 4 < x < 8}

According to the given conditions; A = {2, 4, 6}, B = {4, 5, 6} and C = {5, 6, 7}

(i) A ∩ B = {4, 6}

(ii) A ∪ B = {2, 4, 5, 6}

(iii) (A ∪ B) ∩ C = {5, 6}

Q. 6. If A = {2, 4, 6, 8, 10, 12}, B = {3, 4, 5, 6, 7, 8, 10}, find:

(i) (A – B)

(ii) (B – A)

(iii) (A – B) ∪ (B – A)

Answer : Given; A = {2, 4, 6, 8, 10, 12}, B = {3, 4, 5, 6, 7, 8, 10}

(i) (A – B) = {2, 12}

(ii) (B – A) = {5, 7}

(iii) (A – B) ∪ (B – A) = Φ or {}

Q. 7. If A = {a, b, c, d, e}, B = {a, c, e, g} and C = {b, e, f, g}, find:

(i) A ∩ (B – C)

(ii) A – (B ∪ C)

(iii) A – (B ∩ C)

Answer : Given; A = {a, b, c, d, e}, B = {a, c, e, g} and C = {b, e, f, g}

(i) A ∩ (B – C) = {a, c}

(ii) A – (B ∪ C) = {d}

(iii) A – (B ∩ C) = {a, b, c, d}

Q. 8. If $A = \left\{\frac{1}{x} : x \in N\right\}$ **and x < 8}, and B =** $\left\{\frac{1}{2x} : x \in N \text{ and } x \leq 4\right\}$**, find :**

(i) A ∪ B
(ii) A ∩ B
(iii) A – B
(vi) B – A

Answer : Given; $A = \left\{\frac{1}{x} : x \epsilon N\right\}$ and x < 8 and $B = \left\{\frac{1}{2x} : x \epsilon N\right\}$ and x ≤ 4

According to the given conditions;

$$A = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}\right\} \ and \ B = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}\right\}$$

(i) A ∪ B =

$$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}\right\}$$

(ii) A ∩ B =

$$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}\right\}$$

(iii) A – B =

$$\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}\right\}$$

(vi) B – A =

$$\left\{\frac{1}{8}\right\}$$

Q. 9. If R is the set of all real numbers and Q is the set of all rational numbers then what is the set (R – Q)?

Answer : Given; R is the set of all real numbers and Q is the set of all rational numbers.

Then (R – Q) is the set of all irrational numbers.

Q. 10. If A = {2, 3, 5, 7, 11} and B = ϕ, find:

(i) A ∪ B

(ii) A ∩ B

Answer : Given; A = {2, 3, 5, 7, 11} and B = ϕ

(i) A ∪ B = {2, 3, 5, 7, 11}

(ii) A ∩ B = ϕ

Q. 11. If A and B are two sets such that A ⊆ B then find:

(i) A ∪ B

(ii) A ∩ B

Answer : Given; A and B are two sets such that A ⊆ B.

(i) A ∪ B = A

(ii) A ∩ B = B

Q. 12. Which of the following sets are pairs of disjoint sets? Justify your answer.

(i) A = {3, 4, 5, 6} and B = {2, 5, 7, 9}

(ii) C = {1, 2, 3, 4, 5} and D = {6, 7, 9, 11}

(iii) E = {x : x ϵ N, x is even and x < 8}

F = {x : x = 3n, n ϵ N, and x < 4}

(vi) G = {x : x ϵ N, x is even} and H {x : x ϵ N, x is prime}

(v) J = {x : x ϵ N, x is even} and K = {x : x ϵ N, x is odd}

Answer : Disjoint sets have their intersections as Φ.

(i) A = {3, 4, 5, 6} and B = {2, 5, 7, 9} Are pairs of disjoint sets.

(ii) C = {1, 2, 3, 4, 5} and D = {6, 7, 9, 11} Are pairs of disjoint sets.

(iii) E = {x : x ϵ N, x is even and x < 8} = {2, 4, 6} and

F = {x : x = 3n, n ϵ N, and x < 4} = {3, 6, 9} Are not pairs of disjoint sets.

(iv) G = {x : x ϵ N, x is even} and H {x : x ϵ N, x is prime}

∵ 2 is an even prime number; their intersection is not Φ

Are not pairs of disjoint sets.

(v) J = {x : x ϵ N, x is even} and K = {x : x ϵ N, x is odd}

∵ there is no number which is both odd and even.

∴ J and K are pairs of disjoint sets.

Q. 13. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A A = {1, 2, 3, 4,}, B = {2, 4, 6, 8} and = {1, 4, 5, 6}, find:

(i) A'

(ii) B'

(iii) C'

(iv) (B')'

(v) (A ∪ B)'

(vi) (A ∩ C)'

(vii) (B – C)'

Answer : Given; U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4,}, B = {2, 4, 6, 8} and C = {1, 4, 5, 6}

(i) A' = {5, 6, 7, 8, 9}

(ii) B' = {1, 3, 5, 7, 9}

(iii) C' = {2, 3, 7, 8, 9}

(iv) (B')' = {2, 4, 6, 8}

(v) (A ∪ B)' = {5, 7, 9}

(vi) (A ∩ C)' = {2, 3, 5, 6, 7, 8, 9}

(vi) (B – C)' = {1, 3, 4, 5, 6, 7, 9}

Q. 14. if U = {a, b, c} and A = {a, c, d, e} then verify that:

(i) (A ∪ B)' = (A' ∩ B')

(ii) (A ∩ B)' = (A' ∪ B')

Answer : Given; U = {a, b, c} and B = {a, c, d, e}

(i) (A ∪ B)' = (A' ∩ B')

(ii) (A ∩ B)' = (A' ∪ B')

Q.15. if U is the universal set and A U then fill in the blanks.

(i) A ∪ A' = ….

(ii) A ∩ A' = ….

(iii) ϕ '∩ A = ….

(iv) U' ∩ A = ….

Answer : Given; U is the universal set and A ⊂ U

(i) A ∪ A' = U

(ii) A ∩ A' = Φ or {}

(iii) ϕ '∩ A = Φ

(iv) U' ∩ A = Φ or {}

Exercise 1E

Q. 1. If $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$, verify that:

(i) $A \cup B = B \cup A$
(ii) $A \cup C = C \cup A$
(iii) $B \cup C = C \cup B$
(iv) $A \cap B = B \cap A$
(v) $B \cap C = C \cap B$
(vi) $A \cap C = C \cap A$
(vii) $(A \cup B \cup C = A \cup (B \cup C)$
(viii) $(A \cap B) \cap C = A \cap (B \cap C)$

Answer : (i) LHS = $A \cup B$

$= \{a, b, c, d, e\} \cup \{a, c, e, g\}$

$= \{a, b, c, d, e, g\}$

$= \{a, c, e, g\} \cup \{a, b, c, d, e\}$

$= B \cup A$

= RHS

Hence proved.

(ii) To prove: $A \cup C = C \cup A$

Since the element of set C is not provided,

let x be any element of C.

LHS = $A \cup C$

$= \{a, b, c, d, e\} \cup \{x \mid x \in C\}$

$= \{a, b, c, d, e, x\}$

$= \{x, a, b, c, d, e\}$

$= \{x \mid x \in C\} \cup \{a, b, c, d, e\}$

$= C \cup A$

= RHS

Hence proved.

(iii) To prove: $B \cup C = C \cup B$

Since the element of set C is not provided,

let x be any element of C.

LHS = $B \cup C$

= { a, c, e, g } ∪ { x |x$\in$C}

= { a, c, e, g, x}

= {x, a, c, e, g }

= { x |x$\in$C} ∪ { a, c, e, g }

= $C \cup B$

= RHS

Hence proved.

(iv) LHS = $A \cap B$

= {a, b, c, d, e}∪ {a, c, e, g}

= {a, c, e}

RHS = $B \cap A$

= {a, c, e, g}∩{a, b, c, d, e}

= {a, c, e}

∴ $A \cap B = B \cap A$

(v) Let x be an element of $B \cap C$

⇒ $x \in B \cap C$

⇒ $x \in B$ and $x \in C$

⇒ $x \in C$ and $x \in B$ [by definition of intersection]

⇒ $x \in C \cap B$

⇒ $B \cap C \subset C \cap B$ ....(i)

Now let x be an element of $C \cap B$

Then, $x \in C \cap B$

⇒ $x \in C$ and $x \in B$

⇒ $x \in B$ and $x \in C$ [by definition of intersection]

⇒ $x \in B \cap C$

⇒ $C \cap B \subset B \cap C$ ....(ii)

From (i) and (ii) we have,

$B \cap C = C \cap B$ [every set is a subset of itself]

Hence proved.

(vi) Let x be an element of $A \cap C$

⇒ $x \in A \cap C$

⇒ $x \in A$ and $x \in C$

⇒ $x \in C$ and $x \in A$ [by definition of intersection]

⇒ $x \in C \cap A$

⇒ $A \cap C \subset C \cap A$ ....(i)

Now let x be an element of $C \cap A$

Then, $x \in C \cap A$

$\Rightarrow x \in C$ and $x \in A$

$\Rightarrow x \in A$ and $x \in C$ [by definition of intersection]

$\Rightarrow x \in A \cap C$

$\Rightarrow C \cap A \subset A \cap C$ ....(ii)

From (i) and (ii) we have,

$A \cap C = C \cap A$ [every set is a subset of itself]

Hence proved.

(vii) Let x be any element of $(A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A \cup (B \cup C)$

$\Rightarrow (A \cup B) \cup C \subset A \cup (B \cup C)$ .....(i)

Now, let x be an element of $A \cup (B \cup C)$

Then, $x \in A$ or $(B \cup C)$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in (A \cup B) \cup C$

$\Rightarrow A \cup (B \cup C) \subset (A \cup B) \cup C$ .....(ii)

From i and ii, $(A \cup B) \cup C = A \cup (B \cup C)$

[every set is a subset of itself]

Hence , proved.

(viii) Let x be any element of (A ∩ B) ∩ C

⇒ x ∈ (A ∩ B) and x ∈ C

⇒ x ∈ A and x ∈ B and x ∈ C

⇒ x ∈ A and x ∈ (B ∩ C)

⇒ x ∈ A ∩ (B ∩ C)

⇒ (A ∩ B) ∩ C ⊂ A ∩ (B ∩ C) .....(i)

Now, let x be an element of A ∩ (B ∩ C)

Then, x ∈ A and (B ∩ C)

⇒ x ∈ A and x ∈ B and x ∈ C

⇒ x ∈ (A ∩ B) and x ∈ C

⇒ x ∈ (A ∩ B) ∩ C

⇒ A ∩ (B ∩ C) ⊂ (A ∩ B) ∩ C .....(ii)

From i and ii, (A ∩ B) ∩ C = A ∩ (B ∩ C)

[every set is a subset of itself]

Hence, proved.

Q. 2. If A = {a, b, c, d, e}, B = {a, c, e, g}, and C = {b, e, f, g} verify that:

(i) A ∩ (B – C) = (A ∩ B) – (A ∩ C)

(ii) A – (B ∩ C) = (A – B) ∪ (A – C)

Answer : (i) B - C represents all elements in B that are not in C

B - C = {a, c}

$A \cap (B - C) = \{a, c\}$

$A \cap B = \{a, c, e\}$

$A \cap C = \{b, e\}$

$(A \cap B) - (A \cap C) = \{a, c\}$

$\Rightarrow A \cap (B - C) = (A \cap B) - (A \cap C)$

Hence proved

(ii) $B \cap C = \{e, g\}$

$A - (B \cap C) = \{a, b, c, d\}$

$(A - B) = \{b, d\}$

$(A - C) = \{a, c, d\}$

$(A - B) \cup (A - C) = \{a, b, c, d\}$

$\Rightarrow A - (B \cap C) = (A - B) \cup (A - C)$

Hence proved

Q. 3. If $A = \{x : x \in N, x \leq 7\}$, $B = \{x : x \text{ is prime}, x < 8\}$ and $C = \{x : x \in N, x \text{ is odd and } x < 10\}$, verify that:

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Answer : Natural numbers start from 1

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$

$C = \{1, 3, 5, 7, 9\}$

(i) $B \cap C = \{3, 5, 7\}$

$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 9\}$

$(A \cup B) \cap (A \cup C) = \{1, 23, 4, 5, 6, 7\}$

$\Rightarrow A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence proved

(ii) $B \cup C = \{1, 2, 3, 5, 7, 9\}$

$A \cap (B \cup C) = \{1, 2, 3, 5, 7\}$

$A \cap B = \{2, 3, 5, 7\}$

$A \cap C = \{1, 3, 5, 7\}$

$(A \cap B) \cup (A \cap C) = \{1, 2, 3, 5, 7\}$

$\Rightarrow A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Q. 4. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8}, and = {2, 3, 5, 7} verify that:

(i) (A ∪ B)' = (A' ∩ B')

(ii) (A ∩ C)' = (A' ∪ B')

Answer : (i) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)' = \{1, 9\}$

$A' = \{1, 3, 5, 7, 9\}$

$B' = \{1, 4, 6, 8, 9\}$

$A' \cap B' = \{1, 9\}$

$\Rightarrow (A \cup B)' = A' \cap B'$

Hence proved

(ii) $A \cap B = \{2\}$

$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$\Rightarrow (A \cap B)' = A' \cup B'$

Hence proved

These are also known as De Morgan's theorem

Q. 5. Let A = {a, b, c}, B = {b, c, d, e} and = {c, d, e, f} be subsets of U = {a, b, c, d, e, f}. Then verify that:

(i) (A')' = A
(ii) (A ∪ B)' = (A' ∩ B')
(iii) (A ∩ B)' = (A' ∪ B')

Answer : (i) A' = {d, e, f}

(A')' = {a, b, c} = A

Hence proved

(ii) $A \cup B = \{a, b, c, d, e\}$

$(A \cup B)' = \{f\}$

A' = {d, e, f}

B' = {a, f}

$A' \cap B' = \{f\}$

$\Rightarrow (A \cup B)' = (A' \cap B')$

Hence proved

(iii) $A' \cup B' = \{a, d, e, f\}$

$A \cap B = (b, c\}$

$(A \cap B)' = \{a, d, e, f\}$

$\Rightarrow (A \cap B)' = A' \cup B'$

Hence proved

Q. 6. Given an example of three sets A, B, C such that A ∩ C ≠ ϕ, B ∩ C ≠ ϕ, A ∩ C ≠ ϕ, and A ∩ B ∩ C = ϕ

Answer : Let A = {1, 2}

B = {2, 3}

C = {1, 3, 4}

$A \cap B$ = {2}

$A \cap C$ = {1}

$B \cap C$ = {3}

$A \cap B \cap C$ = {2} $\cap$ {1, 3, 4} = ø

So the three sets are valid and satisfy the given conditions

Q. 7. For any sets A and B, prove that:

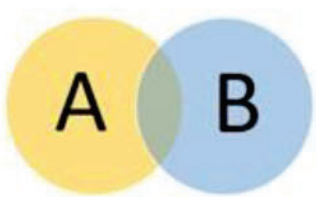
(i) (A – B) ∩ B = ϕ
(ii) A ∪ (B – A) = A ∪ B
(iii) (A – B) ∪ (A ∩ B) = A
(iv) (A ∪ B) – B = A – B
(iv) A – (A ∪ B) = A – B

Answer : Two sets are shown with the following Venn Diagram

The yellow region is denoted by 1.

Blue region is denoted by 2.

The common region is denoted by 3.



(i) A - B denotes region 1

B denotes region (2+3)

So their intersection is a null set

⇒ $(A - B) \cap B$ = ø

(ii) B - A denotes region 2

A denotes region (1+3)

So their union denotes region (1+2+3) which is the union of A and B

$\Rightarrow A \cup (B - A) = A \cup B$

(iii) A - B denotes region 1

$A \cup B$ denotes region 3

Their union denotes region (1+3) which is set A$\Rightarrow (A - B) \cup (A \cap B) = A$

(iv) $A \cup B$ denotes region (1+2+3)

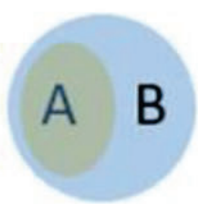
(AUB) - B denotes region (1+2+3) - (2+3) = 1A - B denotes region 1$\Rightarrow (A \cup B) - B = A – B$

(v) Wrong question

Q. 8. For any sets A and B, prove that:

(i) $A \cap B' = \phi \Rightarrow A \subset B$
(ii) $A' \cup B' = U \Rightarrow A \subset B$

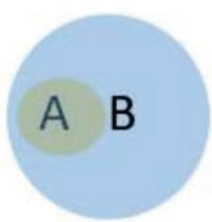
Answer : (i) The Venn Diagram for the given condition is given below



As can be seen from the Venn Diagram, A is a proper subset of B

$\Rightarrow A \subset B$

(ii) Wrong question. If A is a proper subset of B then $A' \cup B' \neq U$



Exercise 1F

Q. 1. Let A = {a, b, c, e, f} B = {c, d, e, g} and C = {b, c, f, g} be subsets of the set U = {a, b, c, d, e, f, g, h}.

(i) A ∩ B
(ii) A ∪ (B ∩ C)
(iii) A – B
(iv) B – A
(v) A – (B ∩ C)
(vi) (B – C) ∪ (C – B)

Answer : (i) A∩B will contain the common elements of A and B

A∩B = {c, e}

(ii) AU(B∩C)

B∩C = {c, d, g}

AU(B∩C) = {a, b, c, d, e, f, g}

(iii) A - B implies the set of all elements in A that are not in B

A - B = {a, b, f}

(iv) B - A implies the set of all elements in B that are not in A

B - A = {d, g}

(v) A - (B∩C) denotes elements of A that are not in B∩C

A - (B∩C) = {a, b, e, f}

(vi) (B - C)U(C - B) implies the union of sets B - C and C – B

B - C = {d, e}

C - B = {b, f}

(B - C)U(C - B) = {b, d, e, f}

Q. 2. Let A = {2, 4, 6, 8, 10}, B = {4, 8, 12, 16} and C = {6, 12, 18, 24}.

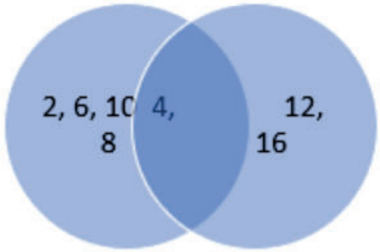
Using Venn diagrams, verify that:

(i) (A ∪ B) ∪ C = A ∪ (B ∪ C)

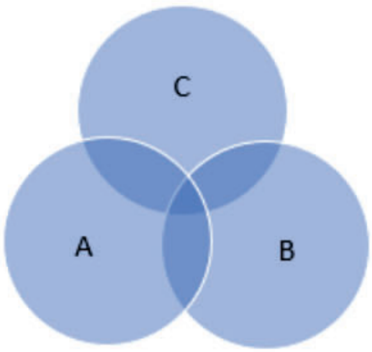
(ii) (A ∩ B) ∩ C = A ∩ (B ∩ C).

Answer : (i) LHS:

A∪B = {2, 4, 6, 8, 10, 12, 16} (A∪B)∪C = {2, 4, 6, 8, 10, 12, 16, 18, 24}



A



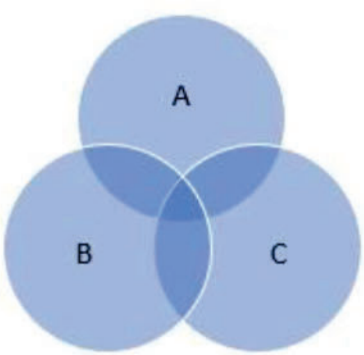
B

RHS:

$B \cup C$ = {4, 6, 8, 10, 12, 16, 18, 24}

B C

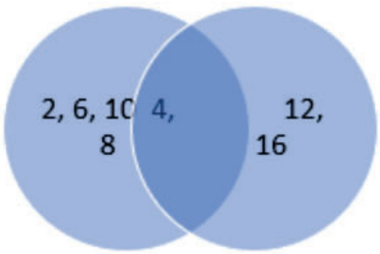
$A \cup (B \cup C)$ = {2, 4, 6, 8, 10, 12, 16, 18, 24}



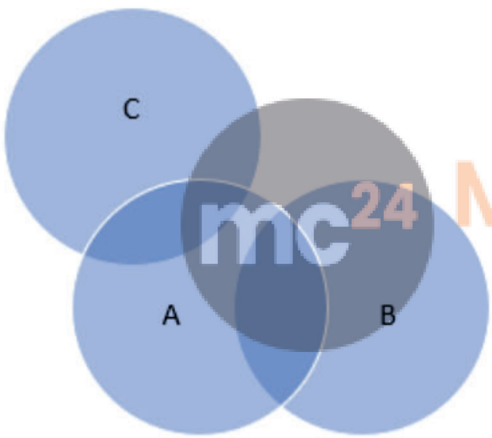
LHS = RHS. [Verified]

(ii) LHS:

$A \cap B = \{4, 8\}$ $(A \cap B) \cap C = \{\}$ or ø



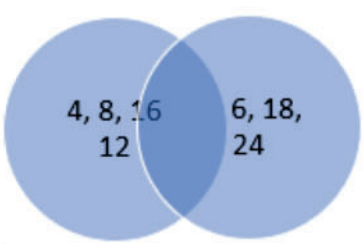
A



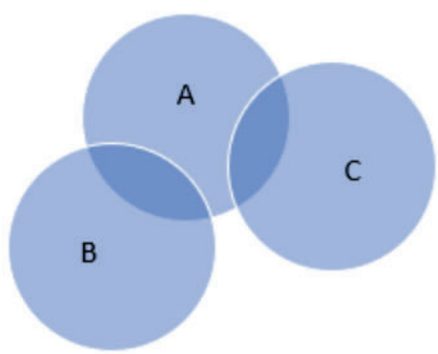
B

RHS:

$B \cap C = \{12\}$ $A \cap (B \cap C) = \{\}$



B



C

Q. 3. Let A = {a, e, I, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}.

Using Venn diagrams, verify the following

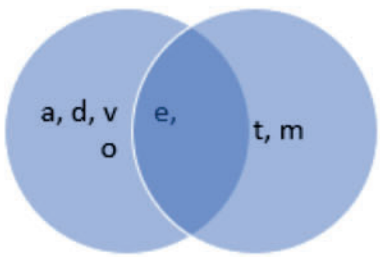
(i) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

(ii) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C).

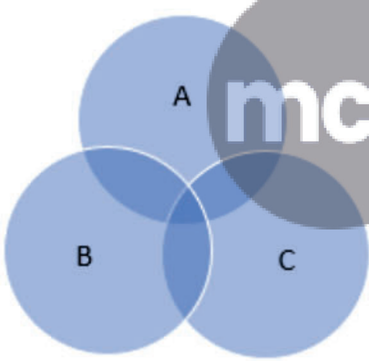
Answer : (i) Given:

A = {a, e, l, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}.

B∩C = {e, o} and A∪(B∩C) = {a, e, l, o, u}



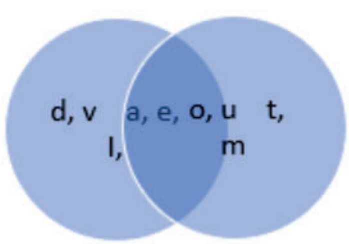
B



C

RHS:

A∪B = {a, d, e, l, o, u, v} and A∪C = {a, e, l, o, u, t, m}



A∪B A∪C

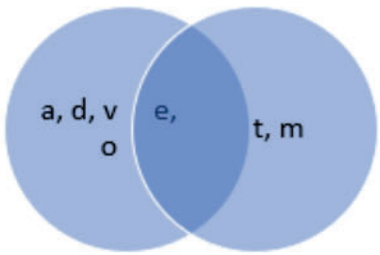
(A ∪ B) ∩ (A ∪ C) = {a, e, l, o, u}

LHS = RHS. [Verified].

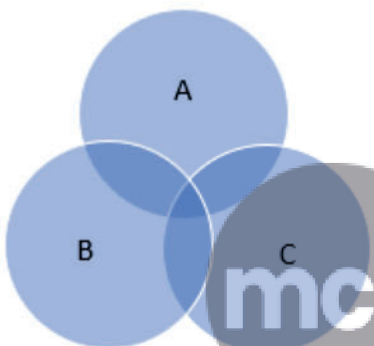
(ii) Given:

A = {a, e, I, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}.

$B \cup C$ = {a, d, v, e, o, t, m} and $A \cap (B \cup C)$ = {a, e, o}



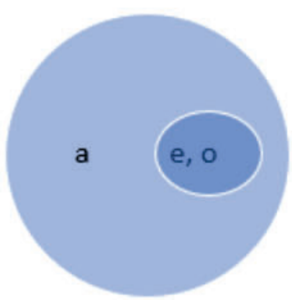
B



C

RHS:

$A \cap B$ = {a, e, o} and $A \cap C$ = {e, o}



A∩B A∩C

(A ∩ B) ∪ (A ∩ C) = {a, e, o}

LHS = RHS. [Verified]

Q. 4. Let A ⊂ B ⊂ U. Exhibit it in a Venn diagram.

Answer : Given: A⊂B⊂U.