Exercise 14(C)

1. Find the equation of a line whose:
y - intercept = 2 and slope = 3.
Solution:

Given,
y - intercept = c = 2 and slope = m = 3.
The line equation is given by: $y = mx + c$
On substituting the values of c and m, we get
$y = 3x + 2$
The above is the required line equation.

2. Find the equation of a line whose:
y - intercept = -1 and inclination = 45°.
Solution:

Given,
y - intercept = c = -1 and inclination = 45°.
So, slope = m = tan 45° = 1
Hence, on substituting the values of c and m in the line equation $y = mx + c$, we get
$y = x - 1$
The above is the required line equation.

3. Find the equation of the line whose slope is -4/3 and which passes through (-3, 4).
Solution:

Given, slope = -4/3
The line passes through the point $(-3, 4) = (x_1, y_1)$
Now, on substituting the values in $y - y_1 = m(x - x_1)$, we have
$y - 4 = -4/3\ (x + 3)$
$3y - 12 = -4x - 12$
$4x + 3y = 0$
Hence, the above is the required line equation.

4. Find the equation of a line which passes through (5, 4) and makes an angle of 60° with the positive direction of the x-axis.
Solution:

The slope of the line, $m = \tan 60^\circ = \sqrt{3}$
And, the line passes through the point $(5, 4) = (x_1, y_1)$
Hence, on substituting the values in $y - y_1 = m(x - x_1)$, we have
$y - 4 = \sqrt{3}\ (x - 5)$
$y - 4 = \sqrt{3}x - 5\sqrt{3}$
$y = \sqrt{3}x + 4 - 5\sqrt{3}$, which is the required line equation.

5. Find the equation of the line passing through:
(i) (0, 1) and (1, 2) (ii) (-1, -4) and (3, 0)
Solution:

(i) Let (0, 1) = (x_1, y_1) and (1, 2) = (x_2, y_2)
So,
Slope of the line = (2 - 1)/ (1 - 0) = 1
Now,
The required line equation is given by,
$y - y_1 = m(x - x_1)$
y - 1 = 1(x - 0)
y - 1 = x
y = x + 1

(ii) Let (-1, -4) = (x_1, y_1) and (3, 0) = (x_2, y_2)
So,
Slope of the line = (0 + 4)/ (3 + 1) = 4/4 = 1
The required line equation is given by,
$y - y_1 = m(x - x_1)$
y + 4 = 1(x + 1)
y + 4 = x + 1
y = x – 3

6. The co-ordinates of two points P and Q are (2, 6) and (-3, 5) respectively. Find:
(i) the gradient of PQ;
(ii) the equation of PQ;
(iii) the co-ordinates of the point where PQ intersects the x-axis.
Solution:

Given,
The co-ordinates of two points P and Q are (2, 6) and (-3, 5) respectively.
(i) Gradient of PQ = (5 - 6)/ (-3 - 2) = -1/-5 = 1/5

(ii) The line equation of PQ is given by
$y - y_1 = m(x - x_1)$
y - 6 = 1/5 (x - 2)
5y - 30 = x - 2
5y = x + 28

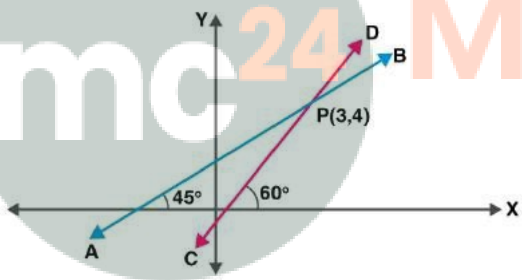
(iii) Let the line PQ intersect the x-axis at point A (x, 0).
So, on putting y = 0 in the line equation of PQ, we get
0 = x + 28
x = -28
Hence, the co-ordinates of the point where PQ intersects the x-axis are A (-28, 0).

7. The co-ordinates of two points A and B are (-3, 4) and (2, -1). Find:
(i) the equation of AB;
(ii) the co-ordinates of the point where the line AB intersects the y-axis.
Solution:

(i) Given, co-ordinates of two points A and B are (-3, 4) and (2, -1).
Slope = (-1 - 4)/ (2 + 3) = -5/5 = -1
The equation of the line AB is given by:
$y - y_1 = m(x - x_1)$
$y + 1 = -1(x - 2)$
$y + 1 = -x + 2$
$x + y = 1$

(ii) Let's consider the line AB intersect the y-axis at point (0, y).
On putting x = 0 in the line equation, we get
$0 + y = 1$
$y = 1$
Thus, the co-ordinates of the point where the line AB intersects the y-axis are (0, 1).

8. The figure given below shows two straight lines AB and CD intersecting each other at point P (3, 4). Find the equation of AB and CD.



Solution:

The slope of line AB = tan 45° = 1
And, the line AB passes through P (3, 4).
Hence, the equation of the line AB is given by
$y - y_1 = m(x - x_1)$
$y - 4 = 1(x - 3)$
$y - 4 = x - 3$
$y = x + 1$

The slope of line CD = tan 60° = √3
And, the line CD passes through P (3, 4).
Hence, the equation of the line CD is given by
$y - y_1 = m(x - x_1)$
$y - 4 = \sqrt{3} (x - 3)$
$y - 4 = \sqrt{3}x - 3\sqrt{3}$

$y = \sqrt{3}x + 4 - 3\sqrt{3}$

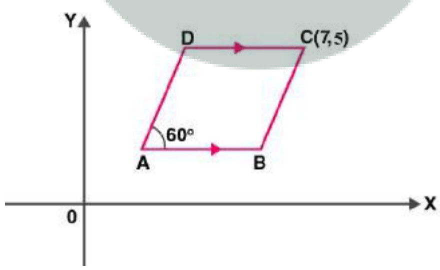
9. In ΔABC, A = (3, 5), B = (7, 8) and C = (1, -10). Find the equation of the median through A.
Solution:

Given,
Vertices of ΔABC, A = (3, 5), B = (7, 8) and C = (1, -10).
Coordinates of the mid-point D of BC = $(x_1 + x_2) / 2, (y_1 + y_2) / 2$

$$= \left(\frac{7+1}{2}, \frac{8+(-10)}{2}\right)$$
$$= \left(\frac{8}{2}, \frac{-2}{2}\right)$$
$$= (4, -1)$$

The slope of AD = $(y_2 - y_1)/ (x_2 - x_1)$
= (-1 - 5)/ (4 - 3)
= -6/1
= -6
Hence, the equation of the median AD through A is given by
$y - y_1 = m (x - x_1)$
y – 5 = -6(x - 3)
y – 5 = -6x + 18
6x + y = 23

10. The following figure shows a parallelogram ABCD whose side AB is parallel to the x-axis, ∠A = 60° and vertex C = (7, 5). Find the equations of BC and CD.



Solution:

Given, ∠A = 60° and vertex C = (7, 5)
As, ABCD is a parallelogram, we have
∠A + ∠B = 180° [corresponding angles]
∠B = 180° - 60° = 120°
Slope of BC = tan 120° = tan (90° + 30°) = cot 30° = $\sqrt{3}$
So, the equation of line BC is given by
$y - y_1 = m(x - x_1)$
$y - 5 = \sqrt{3} (x - 7)$
$y - 5 = \sqrt{3}x - 7\sqrt{3}$

$y = \sqrt{3}x + 5 - 7\sqrt{3}$
As, CD || AB and AB || x-axis
Slope of CD = Slope of AB = 0 [As slope of x-axis is zero]
So, the equation of the line CD is given by
$y - y_1 = m(x - x_1)$
$y - 5 = 0(x - 7)$
$y = 5$

11. Find the equation of the straight line passing through origin and the point of intersection of the lines x + 2y = 7 and x - y = 4.
Solution:

The given line equations are:
$x + 2y = 7$ .... (1)
$x - y = 4$ .... (2)
On solving the above line equations, we can find the point of intersection of the two lines.
So, subtracting (2) from (1), we get
$3y = 3$
$y = 1$
Now,
$x = 4 + y = 4 + 1 = 5$ [From (2)]
It's given that,
The required line passes through (0, 0) and (5, 1).
The slope of the line = (1 - 0)/ (5 - 0) = 1/5
Hence, the required equation of the line is given by
$y – y_1 = m(x – x_1)$
$y – 0 = 1/5(x - 0)$
$5y = x$
$x – 5y = 0$