EXERCISE 23.19

Find the equation of a straight line through the point of intersection of the lines 4x – 3y = 0 and 2x – 5y + 3 = 0 and parallel to 4x + 5 y + 6 = 0.

Solution:

Given:

Lines 4x – 3y = 0 and 2x – 5y + 3 = 0 and parallel to 4x + 5 y + 6 = 0

The equation of the straight line passing through the points of intersection of 4x − 3y = 0 and 2x − 5y + 3 = 0 is given below:

4x − 3y + λ (2x − 5y + 3) = 0

(4 + 2λ)x + (− 3 − 5λ)y + 3λ = 0

$$y = \left(\frac{4+2\lambda}{3+5\lambda}\right)x + \frac{3\lambda}{(3+5\lambda)}$$

The required line is parallel to 4x + 5y + 6 = 0 or, y = -4x/5 – 6/5

$$\frac{4+2\lambda}{3+5\lambda} = -\frac{4}{5}$$

$$\frac{4+2\lambda}{3+5\lambda} = -\frac{4}{5}$$

λ = -16/15

∴ The required equation is

$$\left(4 - \frac{32}{15}\right)x - \left(3 - \frac{80}{15}\right)y - \frac{48}{15} = 0$$

28x + 35y – 48 = 0

1. Find the equation of a straight line passing through the point of intersection of x + 2y + 3 = 0 and 3x + 4y + 7 = 0 and perpendicular to the straight line x – y + 9 = 0.

Solution:

Given:

x + 2y + 3 = 0 and 3x + 4y + 7 = 0

The equation of the straight line passing through the points of intersection of x + 2y + 3 = 0 and 3x + 4y + 7 = 0 is

x + 2y + 3 + λ(3x + 4y + 7) = 0

(1 + 3λ)x + (2 + 4λ)y + 3 + 7λ = 0

$$y = -\left(\frac{1+3\lambda}{2+4\lambda}\right)x - \left(\frac{3+7\lambda}{2+4\lambda}\right)$$

The required line is perpendicular to x − y + 9 = 0 or, y = x + 9

2. Find the equation of the line passing through the point of intersection of 2x – 7y + 11 = 0 and x + 3y – 8 = 0 and is parallel to (i) x = axis (ii) y-axis.

Solution:

Given:

The equations, $2x - 7y + 11 = 0$ and $x + 3y - 8 = 0$

The equation of the straight line passing through the points of intersection of $2x - 7y + 11 = 0$ and $x + 3y - 8 = 0$ is given below:

$2x - 7y + 11 + \lambda(x + 3y - 8) = 0$

$(2 + \lambda)x + (-7 + 3\lambda)y + 11 - 8\lambda = 0$

(i) The required line is parallel to the x-axis. So, the coefficient of x should be zero.

$2 + \lambda = 0$

$\lambda = -2$

Now, substitute the value of λ back in equation, we get

$0 + (-7 - 6)y + 11 + 16 = 0$

$13y - 27 = 0$

∴ The equation of the required line is $13y - 27 = 0$

(ii) The required line is parallel to the y-axis. So, the coefficient of y should be zero.

$-7 + 3\lambda = 0$

$\lambda = 7/3$

Now, substitute the value of λ back in equation, we get

$(2 + 7/3)x + 0 + 11 - 8(7/3) = 0$

$13x - 23 = 0$

∴ The equation of the required line is $13x - 23 = 0$

3. Find the equation of the straight line passing through the point of intersection of 2x + 3y + 1 = 0 and 3x – 5y – 5 = 0 and equally inclined to the axes.

Solution:

Given:

The equations, $2x + 3y + 1 = 0$ and $3x - 5y - 5 = 0$

The equation of the straight line passing through the points of intersection of $2x + 3y + 1 = 0$ and $3x - 5y - 5 = 0$ is

$2x + 3y + 1 + \lambda(3x - 5y - 5) = 0$

$(2 + 3\lambda)x + (3 - 5\lambda)y + 1 - 5\lambda = 0$

$y = -[(2 + 3\lambda) / (3 - 5\lambda)] - [(1 - 5\lambda) / (3 - 5\lambda)]$

The required line is equally inclined to the axes. So, the slope of the required line is either 1 or − 1.

So,
$-[(2 + 3\lambda) / (3 - 5\lambda)] = 1$ and $-[(2 + 3\lambda) / (3 - 5\lambda)] = -1$
$-2 - 3\lambda = 3 - 5\lambda$ and $2 + 3\lambda = 3 - 5\lambda$
$\lambda = 5/2$ and $1/8$

Now, substitute the values of λ in $(2 + 3\lambda)x + (3 - 5\lambda)y + 1 - 5\lambda = 0$, we get the equations of the required lines as:
$(2 + 15/2)x + (3 - 25/2)y + 1 - 25/2 = 0$ and $(2 + 3/8)x + (3 - 5/8)y + 1 - 5/8 = 0$
$19x - 19y - 23 = 0$ and $19x + 19y + 3 = 0$
∴ The required equation is $19x - 19y - 23 = 0$ and $19x + 19y + 3 = 0$

4. Find the equation of the straight line drawn through the point of intersection of the lines x + y = 4 and 2x – 3y = 1 and perpendicular to the line cutting off intercepts 5, 6 on the axes.
Solution:
Given:
The lines $x + y = 4$ and $2x - 3y = 1$
The equation of the straight line passing through the point of intersection of $x + y = 4$ and $2x - 3y = 1$ is
$x + y - 4 + \lambda(2x - 3y - 1) = 0$
$(1 + 2\lambda)x + (1 - 3\lambda)y - 4 - \lambda = 0$ … (1)
$y = -[(1 + 2\lambda) / (1 - 3\lambda)]x + [(4 + \lambda) / (1 - 3\lambda)]$
The equation of the line with intercepts 5 and 6 on the axis is
$x/5 + y/6 = 1$ …. (2)
So, the slope of this line is $-6/5$
The lines (1) and (2) are perpendicular.

∴ $-6/5 \times [(-1+2\lambda) / (1 - 3\lambda)] = -1$
$\lambda = 11/3$
Now, substitute the values of λ in (1), we get the equation of the required line.
$(1 + 2(11/3))x + (1 - 3(11/3))y - 4 - 11/3 = 0$
$(1 + 22/3)x + (1 - 11)y - 4 - 11/3 = 0$
$25x - 30y - 23 = 0$
∴ The required equation is $25x - 30y - 23 = 0$