

Exercise 20(G)

What is the least number of solid metallic spheres, each of 6 cm diameter, that should be melted and recast to form a solid metal cone whose height is 45 cm and diameter is 12 cm?

Solution:

Given,

Diameter of solid metallic sphere = 6 cm

So, its radius = 3 cm

Height of solid metal cone = 45

Diameter of metal cone = 12 cm

So, its radius = 6 cm

Now,

Let the number of solid metallic spheres be 'n'

Volume of 1 sphere = $\frac{4}{3} \pi (3)^3$

Volume of metallic cone = $\frac{1}{3} \pi 6^2 \times 45$

Hence, $n = \text{Volume of metal cone} / \text{Volume of 1 sphere}$

$$n = \frac{(\frac{1}{3} \pi 6^2 \times 45)}{(\frac{4}{3} \pi (3)^3)}$$

$$= \frac{(6 \times 6 \times 45)}{(4 \times 3 \times 3 \times 3)}$$

$$n = 15$$

1. A largest sphere is to be carved out of a right circular cylinder of radius 7 cm and height 14 cm. Find the volume of the sphere. (Answer correct to the nearest integer)

Solution:

Radius of the largest sphere that can be formed inside the cylinder will be equal to the radius of the cylinder.

So, radius of the largest sphere = 7 cm

Volume of sphere = $\frac{4}{3} \pi 7^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= 4312/3$$

$$= 1437 \text{ cm}^3$$

2. A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in identical cones of height 12 cm and diameter 6 cm having a hemi-spherical shape on the top. Find the number of cones required.

Solution:

Given,

Diameter of the cylinder = 12 cm

So, its radius = 6 cm

Height of the cylinder = 15 cm

Diameter of the cone = 6 cm

So, its radius = 3 cm

Height of the cone = 12 cm

Radius of the hemisphere = 3 cm

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Now,

Let the number of cones be 'n'.

$$\text{Volume of the cylinder} = \pi r^2 h = \pi 6^2 \times 15$$

$$\begin{aligned}\text{Volume of an ice-cream cone with ice-cream} &= \text{Volume of cone} + \text{Volume of hemisphere} \\ &= \frac{1}{3} \pi (3)^2 \times 12 + \frac{2}{3} \pi (3)^2 \\ &= 36 \pi + 18 \pi \\ &= 54 \pi \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Number of cones} &= \text{Volume of cylinder} / \text{Volume of ice-cream cone} \\ &= \pi 6^2 \times 15 / 54 \pi \\ &= (36 \times 15) / 54 \\ &= 10\end{aligned}$$

Therefore, the number of cones required = 10

3. A solid is in the form of a cone standing on a hemisphere with both their radii being equal to 8 cm and the height of cone is equal to its radius. Find in terms of π , the volume of the solid.

Solution:

Given,

Radius of both cone and hemisphere = 8 cm

And, height = 8 cm

Hence,

$$\begin{aligned}\text{Volume of the solid} &= \text{Volume of cone} + \text{volume of hemisphere} \\ &= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \\ &= \frac{1}{3} \pi (8)^2 (8) + \frac{2}{3} \pi (8)^3 \\ &= \pi (8)^3 \\ &= 512 \pi \text{ cm}^3\end{aligned}$$

4. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of wire.

Solution:

Given,

Diameter of the sphere = 6 cm

So, its radius = 3 cm

$$\text{And, volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3 = \frac{792}{7} \text{ cm}^3 \dots\dots (i)$$

Diameter of cylindrical wire = 0.2 cm

So, the radius of the wire = $0.2/2 = 0.1 = 1/10$ cm

Now, let length of wire = h

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times h \text{ cm}^3 = \frac{22h}{700} \text{ cm}^3 \dots\dots (ii)$$

According to the question, equating (i) and (ii) we have

$$\frac{22h}{700} = \frac{792}{7}$$

$$h = \frac{792}{7} \times \frac{700}{22}$$

$$h = 3600 \text{ cm} = 36 \text{ m}$$

Therefore, length of the wire = 36 m

5. Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the

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cube.

Solution:

Let edge of the cube = a

Then, volume of the cube = a x a x a = a³

The sphere that exactly fits in the cube will have radius 'a/2'

So,

Volume of sphere = $\frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (a/2)^3 = \frac{4}{3} \times \frac{22}{7} \times a^3/8 = \frac{11}{21} a^3$

Now,

Volume of cube: Volume of sphere

= a³ : $\frac{11}{21} a^3$

= 1 : $\frac{11}{21}$

= 21 : 11

6. An iron pole consisting of a cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that 1 cm³ of iron has 8 gm of mass (approx). (Take $\pi = 355/113$)

Solution:

Given,

Radius of the base of poles (r) = 6 cm

Height of the cylindrical part (h₁) = 110 cm

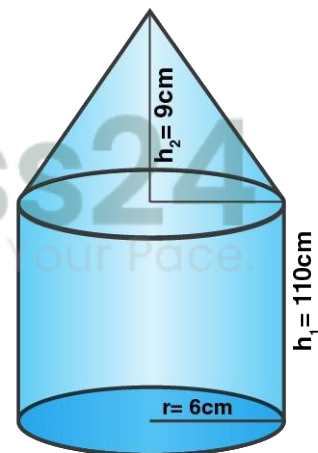
Height of the conical part (h₂) = 9 cm

Now,

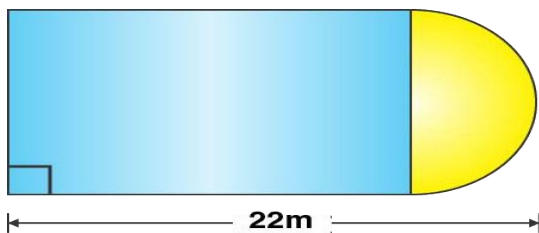
Total volume of the iron pole = $\pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2 = \pi r^2 (h_1 + \frac{1}{3} h_2)$
 = $\frac{355}{113} \times 6 \times 6 (110 + \frac{1}{3} \times 9)$
 = $\frac{355}{113} \times 36 \times 113$
 = 12780 cm³

Weight of 1 cm³ = 8 gm (Given)

Thus, the total weight = 12780 x 8 = 102240 gm = 102.24 kg



7. In the following diagram a rectangular platform with a semicircular end on one side is 22 meters long from one end to the other end. If the length of the half circumference is 11 meters, find the cost of constructing the platform, 1.5 meters high at the rate of Rs 4 per cubic meters.



Solution:

Given,

Length of the platform = 22 m

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Circumference of semi-circle (c) = 11 m

So, its radius = $(c \times 2) / (2 \times \pi) = (11 \times 7) / 22 = 7/2$ m

Thus, the breadth of the part = $7/2 \times 2 = 7$ m

And, length = $22 - 7/2 = 37/2 = 18.5$ m

Now, area of platform = $l \times b + \frac{1}{2} \pi r^2$
 $= 18.5 \times 7 + \frac{1}{2} \times 22/7 \times 7/2 \times 7/2 \text{ m}^2$
 $= 129.6 + 77/4 \text{ m}^2$
 $= 148.75 \text{ m}^2$

Also given, height of the platform = 1.5 m

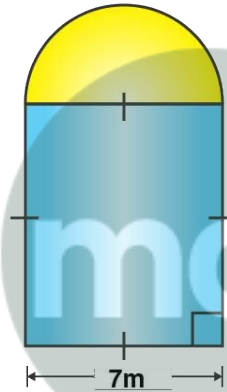
So, the volume = $148.75 \times 1.5 = 223.125 \text{ m}^3$

Rate of construction = Rs 4 per m^3

Therefore,

Total expenditure = Rs 4 x 223.125 = Rs 892.50

8. The cross-section of a tunnel is a square of side 7 m surmounted by a semicircle as shown in the following figure.



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The tunnel is 80 m long. Calculate:

(i) its volume

(ii) the surface area of the tunnel (excluding the floor) and

(iii) its floor area

Solution:

Given,

Side of square (a) = 7m

And, the radius of semi-circle = $7/2$ m

Length of the tunnel = 80 m

Area of cross section of the front part = $a^2 + \frac{1}{2} \pi r^2$
 $= 7 \times 7 + \frac{1}{2} \times 22/7 \times 7/2 \times 7/2$
 $= 49 + 77/4 \text{ m}^2$
 $= (196 + 77) / 4$
 $= 273/4 \text{ m}^2$

(i) Thus, the volume of the tunnel = area of cross section x length of the tunnel

$$= 273/4 \times 80$$

$$= 5460 \text{ m}^3$$

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(ii) Circumference of the front of tunnel (excluding the floor) = $2a + \frac{1}{2} \times 2\pi r$
= $2 \times 7 + \frac{22}{7} \times \frac{7}{2}$
= $14 + 11 = 25$ cm

Hence, the surface area of the inner part of the tunnel = $25 \times 80 = 2000$ m²

(iii) Area of floor = $l \times b = 7 \times 80 = 560$ m²

9. A cylindrical water tank of diameter 2.8m and height 4.2m is being fed by a pipe of diameter 7 cm through which water flows at the rate of 4m/s. Calculate, in minutes, the time it takes to fill the tank.

Solution:

Given,

Diameter of cylindrical tank = 2.8 m

So, its radius = 1.4 m

Height = 4.2 m

Volume of water filled in it = $\pi r^2 h$
= $\frac{22}{7} \times 1.4 \times 1.4 \times 4.2$ m³
= $181.104/7$ m³
= 25.872 m³ (i)

Diameter of the pipe = 7 cm

So, the radius (r) = $7/2$ cm

Now, let the length of water in the pipe = h_1

Volume = $\pi r^2 h_1$
= $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h_1$
= $77/2 h_1$ cm³ (ii)

According to the question, equating (i) and (ii) we have

$77/2 h_1$ cm³ = 25.872×100^3 cm³ (Converted from m³ to cm³)

$$h_1 = \frac{25.872 \times 100^3 \times 2}{77} \text{ cm}$$

$$h_1 = \frac{25.872 \times 100^3 \times 2}{77 \times 100} \text{ m}$$

$$h_1 = 0.672 \times 100^2 \text{ m}$$

$$h_1 = 6720 \text{ m}$$

Thus, the time taken at the speed of 4 m per second = $6720/(4 \times 60)$ min = 28 min