Chapter 2.

EXERCISE 2.4 (Optional)*

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

Solution:

P(x) = $2x^3 + x^2 - 5x + 2$

Now for zeroes, putting the given values in x.

P$\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5 \times \frac{1}{2} + 2 = \left(2 \times \frac{1}{8}\right) + \frac{1}{4} - \left(5 \times \frac{1}{2}\right) + 2 = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = \frac{1}{2} - \frac{5}{2} + 2 = 0$

P(1) = $2 \times 1 + 1 - 5 \times 1 + 2 = 2 + 1 - 5 + 2 = 0$

P(-2) = $2 \times (-2)^3 + (-2)^2 - 5(-2) + 2 = (2 \times -8) + 4 + 10 + 2 = -16 + 16 = 0$

Thus, $\frac{1}{2}, 1$ *and* -2 *are the zeroes of given polynomial.*

Comparing given polynomial with $ax^3 + bx^2 + cx + d$, *we get,*

= $a = 2, b = 1, c = -5, d = 2$ *and* $\alpha = \frac{1}{2}$, $\beta = 1, \gamma = -2$

Now,

= $\alpha + \beta + \gamma = -\frac{b}{a}$

= $\frac{1}{2} + 1 - 2 = \frac{1}{2}$

= $\frac{1}{2} = \frac{1}{2}$

= $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

= $\left(\frac{1}{2} \times 1\right) + (1 \times -2) + \left(-2 \times \frac{1}{2}\right) = -\frac{5}{2}$

= $\frac{1}{2} - 2 - 1 = -\frac{5}{2}$

= $-\frac{5}{2} = -\frac{5}{2}$

= $\alpha\beta\gamma = -\frac{d}{a}$

= $\left(\frac{1}{2} \times 1 \times -2\right) = -\frac{2}{2}$

= $-\frac{2}{2} = -\frac{2}{2}$

Thus, *relationship between zeroes and the coefficients is verified.*

(ii) $x^3 - 4x^2 + 5x - 2$; 2, 1, 1

Solution:

p(x) = $x^3 - 4x^2 + 5x - 2$

Now for zeroes , put the given value in x.

P(2) = $2^3 - 4(2)^2 + 5 \times 2 - 2$ = $8 - 16 + 10 - 2 = 18 - 18 = 0$

P(1) = $1^3 - 4(1)^2 + 5 \times 1 - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$

P(1) = $1^3 - 4(1)^2 + 5 \times 1 - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$

Thus, 2, 1 , 1 are the zeroes of the given polynomial.

Now,

Comparing the given polynomial with $ax^3 + bx^2 + cx + d, we\ get,$

= $a = 1, b = -4, c = 5\ , d = -2\ \ and\ \alpha = 2, \beta = 1\ , \gamma = 1$

$\alpha + \beta + \gamma = \ -\frac{b}{a}$

= $2 + 1 + 1 = \frac{4}{1}$

= 4 = 4

= $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

= $(2 \times 1) + (1 \times 1) + (1 \times 2) = \frac{5}{1}$

= $2 + 1 + 2 = 5$

= 5 = 5

αβγ = $-\frac{d}{a}$

= 2×1×1 = 2

= 2 =2

Thus, $relationship\ between\ zeroes\ and\ coefficients\ is\ verified.$

1. **Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as** $2, -7, -14$ **respectively.**

Solution:

Let the polynomial be $ax^3 + bx^2 + cx + d$, $and\ zeroes\ are\ \alpha, \beta, \gamma$

Then,

= $\alpha + \beta + \gamma = \frac{2}{1} = -\frac{b}{a}$ $\quad given$

= $\alpha\beta + \beta\gamma + \alpha\gamma = -\frac{7}{1} = \frac{c}{a}$ $\quad given$

= $\alpha\beta\gamma = -\frac{14}{1} = -\frac{d}{a}$ $\quad given$

If a = 1 then, $b = -2, c = -7, d = 14$

Hence, $the\ polynomial\ become = x^3 - 2x^2 - 7x + 14.$

2. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b.

Solution:

Given,

p(x) = $x^3 - 3x^2 + x + 1$

zeroes are = a – b , a + b , a

$Comparing\ the\ given\ polynomial\ with\ mx^3 + nx^2 + px + q, we\ get,$

= $m = 1, n = -3, p = 1, q = 1$

Sum of zeroes = $a - b + a + a + b = -\frac{n}{m}$

= $3a = -\frac{-3}{1} = 3$

= a = $\frac{3}{3} = 1$

The zeroes are = $1 - b, 1, 1 + b$

Product of zeroes = $1(1 - b)(1 + b)$

= $1 - b^2 = -\frac{q}{p}$

= $1 - b^2 = -\frac{1}{1} = -1$

= $b^2 = 2$

= $b = \pm\sqrt{2}$

So,

We get, $a = 1$ *and* $b = \pm\sqrt{2}$

3.If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 13.8x - 35$ **are** $2 \pm \sqrt{3}$**, find other zeroes.**

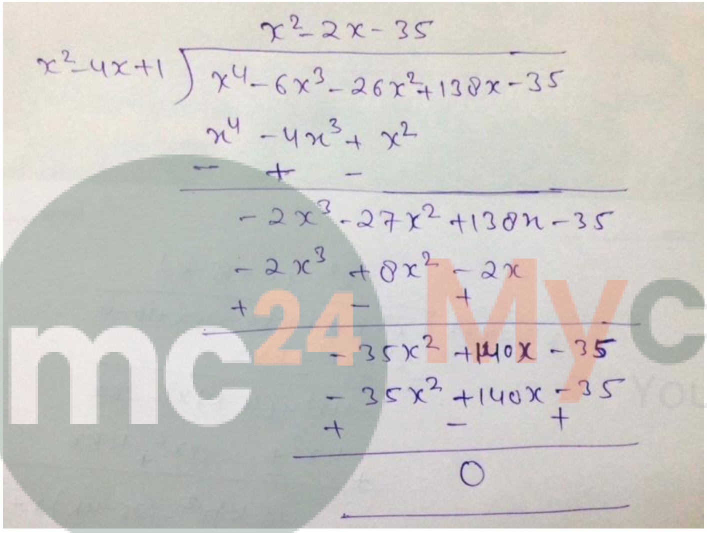
Solution:

Given,

2+$\sqrt{3}$ *and* $2 - \sqrt{3}$ *are zeroes of polynomial*

So, $\left(x - 2 - \sqrt{3}\right) \times \left(x - 2 + \sqrt{3}\right) = x^2 - 4x + 4 + 3 = x^2 - 4x + 1$ *is a factor of polynomial.*

Hence, dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ *by* $x^2 - 4x +$
1 *to determine the remaining roots.*



We get ,

= $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)\ (x^2 - 2x - 35)$

= $x^2 - 2x - 35$ *is also a factor of given polynomial and* $x^2 - 2x - 35 = (x - 7)(x + 5)$

The value of polynomial is also zero when ,

= $x - 7 = 0$, $x = 7$

= $x + 5 = 0$,= $x = -5$

Hence, 7 and -5 are also zeroes of this polynomial

5. **If the polynomial** $x^4 - 6x^3 + 16x^2 - 25x + 10$ **is divided by another polynomial** $x^2 - 2x + k$**, the remainder comes out to be** $x + a$**, find** k **and** a.

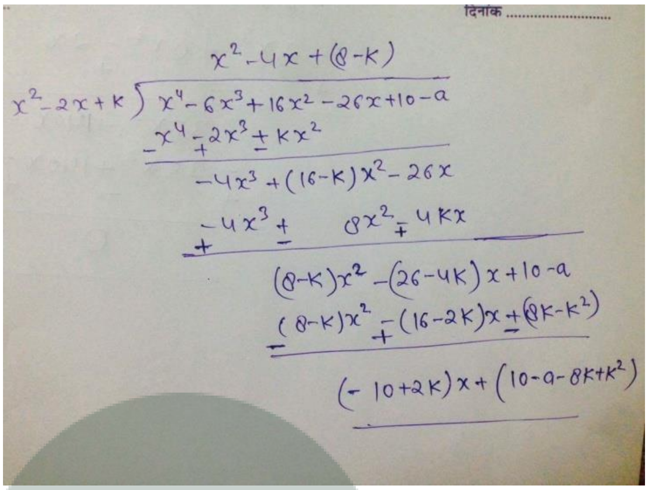
Solution:

We know that,

= $Dividend = Divisor \times quoient + remainder$

And , $Dividend - remainder = Quotient \times Divisor$

= $x^4 - 6x^3 + 16x^2 - 25x + 10 - (x + a) = \ x^4 - 6x^3 + 16x^2 - 26x + 10 - a \ will \ be \ divisible \ by \ x^2 - 2x + k$



So, $(-10 + 2k)x + (10 - a - 8k + k^2) = \ 0$

Either $(-10 + 2k) = \ 0 \ or \ (10 - a - 8k + k^2) = \ 0$

Put $(-10 + 2k) = 0$

= 2k = 10

= k = $\frac{10}{2}$ = 5

Put $(10 - a - 8k + k^2) = \ 0$

= 10 – a – 8 × 5 + 25 = 0

= 10 – a – 40 + 25 = 0

= - 5 – a = 0

= a = -5

Hence , $k = 5 \ and \ a = \ -5$