Exercise 5(E)

Solution 1:

$$x^2+\frac{1}{4x^2}+1-7x-\frac{7}{2x}=(x)^2+\frac{1}{(2x)^2}+2\times x\times\frac{1}{2x}-7\left(x+\frac{1}{2x}\right)$$

$$=\left(x+\frac{1}{2x}\right)^2-7\left(x+\frac{1}{2x}\right)$$

$$=\left(x+\frac{1}{2x}\right)\left(x+\frac{1}{2x}-7\right)$$

$$=\left(x+\frac{1}{2x}\right)\left(x-7+\frac{1}{2x}\right)$$

Solution 2:

$$9a^2+\frac{1}{9a^2}-2-12a+\frac{4}{3a}=(3a)^2+\frac{1}{(3a)^2}-2\times 3a\times\frac{1}{3a}-4\left(3a-\frac{1}{3a}\right)$$

$$=\left(3a-\frac{1}{3a}\right)^2-4\left(3a-\frac{1}{3a}\right)$$

$$=\left(3a-\frac{1}{3a}\right)\left(\left(3a-\frac{1}{3a}\right)-4\right)$$

$$=\left(3a-\frac{1}{3a}\right)\left(3a-4-\frac{1}{3a}\right)$$

Solution 3:

$$x^2+\frac{a^2+1}{a}x+1=x^2+ax+\frac{1}{a}x+1$$

$$=x(x+a)+\frac{1}{a}(x+a)$$

$$=(x+a)\left(x+\frac{1}{a}\right)$$

Solution 4:

$$x^4+y^4-27x^2y^2=\left(x^2\right)^2+\left(y^2\right)^2-2x^2y^2-25x^2y^2$$

$$=\left(x^2-y^2\right)^2-25x^2y^2$$

$$=\left(x^2-y^2\right)^2-(5xy)^2\quad[\because a^2-b^2=(a+b)(a-b)]$$

$$=\left[\left(x^2-y^2\right)+5xy\right]\left[\left(x^2-y^2\right)-5xy\right]$$

$$=\left[x^2+5xy-y^2\right]\left[x^2-5xy-y^2\right]$$

Solution 5:

$$4x^4 + 9y^4 + 11x^2y^2 = (2x^2)^2 + (3y^2)^2 + 12x^2y^2 - x^2y^2$$
$$= (2x^2 + 3y^2)^2 - x^2y^2$$
$$= (2x^2 + 3y^2)^2 - (xy)^2$$
$$= (2x^2 + 3y^2 - xy)(2x^2 + 3y^2 + xy)$$
$$[\because a^2 - b^2 = (a+b)(a-b)]$$

Solution 6:

$$x^2 + \frac{1}{x^2} - 3 = x^2 + \frac{1}{x^2} - 2 - 1$$
$$= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} - 1$$
$$= \left(x - \frac{1}{x}\right)^2 - 1$$
$$= \left(x - \frac{1}{x}\right)^2 - (1)^2$$
$$= \left(x - \frac{1}{x} - 1\right)\left(x - \frac{1}{x} + 1\right) \quad [\because a^2 - b^2 = (a+b)(a-b)]$$

Solution 7:

$$a - b - 4a^2 + 4b^2 = (a-b) - 4(a^2 - b^2)$$
$$= (a-b) - 4(a-b)(a+b) \; [\because a^2 - b^2 = (a+b)(a-b)]$$
$$= (a-b)[1 - 4(a+b)]$$
$$= (a-b)[1 - 4a - 4b]$$

Solution 8:

$$(2a-3)^2 - 2(2a-3)(a-1) + (a-1)^2$$
$$= [(2a-3) - (a-1)]^2$$
$$= [2a - 3 - a + 1]^2$$
$$= (a-2)^2$$

Solution 9:

Let us assume, $a^2 - 3a = x$

Then the given expression is,

$$(a^2 - 3a)(a^2 - 3a + 7) + 10 = x(x + 7) + 10$$

$$= x^2 + 7x + 10$$

$$= x^2 + 5x + 2x + 10$$

$$= x(x + 5) + 2(x + 5)$$

$$= (x + 5)(x + 2)$$

$$= (a^2 - 3a + 5)(a^2 - 3a + 2)$$

[resubstitute the value of x]

$$= (a^2 - 3a + 5)(a^2 - 2a - a + 2)$$

$$= (a^2 - 3a + 5)(a(a - 2) - 1(a - 2))$$

$$= (a^2 - 3a + 5)[(a - 1)(a - 2)]$$

Solution 10:

Let us assume $a^2 - a = x$

Then the given expression is

$$(a^2 - a)(4a^2 - 4a - 5) - 6 = x(4x - 5) - 6$$

$$= 4x^2 - 5x - 6$$

$$= 4x^2 - 8x + 3x - 6$$

$$= 4x(x - 2) + 3(x - 2)$$

$$= (4x + 3)(x - 2)$$

$$= (4(a^2 - a) + 3)(a^2 - a - 2)$$

[resubstitute the value of x]

$$= (4a^2 - 4a + 3)(a^2 - a - 2)$$

$$= (4a^2 - 4a + 3)(a^2 - 2a + a - 2)$$

$$= (4a^2 - 4a + 3)(a(a - 2) + 1(a - 2))$$

$$= (4a^2 - 4a + 3)(a - 2)(a + 1)$$

Solution 11:

$$x^4 + y^4 - 3x^2y^2 = x^4 + y^4 - 2x^2y^2 - x^2y^2$$

$$= (x^2)^2 + (y^2)^2 - 2x^2y^2 - x^2y^2$$

$$= (x^2 - y^2)^2 - (xy)^2$$

$$= (x^2 - y^2 - xy)(x^2 - y^2 + xy)$$

$[\because a^2 - b^2 = (a + b)(a - b)]$

Solution 12:

$5a^2 - b^2 - 4ab + 7a - 7b$

$= 4a^2 + a^2 - b^2 - 4ab + 7a - 7b$

$= a^2 - b^2 + 4a^2 - 4ab + 7a - 7b$

$= (a^2 - b^2) + 4a(a - b) + 7(a - b)$

$= (a - b)(a + b) + 4a(a - b) + 7(a - b)$ $[\because a^2 - b^2 = (a + b)(a - b)]$

$= (a - b)[(a + b) + 4a + 7]$

$= (a - b)[a + b + 4a + 7]$

$= (a - b)[5a + b + 7]$

Solution 13:

$12(3x - 2y)^2 - 3x + 2y - 1 = 12(3x - 2y)^2 - (3x - 2y) - 1$

Let us assume that $3x - 2y = a$

Then the given expression is

$12(3x - 2y)^2 - 3x + 2y - 1 = 12a^2 - 3a - 1$

$= 12a^2 - 4a + 3a - 1$

$= 4a(3a - 1) + 1(3a - 1)$

$= (4a + 1)(3a - 1)$

$= (4(3x - 2y) + 1)(3(3x - 2y) - 1)$

[resubstitute the value of a]

$= (12x - 8y + 1)(9x - 6y - 1)$

Solution 14:

$4(2x - 3y)^2 - 8x + 12y - 3 = 4(2x - 3y)^2 - 4(2x - 3y) - 3$

Let us assume that $2x - 3y = a$

Then the given expression is

$4(2x - 3y)^2 - 8x + 12y - 3 = 4a^2 - 4a - 3$

$= 4a^2 - 6a + 2a - 3$

$= 2a(2a - 3) + 1(2a - 3)$

$= (2a - 3)(2a + 1)$

$= (2(2x - 3y) - 3)(2(2x - 3y) + 1)$

$= (4x - 6y - 3)(4x - 6y + 1)$

Solution 15:

$3-5x+5y-12(x-y)^2 = 3-5(x-y)-12(x-y)^2$

Let us assume that $x-y=a$

Then the given expression is

$3-5x+5y-12(x-y)^2 = 3-5a-12a^2$

$= 3-9a+4a-12a^2$

$= 3(1-3a)+4a(1-3a)$

$= (3+4a)(1-3a)$

[resubstitute the value of a]

$= (3+4(x-y))(1-3(x-y))$

$= (3+4x-4y)(1-3x+3y)$

Solution 16:

$9x^2+3x-8y-64y^2$

$= 9x^2-64y^2+3x-8y$

$= [(3x)^2-(8y)^2]+(3x-8y)$

$= [(3x+8y)(3x-8y)]+(3x-8y)$

$= (3x-8y)(3x+8y+1)$

Solution 17:

$2\sqrt{3}x^2+x-5\sqrt{3}$

$= 2\sqrt{3}x^2+6x-5x-5\sqrt{3}$

$= 2\sqrt{3}x(x+\sqrt{3})-5(x+\sqrt{3})$

$= (2\sqrt{3}x-5)(x+\sqrt{3})$

Solution 18:

$\frac{1}{4}(a+b)^2-\frac{9}{16}(2a-b)^2$

$= \frac{1}{4}\left[(a+b)^2-\frac{9}{4}(2a-b)^2\right]$

$= \frac{1}{4}\left[(a+b)^2-\left(\frac{3}{2}(2a-b)\right)^2\right]$

$= \frac{1}{4}\left[\left(a+b+\frac{3}{2}(2a-b)\right)\left(a+b-\frac{3}{2}(2a-b)\right)\right]$

$= \frac{1}{4}\left[\left(a+b+3a-\frac{3b}{2}\right)\left(a+b-3a+\frac{3b}{2}\right)\right]$

$= \frac{1}{4}\left[\left(4a-\frac{b}{2}\right)\left(\frac{5b}{2}-2a\right)\right]$

$= \frac{1}{4}\left[\left(\frac{8a-b}{2}\right)\left(\frac{5b-4a}{2}\right)\right]$

$= \frac{1}{4}\left[\frac{1}{4}(8a-b)(5b-4a)\right]$

$= \frac{1}{16}(8a-b)(5b-4a)$

Solution 19:

$2(ab + cd) - a^2 - b^2 + c^2 + d^2$

$= 2ab + 2cd - a^2 - b^2 + c^2 + d^2$

$= c^2 + d^2 + 2cd - a^2 - b^2 + 2ab$

$= (c^2 + d^2 + 2cd) - (a^2 + b^2 - 2ab)$

$= (c + d)^2 - (a - b)^2$

$= (c + d + a - b)(c + d - a + b)$

Solution 20:

(i) $(987)^2 - (13)^2$

$= (987 + 13)(987 - 13)$

$= 1000 \times 974$

$= 974000$

(ii) $(67.8)^2 - (32.2)^2$

$= (67.8 + 32.2)(67.8 - 32.2)$

$= 100 \times 35.6$

$= 3560$

(iii) $\dfrac{(6.7)^2 - (3.3)^2}{6.7 - 3.3}$

$= \dfrac{(6.7 + 3.3)(6.7 - 3.3)}{(6.7 - 3.3)}$

$= 10$

(iv) $\dfrac{(18.5)^2 - (6.5)^2}{18.5 + 6.5}$

$= \dfrac{(18.5 + 6.5)(18.5 - 6.5)}{(18.5 + 6.5)}$

$= 12$