

## Exercise 25.1

### 1. Solution:

Given,

$$\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k} \text{ and } \vec{b} = -\hat{i} + 3\hat{k}$$

We know that, if

$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k} \text{ and } \vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}, \text{ then}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

So, here

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix} \\ &= \hat{i}(9 - 0) - \hat{j}(3 - 2) + \hat{k}(0 + 3) \end{aligned}$$

$$\therefore \vec{a} \times \vec{b} = 9\hat{i} - \hat{j} + 3\hat{k}$$

Now,

$$\begin{aligned} |\vec{a} \times \vec{b}| &= \sqrt{(9)^2 + (-1)^2 + (3)^2} \\ &= \sqrt{81 + 1 + 9} \end{aligned}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{91}$$

### 2.

#### (i) Solution:

$$\vec{a} = 3\hat{i} + 4\hat{j} \text{ and } \vec{b} = \hat{i} + \hat{j} + \hat{k}$$

We know that, if

$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k} \text{ and } \vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}, \text{ then}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

So, here

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix} \\ &= \hat{i}(4 - 0) - \hat{j}(3 - 0) + \hat{k}(3 - 4) \end{aligned}$$

$$\therefore \vec{a} \times \vec{b} = 4\hat{i} - 3\hat{j} - \hat{k}$$

Now,

$$|\vec{a} \times \vec{b}| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{16 + 9 + 1}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{26}$$

**(ii) Solution:**

$$\vec{a} = 2\hat{i} + \hat{j} \text{ and } \vec{b} = \hat{i} + \hat{j} + \hat{k}$$

We know that, if

$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k} \text{ and } \vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}, \text{ then}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

So, here

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(2-0)$$

$$\therefore \vec{a} \times \vec{b} = -\hat{i} - \hat{j} + 2\hat{k}$$

Now,

$$|\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{1+1+4}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{6}$$

**3.**

**(i) Solution:**

$$\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$$

Now,

A vector perpendicular to both  $\vec{a}$  and  $\vec{b} = \vec{a} \times \vec{b}$ .

$$\vec{a} \times \vec{b} = \vec{c} \text{ (say)} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$\vec{c} = \hat{i}(2-3) - \hat{j}(-8+6) + \hat{k}(4-2)$$

$$\vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}$$

We know that,

$\vec{c}$  is a vector perpendicular to both  $\vec{a}$  and  $\vec{b}$ .

So, it's unit vector is given by

$$\begin{aligned}\hat{c} &= \frac{\vec{c}}{|\vec{c}|} \\ &= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{(-1)^2 + (2)^2 + (2)^2}} \\ &= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1+4+4}} \\ &= \frac{1}{3}(-\hat{i} + 2\hat{j} + 2\hat{k})\end{aligned}$$

Therefore, unit vector perpendicular to both  $\vec{a}$  and  $\vec{b} = \frac{1}{3}(-\hat{i} + 2\hat{j} + 2\hat{k})$ .

**(ii) Solution:**

Given vectors are,

$$\vec{a} = 2\hat{i} + \hat{j} + \hat{k} \text{ and } \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$$

Now,

A vector perpendicular to the plane containing the vector  $\vec{a}$  and  $\vec{b} = \vec{a} \times \vec{b}$ .

$$\vec{a} \times \vec{b} = \vec{c} \text{ (say)} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\vec{c} = \hat{i}(1-2) - \hat{j}(2-1) + \hat{k}(4-1)$$

$$\vec{c} = -\hat{i} - \hat{j} + 3\hat{k}$$

We know that,

$\vec{c}$  is a vector perpendicular to both  $\vec{a}$  and  $\vec{b}$ .

So, it's unit vector is given by

$$\begin{aligned}\hat{c} &= \frac{\vec{c}}{|\vec{c}|} \\ &= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{(-1)^2 + (-1)^2 + (3)^2}} \\ &= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1+1+9}} \\ &= \frac{1}{\sqrt{11}}(-\hat{i} - \hat{j} + 3\hat{k})\end{aligned}$$

Therefore, unit vector perpendicular to the plane containing the vector both  $\vec{a}$  and  $\vec{b} = \frac{1}{\sqrt{11}}(-\hat{i} - \hat{j} + 3\hat{k})$

**4. Solution:**

Given,

$$\vec{a} = (3\hat{k} + 4\hat{j}) \times (\hat{i} + \hat{j} - \hat{k})$$

We know that, if

$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k} \text{ and } \vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}, \text{ then}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

So, here

$$\begin{aligned} \vec{a} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 3 \\ 1 & 1 & -1 \end{vmatrix} \\ &= \hat{i}(-4-3) - \hat{j}(0-3) + \hat{k}(0-4) \end{aligned}$$

$$\therefore \vec{a} = -7\hat{i} + 3\hat{j} - 4\hat{k}$$

Now,

$$\begin{aligned} |\vec{a}| &= \sqrt{(-7)^2 + (3)^2 + (-4)^2} \\ &= \sqrt{49 + 9 + 16} \end{aligned}$$

$$\therefore |\vec{a}| = \sqrt{74}$$

**5. Solution:**

Given,

$$\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{k}$$

Now, the unit vector of  $\hat{b}$  is given by

$$\begin{aligned} \hat{b} &= \frac{\vec{b}}{|\vec{b}|} \\ &= \frac{\hat{i} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2}} \\ &= \frac{\hat{i} - 2\hat{k}}{\sqrt{1+4}} \\ &= \frac{\hat{i} - 2\hat{k}}{\sqrt{5}} \end{aligned}$$

So,

$$2\hat{b} = \frac{2}{\sqrt{5}}\hat{i} - \frac{4}{\sqrt{5}}\hat{k}$$

And,  $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$

The cross product is given by

$$2\hat{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{2}{\sqrt{5}} & 0 & -\frac{4}{\sqrt{5}} \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \hat{i} \left( 0 + \frac{12}{\sqrt{5}} \right) - \hat{j} \left( \frac{2}{\sqrt{5}} + \frac{16}{\sqrt{5}} \right) + \hat{k} \left( \frac{6}{\sqrt{5}} - 0 \right)$$

$$\therefore 2\hat{b} \times \vec{a} = \frac{12}{\sqrt{5}}\hat{i} - \frac{18}{\sqrt{5}}\hat{j} + \frac{6}{\sqrt{5}}\hat{k}$$

Now,

$$|2\hat{b} \times \vec{a}| = \sqrt{\left(\frac{12}{\sqrt{5}}\right)^2 + \left(-\frac{18}{\sqrt{5}}\right)^2 + \left(\frac{6}{\sqrt{5}}\right)^2}$$

$$= \sqrt{\frac{144}{5} + \frac{324}{5} + \frac{36}{5}}$$

$$\therefore |2\hat{b} \times \vec{a}| = \sqrt{\frac{504}{5}}$$

### 6. Solution:

Given,

$$\vec{a} = 3\hat{i} - \hat{j} - 2\hat{k} \text{ and } \vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$$

Required to find  $(\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b})$

Now,

$$\vec{a} + 2\vec{b} = (3\hat{i} - \hat{j} - 2\hat{k}) + 2(2\hat{i} + 3\hat{j} + \hat{k})$$

$$= 3\hat{i} - \hat{j} - 2\hat{k} + 4\hat{i} + 6\hat{j} + 2\hat{k}$$

$$\Rightarrow \vec{a} + 2\vec{b} = 7\hat{i} + 5\hat{j}$$

And,

$$2\vec{a} - \vec{b} = 2(3\hat{i} - \hat{j} - 2\hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k})$$

$$= 6\hat{i} - 2\hat{j} - 4\hat{k} - 2\hat{i} - 3\hat{j} - \hat{k}$$

$$\Rightarrow 2\vec{a} - \vec{b} = 4\hat{i} - 5\hat{j} - 5\hat{k}$$

We know that if  $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$  and  $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$  then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

Thus,

$$\begin{aligned} (\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b}) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 5 & 0 \\ 4 & -5 & -5 \end{vmatrix} \\ &= \hat{i}(-25 - 0) - \hat{j}(-35 - 0) + \hat{k}(-35 - 20) \\ &= -25\hat{i} + 35\hat{j} - 55\hat{k} \\ \therefore (\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b}) &= -25\hat{i} + 35\hat{j} - 55\hat{k} \end{aligned}$$



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