

EXERCISE 23.14

Find the values of α so that the point $P(\alpha^2, \alpha)$ lies inside or on the triangle formed by the lines $x - 5y + 6 = 0$, $x - 3y + 2 = 0$ and $x - 2y - 3 = 0$.

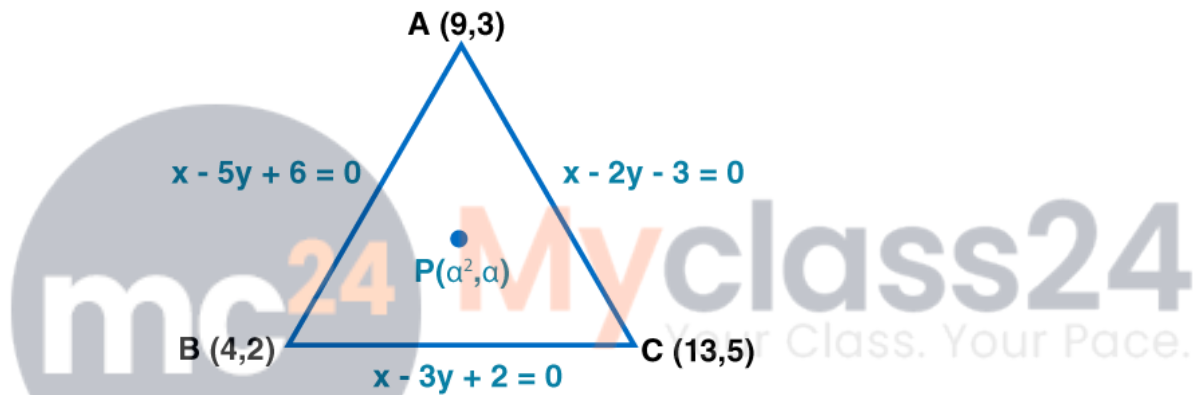
Solution:

Given:

$x - 5y + 6 = 0$, $x - 3y + 2 = 0$ and $x - 2y - 3 = 0$ forming a triangle and point $P(\alpha^2, \alpha)$ lies inside or on the triangle

Let ABC be the triangle of sides AB, BC and CA whose equations are $x - 5y + 6 = 0$, $x - 3y + 2 = 0$ and $x - 2y - 3 = 0$, respectively.

On solving the equations, we get A (9, 3), B (4, 2) and C (13, 5) as the coordinates of the vertices.



It is given that point $P(\alpha^2, \alpha)$ lies either inside or on the triangle. The three conditions are given below.

- (i) A and P must lie on the same side of BC.
- (ii) B and P must lie on the same side of AC.
- (iii) C and P must lie on the same side of AB.

If A and P lie on the same side of BC, then

$$(9 - 9 + 2)(\alpha^2 - 3\alpha + 2) \geq 0$$

$$(\alpha - 2)(\alpha - 1) \geq 0$$

$$\alpha \in (-\infty, 1] \cup [2, \infty) \dots (1)$$

If B and P lie on the same side of AC, then

$$(4 - 4 - 3)(\alpha^2 - 2\alpha - 3) \geq 0$$

$$(\alpha - 3)(\alpha + 1) \leq 0$$

$$\alpha \in [-1, 3] \dots (2)$$

If C and P lie on the same side of AB, then

$$(13 - 25 + 6)(\alpha^2 - 5\alpha + 6) \geq 0$$

$$(\alpha - 3)(\alpha - 2) \leq 0$$

$$\alpha \in [2, 3] \dots (3)$$

From equations (1), (2) and (3), we get

$$\alpha \in [2, 3]$$

$$\therefore \alpha \in [2, 3]$$

1. Find the values of the parameter a so that the point (a, 2) is an interior point of the triangle formed by the lines $x + y - 4 = 0$, $3x - 7y - 8 = 0$ and $4x - y - 31 = 0$.

Solutions:

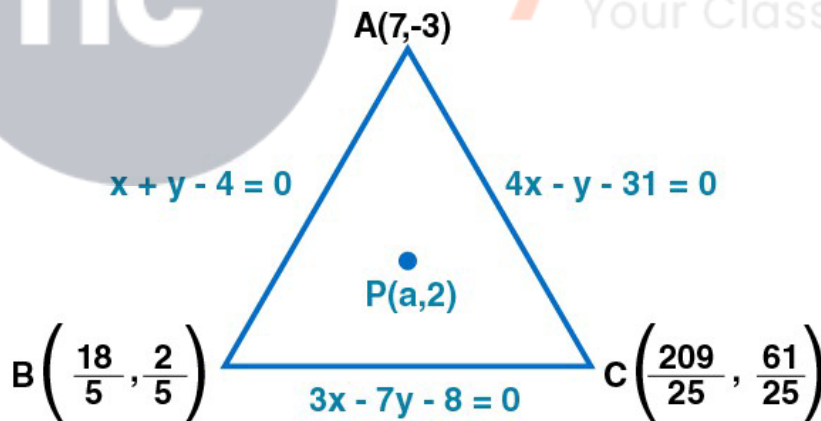
Given:

$x + y - 4 = 0$, $3x - 7y - 8 = 0$ and $4x - y - 31 = 0$ forming a triangle and point (a, 2) is an interior point of the triangle

Let ABC be the triangle of sides AB, BC and CA whose equations are $x + y - 4 = 0$, $3x - 7y - 8 = 0$ and $4x - y - 31 = 0$, respectively.

On solving them, we get A (7, -3), B (18/5, 2/5) and C (209/25, 61/25) as the coordinates of the vertices.

Let P (a, 2) be the given point.



It is given that point P (a, 2) lies inside the triangle. So, we have the following:

- (i) A and P must lie on the same side of BC.
- (ii) B and P must lie on the same side of AC.
- (iii) C and P must lie on the same side of AB.

Thus, if A and P lie on the same side of BC, then

$$21 + 21 - 8 - 3a - 14 - 8 > 0$$

$$a > 22/3 \dots (1)$$

If B and P lie on the same side of AC, then

$$4 \times \frac{18}{5} - \frac{2}{5} - 31 - 4a - 2 - 31 > 0$$

$$a < 33/4 \dots (2)$$

If C and P lie on the same side of AB, then

$$\frac{209}{25} + \frac{61}{25} - 4 - a + 2 - 4 > 0$$

$$\frac{34}{5} - 4 - a + 2 - 4 > 0$$

$$a > 2 \dots (3)$$

From (1), (2) and (3), we get:

$$A \in (22/3, 33/4)$$

$$\therefore A \in (22/3, 33/4)$$

2. Determine whether the point (-3, 2) lies inside or outside the triangle whose sides are given by the equations $x + y - 4 = 0$, $3x - 7y + 8 = 0$, $4x - y - 31 = 0$.

Solution:

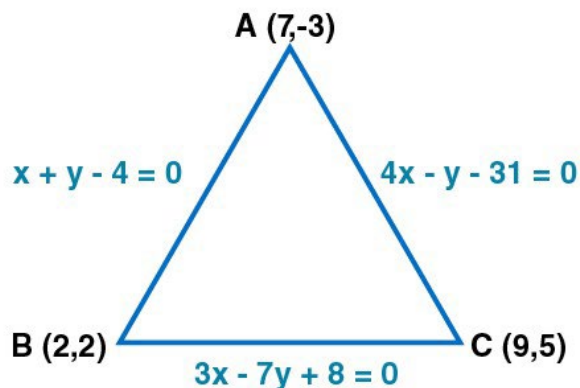
Given:

$x + y - 4 = 0$, $3x - 7y + 8 = 0$, $4x - y - 31 = 0$ forming a triangle and point (-3, 2)

Let ABC be the triangle of sides AB, BC and CA, whose equations $x + y - 4 = 0$, $3x - 7y + 8 = 0$ and $4x - y - 31 = 0$, respectively.

On solving them, we get A (7, -3), B (2, 2) and C (9, 5) as the coordinates of the vertices.

Let P (-3, 2) be the given point.



The given point P (-3, 2) will lie inside the triangle ABC, if

(i) A and P lies on the same side of BC

- (ii) B and P lies on the same side of AC
- (iii) C and P lies on the same side of AB

Thus, if A and P lie on the same side of BC, then

$$21 + 21 + 8 - 9 - 14 + 8 > 0$$

$$50 - 15 > 0$$

$$-750 > 0,$$

This is false

∴ The point $(-3, 2)$ lies outside triangle ABC.



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