Exercise -2(C)

1. **Write three equations of uniformly accelerated motion relating the initial velocity (u), final velocity (v), time (t), acceleration (a) and displacement (S).**

Solution:

The three equations of uniformly accelerated motion relating to initial velocity(u), final velocity(v), time(t), acceleration(a) and displacement(S) are as follows:

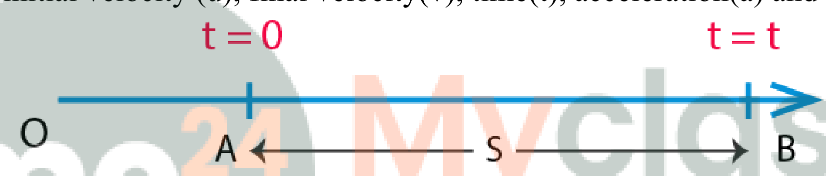
(i) $v = u + at$
(ii) $S = ut + \frac{1}{2} at^2$
(iii) $v^2 = u^2 + 2As$

2. **Derive the following equations for a uniformly accelerated motion:**

(i) $v = u + at$
(ii) $S = ut + \frac{1}{2} at^2$
(iii) $v^2 = u^2 + 2As$
where the symbols have their usual meanings.

Solution:

Let initial velocity (u), final velocity(v), time(t), acceleration(a) and displacement(S)



First law of Motion

(i) We know that acceleration is the rate of change of velocity;

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$a = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{time taken}}$$

$$a = \frac{v-u}{t}$$

$$at = v - u$$

$\therefore v = u + at$ -- **equation (1)**

(ii) We know that distance travelled = average velocity x time

$$S = \frac{\text{Initial velocity} + \text{Final velocity}}{2} \text{ x time}$$

$$S = \frac{u+v}{2} \text{ x time}$$

From equation (1), **$v = u + at$**, substituting the value of v in the above equation;

$$S = \frac{u + u + at}{2} \times \text{time}$$

$$S = \frac{2u + at}{2} \times$$

$\therefore S = ut + \frac{1}{2} at^2$-- **equation (2)**

(iii) We know that distance travelled = average velocity x time

Or $S = \frac{u + v}{2} \times t$

From equation (1), v = u + at or $t = \frac{v - u}{a}$

$$\therefore S = \frac{u + v}{2} \times \frac{v - u}{a} = \frac{v^2 - u^2}{2a}$$

$= v^2 - u^2 = 2aS$

$\mathbf{v^2 = u^2 + 2aS}$

3. Write an expression for the distance S covered in time T by a body which is initially at rest and starts moving with a constant acceleration a.

Solution:

Let,

distance be ‘s’
acceleration ‘a’
time taken ‘t’
initial velocity u=0

We know that distance can be expressed as:

$S = ut + \frac{1}{2} at^2$

Since u=0 here, equation becomes;

$S = \frac{1}{2} at^2$

Multiple choice type:

1. The correct equation of motion is:
(a) v = u + 2aS
(b) v = ut + a
(c) S = ut + ½ at
(d) v = u + at

Solution:

(d) $v = u + at$

Where u is the initial velocity, v is the final velocity, a is the acceleration and t is the time

2. **A car starting from rest accelerates uniformly to acquire a speed 20km/h in 30 min. The distance travelled by car in this time interval will be:**
   (a) 600km
   (b) 5km
   (c) 6km
   (d) 10km

Solution:

(b) 5km

$v = u + at$

$20 = 0 + a(1/2)$

$a = 40km/h$

$S = ut + \frac{1}{2} at^2$

$= 0(1/2) + \frac{1}{2} (40)(1/2)^2$

$= 0 + 5 = 5km$

Numericals:

1. **A body starts from rest with a uniform acceleration of 2m/s. Find the distance covered by the body in 2s.**

Solution:

Given:

$u=0$; $a=2m/s^2$; $t=2s$

$v = u + at$

$= 0 + 2 \times 2$

$v = 4m/s$

distance travelled, $s = ut + \frac{1}{2} at^2$

$= 0(2) + \frac{1}{2} (2) (2)^2$

$= 0 + 4$

$= 4m$

2. **A body starts with an initial velocity of 10m/s and acceleration $5m/s^2$. Find the distance covered by it in 5s.**

Solution:

Given:

$u=10m/s$; $a=5m/s^2$; $t=5s$; $s=?$

distance travelled, $s = ut + \frac{1}{2} at^2$

$= 10 \times 5 + \frac{1}{2} (5) (5)^2$

$= 50 + 62.5$

$= 112.5m$

3. **A vehicle is accelerating on a straight road. Its velocity at any instant is 30km/h, after 2s, it is 33.6km/h and after further 2s, it is 37.2km/h. Find the acceleration of vehicle in m/s^2. Is the acceleration uniform?**

Solution:

We know that acceleration = rate of change of velocity/time taken

Velocity in the first 2 seconds is 33.6km/h-30km/h = 3.6km/h

∴ Acceleration = 3.6/2 = $1.8km/h^2$

Converting km/h^2 to m/s^2

1.8 x 1000/60x60 = $0.5m/s^2$

Velocity after 2 seconds is 37.2km/h-33.6km/h = 3.6 km/h

∴ Acceleration = 3.6/2 = $1.8km/h^2$

Converting km/h^2 to m/s^2

1.8 x 1000/60x60 = $0.5m/s^2$

Comparing acceleration from both instances it can be said that the acceleration is uniform.

4. **A body, initially at rest, starts moving with a constant acceleration $2m/s^2$. Calculate: (i) the velocity acquired and (ii) and the distance travelled in 5s.**

Solution:

Given: $a=2m/s^2$, u=0m/s, v=?; t=5s; s=?

(i) v=u+at

= 0 + 2x5

v = 10m/s

(ii) s =ut+ ½ at^2

= 0(5) + ½ $(2)(5)^2$

= 0 + 25

Distance travelled = 25m

5. **A body initially moving with a velocity 20m/s strikes a target and comes to rest after penetrating a distance 10cm in the target. Calculate the retardation caused by the target.**

Solution:

Given: u=20m/s, v=0, s=10cm or 0.1m

We know that;

$$v^2= u^2 + 2aS$$

$$(0)^2=(20)^2 + 2 (a)(0.1)$$

$$0 = 400 + 0.2a$$

$$-a = 400/0.2$$

$$a = -2000m/s^2$$

∴ Retardation is $2000m/s^2$

6. **A train moving with a velocity of 20m/s is brought to rest by applying brakes in 5s. Calculate the retardation.**

Solution:

Given: u=20m/s; v=0; t=5s, a=?

We know that acceleration = (final velocity – initial velocity) / time taken

= (0-20)/5 = $-4 m/s^2$
∴ Retardation is $4 m/s^2$

7. A train travels with a speed of 60km/h from station A to station B and then comes back with a speed 80km/h from station B to station A. Find: (i) the average speed, and (ii) the average velocity of train.

Solution:

Assume 'd' to be the distance between station A and station B.

(i) Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

Total distance travelled = d + d = 2d

Total time taken = travel time from station A to B + travel time from station B to A

$= \frac{\text{(distance travelled from station A to B)}}{\text{speed}} + \frac{\text{(distance travelled from station B to A)}}{\text{speed}}$

= (d/60) seconds + (d/80) seconds

= 140d/4800

Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{2d}{\frac{140d}{4800}}$ = (2 x 4800)/140

= 68.57km/h

(ii) To find the average velocity, displacement should be known;
As we know, Average velocity = displacement/total time
Since the train starts the journey and end the journey at the same station, the displacement is zero, therefore, the average velocity is also zero.

8. A train is moving with a velocity of 90km/h. It is brought to stop by applying the brakes which produce a retardation of $0.5 m/s^2$. Find: (i) the velocity after 10s, and (ii) the time taken by the train to come to rest.

Solution:

Given:

u=90km/h , expressing in m/s = (90x1000)/60x60=25m/s

v=0m/s; $a=-0.5 m/s^2$

(i) t=10s; We know that v=u+at
substituting values:
v=u+at
= 25+(-1/2)(10)
= 25 – 5
= 20m/s

(ii) v=0; u=25m/s
v = u + at
t = (v-u)/a
= (0-25)/(-0.5)
= 50s

9. A car travels a distance 100m with a constant acceleration and average velocity of 20m/s. The final velocity acquired by the car is 25m/s. Find: (i) the initial velocity and (ii)

acceleration of car.

Solution:

Given: v=25m/s; s=100m; average velocity=20m/s

(i) We know that average velocity = (u+v)/2

20 = (u+25)/2

u = 40-25 = 15m/s

(ii) $v^2 - u^2 = 2aS$

$(25)^2 - (15)^2 = 2 \times a \times 100$

625-225=200a

$a = 2m/s^2$

10. When brakes are applied to a bus, the retardation produced is 25cm/s and the bus takes 20s to stop. Calculate: (i) the initial velocity of bus, and (ii) the distance travelled by the bus during this time.

Solution:

Given: retardation=$-25cm/s^2$. Expressing in m/s^2 it is -0.25 m/s^2; t=20s

(i) We know that v=u+at

u= v-at = (0-(-0.25) x 20 = 5 m/s

(ii) To find the distance travelled;

$v^2 - u^2 = 2aS$

$(0)^2 - (5)^2 = 2 (-0.25)(s)$

s = 25/0.5 = 50m

11. A body moves from rest with a uniform acceleration and travels 270m in 3s. Find the velocity of the body at 10s after the start.

Solution:

Given: u=0m/s; s=270m; t=3s;

We know that;

$S = ut + ½ at^2$

Substituting values;

$270 = 0 + (1/2)a(3)^2$

270 = 9/2a

$a = 60 m/s^2$

after time = 10s, velocity is

v = u + at

= 0 + 60 x 10 = 600m/s

12. A body moving with a constant acceleration travels the distances 3m and 8m respectively in 1s and 2s. Calculate: (i) the initial velocity, and (ii) the acceleration of body.

Solution:

Let distance travelled in time t1=1s be s1=3m

Let distance travelled in time t2=2s be s2=8m

We know that $s = ut + ½ at^2$, substituting for s1 and s2 is:

$S1=ut1 + ½ at1^2$ and $S2=ut2 + ½ at2^2$

Subtracting s1 from s2, we get;

$S2-S1 = u(t2-t1) + 1/2a\,(t2^2 - t1^2)$
$8-3 = u(2-1) + 1/2\,a(4-1)$
$5 = u + 3/2a$
$a = (10-2u)/3$ - equation 1

use this value of 'a' obtained in equation for S1, we get;

$S1=ut1 + ½\,at1^2$
$3=u(1)^2 + ½\,[(10-2u)/3](1)^2$
$3 = u + \frac{10-2u}{6}$
$18= 6u + 10 - 2u$
$u = 2m/s$

using the value of u obtained in equation 1, we get;

$a= (10-2u)/3$
$a=6/3$
$a=2m/s^2$

13. A car travels with a uniform velocity of 25m/s for 5s. The brakes are then applied and the car is uniformly retarded and comes to rest in further 10s. Find: (i) the distance which the car travels before the brakes are applied, (ii) the retardation and (iii) the distance travelled by the car after applying the brakes.

Solution:
Given: u=25m/s; v=0;

(i) The distance which the car travels before brakes are applied is:
Distance = speed x time
= 25 x 5
= 125m

(ii) Retardation = (final velocity-initial velocity)/time taken
= $-(5/2) = -2.5m/s^2$

(iii) To find distance travelled by the car after applying brakes
Time taken by the car to stop after applying brakes =10s
Assume 's' to be the distance the car travels after brakes are applied
We know that $v^2 - u^2 = 2as$
$(0)^2-(25)^2 = 2(-2.5)(s)$
$-625 = -5s$
s=125m

14. A space craft flying in a straight course with a velocity of 75km/s fires its rocket motors for 6.0s. At the end of this time, its speed is 120km/s in the same direction. Find: (i) the space craft's average acceleration while the motors were firing, (ii) the distance travelled by the space craft in the first 10s after the rocket motors were started, the motors having been in action for only 6.0s.

Solution:

Given: u=75km/s; v=120km/s; t=6s

(i) To find acceleration

acceleration = (final velocity-initial velocity)/time taken

= (120-75)/6

$=7.5 m/s^2$

(ii) Distance travelled by the space craft in t=10s

=Distance travelled in first 6s + distance travelled in next 4s

Distance travelled in first 6 seconds: $s1=ut + ½ at^2$

Distance travelled in the next 4 seconds: $s2 = ut + ½ at^2$

$s1 = ut + ½ at^2$

$= (75)(6) + ½ (7.5)(6)^2$

= 450 + 135

= 585km

At the end of 6 seconds, the speed is 120km/s

Distance covered in the next 4s – s2 = speed x time

S2 = 120 x 4 = 480km

∴ the distance covered by the aircraft in 10s = s1+s2 = 585km+480km=1065km

15. A train starts from rest and accelerates uniformly at a rate of $2m/s^2$ for 10s. It then maintains a constant speed for 200s. The brakes are then applied and the train is uniformly retarded and comes to rest in 50s. Find: (i) the maximum velocity reached, (ii) the retardation in the last 50s, (iii) the total distance travelled, and (iv)the average velocity of the train.

Solution:

(i) u=0, t=10s, $a=2m/s^2$

The maximum velocity reached is v=u+at

v = 0 + 2x10

= 20m/s

(ii) v=0m/s; t=50s; u=20m/s

Retardation = -(final velocity-initial velocity)/time taken

= (0-20)/50

$=-0.4m/s^2$

(iii) total distance=distance travelled in 10s+distance travelled in 200s + distance covered in last 50s

Distance travelled in 10s => $s1=ut+1/2at^2$

$=0+1/2(2)(10)^2$

=100m

Distance covered in 200s; s2 => speed x time

= 20 x 200 = 4000m

Distance travelled in last 50s => $s3=ut+1/2at^2$

$=(20)(50) + \frac{1}{2}(-0.4)(50)^2$

s3=500m

Total distance = s1+s2+s3=100m+4000m+500m=4600m

(iv) average velocity = total distance covered/time taken

=4600/260=17.69m/s