NCERT Solutions for Class-XI Maths

Chapter-12 Exercise-12.2

NCERT Math Class 11

1. Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1)

(ii) (–3, 7, 2) and (2, 4, –1)

(iii) (–1, 3, –4) and (1, –3, 4)

(iv) (2, –1, 3) and (–2, 1, 3)

1. (i) (2, 3, 5) and (4, 3, 1)

Explanation:-

Let P be (2, 3, 5) and Q be (4, 3, 1)

Distance PQ $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

$x_1 = 2, y_1 = 3, z_1 = 5$

$x_2 = 4, y_2 = 3, z_2 = 1$

Distance PQ $=\sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$

$=\sqrt{(2)^2+(0)^2+(-4)^2}$

$=\sqrt{4+0+16}$

$=\sqrt{20}$

$=2\sqrt{5}$

Thus, the required distance is $2\sqrt{5}$ units.

(ii) (–3, 7, 2) and (2, 4, –1)

Explanation:-

Let P be (-3, 7, 2) and Q be (2, 4, -1)

Distance PQ $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

$x_1 = -3, y_1 = 7, z_1 = 2$

$x_2 = 2, y_2 = 4, z_2 = -1$

Distance PQ $=\sqrt{(2-(-3))^2+(4-7)^2+(-1-2)^2}$

$$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$$

$$= \sqrt{25 + 9 + 9}$$

$$= \sqrt{43}$$

Thus, the required distance is $\sqrt{43}$ units.

(iii) (–1, 3, – 4) and (1, –3, 4)

Explanation:-

Let P be (-1, 3, -4) and Q be (1, -3, 4)

Distance PQ $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$x_1 = -1, y_1 = 3, z_1 = -4$

$x_2 = 1, y_2 = -3, z_2 = 4$

Distance PQ $= \sqrt{(1-(-1))^2 + (-3-3)^2 + (4-(-4))^2}$

$$= \sqrt{(2)^2 + (-6)^2 + (8)^2}$$

$$= \sqrt{4 + 36 + 64}$$

$$= \sqrt{104}$$

$$= 2\sqrt{26}$$

Thus, the required distance is $2\sqrt{26}$ units.

(iv) (2, –1, 3) and (–2, 1, 3).

Explanation:-

Let P be (2, -1, 3) and Q be (-2, 1, 3)

Distance PQ $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$x_1 = 2, y_1 = -1, z_1 = 3$

$x_2 = -2, y_2 = 1, z_2 = 3$

Distance PQ $= \sqrt{(-2-2)^2 + (1-(-1))^2 + (3-3)^2}$

$$= \sqrt{(-4)^2 + (2)^2 + (0)^2}$$

$=\sqrt{16+4+0}$

$=\sqrt{20}$

$=2\sqrt{5}$

Thus, the required distance is $2\sqrt{5}$ units.

2. Show that the points $(-2,3,5),(1,2,3)$ and $(7,0,-1)$ are collinear.

2. Let points $(-2,3,5),(1,2,3)$, and $(7,0,-1)$ be denoted by P,Q, and R respectively. Points P,Q, and R are collinear if they lie on a line.

$PQ=\sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$

$=\sqrt{(3)^2+(-1)^2+(-2)^2}$

$=\sqrt{9+1+4}$

$=\sqrt{14}$

$QR=\sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$

$=\sqrt{(6)^2+(-2)^2+(-4)^2}$

$=\sqrt{36+4+16}$

$=\sqrt{56}$

$=2\sqrt{14}$

$PR=\sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$

$=\sqrt{(9)^2+(-3)^2+(-6)^2}$

$=\sqrt{81+9+36}$

$=\sqrt{126}$

$=3\sqrt{14}$

Here, $\mathrm{PQ}+\mathrm{QR}=\sqrt{14}+2\sqrt{14}=3\sqrt{14}=\mathrm{PR}$

Hence, points $\mathrm{P}(-2,3,5),\mathrm{Q}(1,2,3)$, and $\mathrm{R}(7,0,-1)$ are collinear.

3. Verify the following:

(i) (0, 7, –10), (1, 6, – 6) and (4, 9, – 6) are the vertices of an isosceles triangle.

3. Explanation:-

Let points be

P(0, 7, –10), Q(1, 6, – 6) & R(4, 9, – 6)

If any 2 sides are equal, it will be an isosceles triangle

Calculating PQ

P ≡ (0, 7, -10) and Q ≡ (1, 6, -6)

Distance PQ $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

$x_1 = 0$, $y_1 = 7$, $z_1 = -10$

$x_2 = 1$, $y_2 = 6$, $z_2 = -6$

Distance PQ $=\sqrt{(1-0)^2+(6-7)^2+(-6-(-10))^2}$

$=\sqrt{(1)^2+(-1)^2+(4)^2}$

$=\sqrt{1+1+16}$

$=\sqrt{18}$

Calculating QR

Q ≡ (1, 6, -6) and R ≡ (4, 9, -6)

Distance QR $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

$x_1 = 1$, $y_1 = 6$, $z_1 = -6$

$x_2 = 4$, $y_2 = 9$, $z_2 = -6$

Distance QR $=\sqrt{(4-1)^2+(9-6)^2+(-6-(-6))^2}$

$=\sqrt{(3)^2+(3)^2+(-6+6)^2}$

$=\sqrt{9+9+0}$

$=\sqrt{18}$

Since PQ = QR

∴ 2 sides are equal

Hence PQR is an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6) and (– 4, 9, 6) are the vertices of a right angled triangle.
Explanation:-
Let points be
P(0, 7, 10), Q(-1, 6, 6) & R(-4, 9, 6)

Calculating PQ

P ≡ (0, 7, 10) and Q ≡ (-1, 6, 6)

Distance PQ $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,
$x_1 = 0$, $y_1 = 7$, $z_1 = 10$
$x_2 = -1$, $y_2 = 6$, $z_2 = 6$

Distance PQ $=\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$
$=\sqrt{(-1)^2+(-1)^2+(-4)^2}$
$=\sqrt{1+1+16}$
$=\sqrt{18}$

Calculating QR

Q ≡ (-1, 6, 6) and R ≡ (-4, 9, 6)

Distance QR $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,
$x_1 = -1$, $y_1 = 6$, $z_1 = 6$
$x_2 = -4$, $y_2 = 9$, $z_2 = 6$

Distance QR $=\sqrt{(-4-(-1))^2+(9-6)^2+(6-6)^2}$
$=\sqrt{(-3)^2+(3)^2+(0)^2}$
$=\sqrt{9+9+0}$
$=\sqrt{18}$

Calculating PR

P ≡ (0, 7, 10) and R ≡ (-4, 9, 6)

Distance PR $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$x_1 = 0, y_1 = 7, z_1 = 10$

$x_2 = -4, y_2 = 9, z_2 = 6$

Distance PR $= \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2}$

$= \sqrt{(-4)^2 + (2)^2 + (-4)^2}$

$= \sqrt{16+4+16}$

$= \sqrt{36}$

Now,

$PQ^2 + QR^2 = 18 + 18 = 36 = PR^2$

Hence, According to converse of pythagoras theorem,
the given vertices P, Q & R are the vertices of a right-angled triangle at Q.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.
Explanation:-
Let A(–1, 2, 1), B(1, –2, 5), C(4, –7, 8) & D(2, –3, 4)

ABCD can be vertices of parallelogram only if opposite sides are equal.
i.e. AB = CD & BC = AD

Calculating AB

A ≡ (-1, 2, 1) and B ≡ (1, -2, 5)

Distance AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$x_1 = -1, y_1 = 2, z_1 = 1$

$x_2 = 1, y_2 = -2, z_2 = 5$

Distance AB $= \sqrt{(1-(-1))^2 + (-2-2)^2 + (5-1)^2}$

$$= \sqrt{(2)^2 + (-4)^2 + (4)^2}$$

$$= \sqrt{4 + 16 + 16}$$

$$= \sqrt{36}$$

$$= 6$$

<u>Calculating BC</u>

B ≡ (1, -2, 5) and C ≡ (4, -7, 8)

Distance BC $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$x_1 = 1$, $y_1 = -2$, $z_1 = 5$

$x_2 = 4$, $y_2 = -7$, $z_2 = 8$

Distance BC $= \sqrt{(4-1)^2 + (-7-(-2))^2 + (8-5)^2}$

$$= \sqrt{(3)^2 + (-5)^2 + (3)^2}$$

$$= \sqrt{9 + 25 + 9}$$

$$= \sqrt{43}$$

<u>Calculating CD</u>

C ≡ (4, -7, 8) and D ≡ (2, -3, 4)

Distance CD $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$x_1 = 4$, $y_1 = -7$, $z_1 = 8$

$x_2 = 2$, $y_2 = -3$, $z_2 = 4$

Distance CD $= \sqrt{(2-4)^2 + (-3-(-7))^2 + (4-8)^2}$

$$= \sqrt{(-2)^2 + (4)^2 + (-4)^2}$$

$$= \sqrt{4 + 16 + 16}$$

$$= \sqrt{36}$$

$= 6$

Calculating DA

D ≡ (2, -3, 4) and A ≡ (-1, 2, 1)

Distance DA $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$x_1 = 2, y_1 = -3, z_1 = 4$

$x_2 = -1, y_2 = 2, z_2 = 1$

Distance DA $= \sqrt{(-1-2)^2 + (2-(-3))^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (5)^2 + (-3)^2}$

$= \sqrt{9+25+9}$

$= \sqrt{43}$

Since AB = CD & BC = DA

So, In ABCD both pairs of opposite sides are equal.

Thus, ABCD is a parallelogram.

4. Find the equation of the set of points which are equidistant from the points $(1,2,3)$ and $(3,2,-1)$

4. Let $P(x, y, z)$ be the point that is equidistant from points $A(1,2,3)$ and $B(3,2,-1)$.

Accordingly, PA $=$ PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

5. Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (– 4, 0, 0) is equal to 10.

5. Let A (4, 0, 0) & B (-4, 0, 0)

Let the coordinates of point P be (x, y, z)

Calculating PA

P ≡ (x, y, z) and A ≡ (4, 0, 0)

Distance PA $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$x_1 = x$, $y_1 = y$, $z_1 = z$

$x_2 = 4$, $y_2 = 0$, $z_2 = 0$

Distance PA $= \sqrt{(4 - x)^2 + (0 - y)^2 + (0 - z)^2}$

Calculating PB

P ≡ (x, y, z) and B ≡ (-4, 0, 0)

Distance PB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$x_1 = x$, $y_1 = y$, $z_1 = z$

$x_2 = -4$, $y_2 = 0$, $z_2 = 0$

Distance PB $= \sqrt{(-4 - x)^2 + (0 - y)^2 + (0 - z)^2}$

Given that -

PA + PB = 10

⇒ PA = 10 - PB

Squaring both sides, we get-

$PA^2 = (10-PB)^2$

$\Rightarrow PA^2 = 100 + PB^2 - 20\, PB$

$\Rightarrow (4-x)^2 + (0-y)^2 + (0-z)^2$

$= 100 + (-4-x)^2 + (0-y)^2 + (0-z)^2 - 20\, PB$

$\Rightarrow (16+x^2-8x) + (y^2) + (z^2)$

$= 100 + (16+x^2+8x) + (y^2) + (z^2) - 20\ PB$

$\Rightarrow 20\ PB = 16x+100$

$\Rightarrow 5\ PB = (4x+25)$

Squaring both sides again, we get-

$\Rightarrow 25\ PB^2 = 16x^2+200x+625$

$\Rightarrow 25\ [(-4-x)^2 + (0-y)^2 + (0-z)^2] = 16x^2+200x+625$

$\Rightarrow 25\ [x^2+y^2+z^2+8x+16] = 16x^2+200x+625$

$\Rightarrow 25x^2+25y^2+25z^2+200x+400 = 16x^2+200x+625$

$\Rightarrow 9x^2+25y^2+25z^2-225 = 0$

This is the required equation.