NCERT Solutions for Class-XII Maths

Chapter-6.2

NCERT Math Class 12

1. Show that the function given by f (x) = 3x + 17 is strictly increasing on R.

1. Let x_1 and x_2 be any two numbers in $\mathbf{R}$.

Then, we have:

$$x_1 < x_2 \Rightarrow 3x_1 < 3x_2 \Rightarrow 3x_1 + 17 < 3x_2 + 17 \Rightarrow f(x_1) < f(x_2)$$

Hence, f is strictly increasing on $\mathbf{R}$.

2. Show that the function given by f (x) = e^{2x} is strictly increasing on R.

2. Let x_1 and x_2 be any two numbers in R.

The, we have,

$x_1 < x_2$

$\Rightarrow 2x_1 < 2x_2$

$\Rightarrow e^{2x_1} < e^{2x_2}$

$\Rightarrow f(x_1) < f(x_2)$

Therefore, f is strictly increasing on R.

3. Show that the function given by f (x) = sin x is

A. strictly increasing in $\left(0, \frac{\pi}{2}\right)$

B. strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

C. neither increasing nor decreasing in (0, π)

3. The given function is $f(x) = \sin x$.

$\therefore f'(x) = \cos x$

(a) Since for each $x \in \left(0, \frac{\pi}{2}\right), \cos x > 0$, we have $f'(x) > 0$.

Hence, f is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

(b) Since for each $x \in \left(\frac{\pi}{2}, \pi\right), \cos x < 0$, we have $f'(x) < 0$.

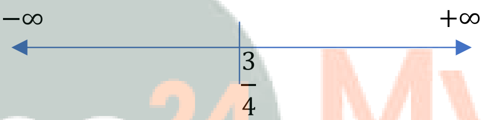
Hence, f is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

(c) From the results obtained in (a) and (b), it is clear that f is neither increasing nor decreasing in $(0, \pi)$.

4. Find the intervals in which the function f given by f (x) = $2x^2 - 3x$ is
A. strictly increasing B. strictly decreasing

4. It is given that function f(x) = $2x^2 - 3x$
⇒ f'(x) = 4x – 3
If f'(x) = 0, then we get,
$x = \frac{3}{4}$

So, the points $\frac{3}{4}$ divides the real line into two disjoint intervals, $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$



So, in interval $\left(\frac{3}{4}, \infty\right)$, f'(x) = 4x –3 >0

Therefore, the given function (f) is strictly increasing in interval $\left(\frac{3}{4}, \infty\right)$**.**

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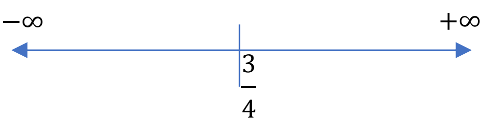

So, in interval $\left(-\infty, \frac{3}{4}\right)$ f'(x) = 4x –3 < 0

Therefore, the given function (f) is strictly decreasing in interval $\left(\frac{3}{4}, \infty\right)$.

5. Find the intervals in which the function f given by f (x) = $2x^3 - 3x^2 - 36x + 7$ is
A. strictly increasing B. strictly decreasing

5. The given function is $f(x) = 2x^3 - 3x^2 - 36x + 7$

$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x+2)(x-3)$

$\therefore f'(x) = 0 \Rightarrow x = -2, 3$

The points $x = -2$ and $x = 3$ divide the real line into three disjoint intervals i.e., $(-\infty, -2), (-2, 3)$, and $(3, \infty)$

In intervals $(-\infty, -2)$ and $(3, \infty)$, $f'(x)$ is positive while in interval $(-2, 3)$, $f'(x)$ is negative.

Hence, the given function (f) is strictly increasing in intervals $(-\infty, -2)$ and $(3, \infty)$, while function (f) is strictly decreasing in interval $(-2, 3)$

6. Find the intervals in which the following functions are strictly increasing or decreasing:
A. $x^2 + 2x - 5$ B. $10 - 6x - 2x^2$
C. $-2x^3 - 9x^2 - 12x + 1$ D. $6 - 9x - x^2$
E. $(x + 1)^3 (x - 3)^3$

6. (A) It is given that function f(x) = $x^2 + 2x - 5$
f'(x) = 2x + 2
If f'(x) = 0, then we get,
$\Rightarrow$ x = –1
So, the point x = –1 divides the real line into two disjoint intervals, $(-\infty, -1)$ and $(1, \infty)$
So, in interval $(-\infty, -1)$
f'(x) = 2x + 2 < 0
Therefore, the given function (f) is strictly decreasing in interval $(-\infty, -1)$.
And in interval $(1, \infty)$
f'(x) = 2x + 2 > 0
Therefore, the given function (f) is strictly increasing in interval $(1, \infty)$.
Thus, f is strictly increasing for x > –1.

(B) It is given that function f(x) = $10 - 6x - 2x^2$
f'(x) = –6 – 4x

If f'(x) = 0, then we get,

$\Rightarrow x = \frac{-3}{2}$

So, the point $x = \frac{-3}{2}$ divides the real line into two disjoint intervals,

$\left(-\infty, \frac{-3}{2}\right) and \left(\frac{-3}{2}, \infty\right)$

So, in interval $\left(-\infty, \frac{-3}{2}\right)$

f'(x) = –6 – 4x < 0

Therefore, the given function (f) is strictly decreasing in interval $\left(-\infty, \frac{-3}{2}\right)$.

And in interval $\left(\frac{-3}{2}, \infty\right)$

f'(x) = –6 – 4x > 0

Therefore, the given function (f) is strictly increasing in interval $\left(\frac{-3}{2}, \infty\right)$.

Thus, f is strictly decreasing for x> $\frac{-3}{2}$.

(C) It is given that function $f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow$ $f'(x) = -6x^2 - 18x + 12$

$\Rightarrow$ $f'(x) = -6(x^2 + 3x + 6)$

$\Rightarrow$ $f'(x) = -6(x + 1)(x + 2)$

If f'(x) = 0, then we get,

$\Rightarrow$ x = –1 and –2

So, the points x = –1 and x = –2 divides the real line into two disjoint intervals,

(–∞,–2), (–2,–1) and (–1, ∞)

So, in interval (–∞,–2),(–1, ∞)

f'(x) = –6(x + 1) (x +2) < 0

Therefore, the given function (f) is strictly decreasing for x < –2 and x>–1.

So, in interval (–2.–1)

f'(x) = –6(x + 1)(x+2) > 0

Therefore, the given function (f) is strictly increasing for –2 < x < –1.

(D) It is given that function $f(x) = 6 - 9x - x^2$

f'(x) = –9 – 2x

If f'(x) = 0, then we get,

$\Rightarrow\ x = \frac{-9}{2}$

So, the point $x = \frac{-9}{2}$ divides the real line into two disjoint intervals,

$\left(-\infty, \frac{-9}{2}\right) and \left(\frac{-9}{2}, \infty\right)$

So, in interval $\left(-\infty, \frac{-9}{2}\right)$

$f'(x) = -9 - 2x > 0$

Therefore, the given function (f) is strictly increasing for x < $\frac{-9}{2}$.

And in interval $\left(\frac{-9}{2}, \infty\right)$

$f'(x) = -9 - 2x < 0$

Therefore, the given function (f) is strictly decreasing for x> $\frac{-9}{2}$.

Thus, f is strictly decreasing for x> $\frac{-9}{2}$.

(E) It is given that function $f(x) = (x + 1)^3 (x - 3)^3$
$\Rightarrow f'(x) = 3(x + 1)^2 (x - 3)^3 + 3(x + 1)^3 (x - 3)^2$
$\Rightarrow f'(x) = 3(x + 1)^2 (x - 3)^2[x - 3 + x + 1]$
$\Rightarrow f'(x) = 6(x + 1)^2 (x - 3)^2(x-1)$
If $f'(x) = 0$, then we get,
$\Rightarrow x = -1, 3$ and 1
So, the points x = –1, x =1 and x = 3 divides the real line into four disjoint intervals,
$(-\infty,-1),(-1,1),(1,3)\ and\ (3,\infty)$

So, in interval $(-\infty,-1),(-1,1)$

$f'(x) = 6(x + 1)^2 (x - 3)^2(x-1) < 0$

Therefore, the given function (f) is strictly decreasing in intervals $(-\infty,-1),(-1,1)$.

So, in interval $(1,3)\ and\ (3,\infty)$

$f'(x) = 6(x + 1)^2 (x - 3)^2(x-1) > 0$

Therefore, the given function (f) is strictly increasing in intervals $(1,3)$ and $(3,\infty)$.

7. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1,$ is an increasing function of x throughout its domain.

7. We have,

$$y = \log(1+x) - \frac{2x}{2+x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{1+x} - \frac{(2+x)(2) - 2x(1)}{(2+x)^2} = \frac{1}{1+x} - \frac{4}{(2+x)^2} = \frac{x^2}{(2+x)^2}$$

Now, $\frac{dy}{dx} = 0$

$\Rightarrow \frac{x^2}{(2+x)^2} = 0$

$\Rightarrow x^2 = 0$

$\Rightarrow x = 0$

Since $x > -1$, point $x = 0$ divides the domain $(-1, \infty)$ in two disjoint intervals i.e., $-1 < x < 0$ and $x > 0$.

When $-1 < x < 0$, we have:

$x\langle 0 \Rightarrow x^2 \rangle 0$

$x > -1 \Rightarrow (2+x) > 0 \Rightarrow (2+x)^2 > 0$

$\therefore y' = \frac{x^2}{(2+x)^2} > 0$

Also, when $x > 0$:

$x > 0 \Rightarrow x^2 > 0, (2+x)^2 > 0$

$\therefore y' = \frac{x^2}{(2+x)^2} > 0$

Hence, function f is increasing throughout this domain.

8. Find the values of x for which y = $[x(x-2)]^2$ is an increasing function.

8. It is given that y = $[x(x-2)]^2$, then,

$\frac{dy}{dx} = y' = 2(x^2 - 2x)(2x - 2)$

= 4x(x - 2)(x - 1)

Now if $\frac{dy}{dx} = 0$

$\Rightarrow$ x = 0, 1,2

So, the points x = 0, x =1 and x = 2 divides the real line into four disjoint intervals,

(-∞,0), (0,1), (1, 2) and (2,∞).
So, in interval(-∞,0),(1,2)

$$\frac{dy}{dx}<0$$

Therefore, the given function (f) is strictly decreasing in intervals $(-\infty,0),(1,2)$.

So, in interval (0,1) and (2,∞)

$$\frac{dy}{dx}>0$$

Therefore, the given function (f) is strictly increasing for 0 < x < 1 and x>2

9. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$

9. We have,

$$y=\frac{4\sin\theta}{\left(2+\cos\theta\right)}-\theta$$

$$\therefore\frac{dy}{dx}=\frac{\left(2+\cos\theta\right)\left(4\cos\theta\right)-4\sin\theta\left(-\sin\theta\right)}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4\cos^2\theta+4\sin^2\theta}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4}{(2+\cos\theta)^2}-1$$

Now, $\frac{dy}{dx}=0$

$$\Rightarrow\frac{8\cos\theta+4}{(2+\cos\theta)^2}=1$$

$$\Rightarrow 8\cos\theta+4=4+\cos2+4\cos\theta$$

$$\Rightarrow\cos\theta-4\cos\theta=0$$

$$\Rightarrow\cos\theta\left(\cos\theta-4\right)=0$$

$$\Rightarrow\cos\theta=0\text{ or }\cos\theta=4$$

Since $\cos\theta\neq4,\cos\theta=0.$

$$\cos\theta=0\Rightarrow\theta=\frac{\pi}{2}$$

Now,

$$\frac{dy}{dx}=\frac{8\cos\theta+4-\left(4+\cos^2\theta+4\cos\theta\right)}{(2+\cos\theta)^2}=\frac{4\cos\theta-\cos^2\theta}{(2+\cos\theta)^2}=\frac{\cos\theta\left(4-\cos\theta\right)}{(2+\cos\theta)^2}$$

In interval $\left(0,\frac{\pi}{2}\right)$, we have $\cos\theta>0$. Also, $4>\cos\theta\Rightarrow 4-\cos\theta>0$

$\therefore \cos\theta\left(4-\cos\theta\right)>0$ and also $(2+\cos\theta)^2>0$

$$\Rightarrow \frac{\cos\theta\left(4-\cos\theta\right)}{(2+\cos\theta)^2}>0$$

$$\Rightarrow \frac{dy}{dx}>0$$

Therefore, y is strictly increasing in interval $\left(0,\frac{\pi}{2}\right)$.

Also, the given function is continuous at $x=0$ and $x=\frac{\pi}{2}$,

Hence, y is increasing in interval $\left[0,\frac{\pi}{2}\right]$.

10. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

10. The given function is f(x) = logx

$$\Rightarrow f'=\frac{1}{x}$$

It is clear that for x>0, $f'=\frac{1}{x}>0$

Therefore, f(x) = log x is strictly increasing in interval $(0,\infty)$.

11. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on (– 1, 1).

11. The given function is $f\left(x\right)=x^2-x+1$

$\therefore f'\left(x\right)=2x-1$

Now, $f'\left(x\right)=0\Rightarrow x=\frac{1}{2}$.

The point $\frac{1}{2}$ divides the interval $\left(-1,1\right)$ into two disjoint intervals

i.e., $\left(-1,\frac{1}{2}\right)$ and $\left(\frac{1}{2},1\right)$.

Now, in interval $\left(-1,\frac{1}{2}\right), f'\left(x\right)=2x-1<0$

Therefore, f is strictly decreasing in interval $\left(-1,\frac{1}{2}\right)$.

However, in interval $\left(\frac{1}{2},1\right)$, $f'(x)=2x-1>0$

Therefore, f is strictly increasing in interval $\left(\frac{1}{2},1\right)$.

Hence, f is neither strictly increasing nor decreasing in interval $(-1,1)$.

12. Which of the following functions are strictly decreasing on $\left(0,\frac{\pi}{2}\right)$?

A. cos x B. cos 2x
C. cos 3x D. tan x

12. (A) Let $f_1(x)$ = cosx

$\therefore f_1'(x)=-sinx$

In interval $\left(0,\frac{\pi}{2}\right)$, $f_1'(x)=-sinx<0$.

Therefore, $f_1(x)$ = cosx is strictly decreasing in interval $\left(0,\frac{\pi}{2}\right)$.

(B) Let $f_2(x)$ = cos2x

$\therefore f_2'(x)=-2sin2x$

Now, $0 < x < \frac{\pi}{2}$

$\Rightarrow 0 < 2x < \pi$

$\Rightarrow \sin 2x > 0$

$\Rightarrow -2\sin 2x < 0$

$\therefore f_2'(x)=-2sin2x<0\,on\left(0,\frac{\pi}{2}\right)$

Therefore, $f_2(x)$ = cos2x is strictly decreasing in interval $\left(0,\frac{\pi}{2}\right)$.

(C) Let $f_3(x)$ = cos3x

$\therefore f_3'(x)=-3sin3x$

Now, $f_3'=0$

$\Rightarrow \sin 3x = 0$

$\Rightarrow 3x = \pi$, as $x \in \left(0,\frac{\pi}{2}\right)$

$\Rightarrow x = \frac{\pi}{3}$

The point $x = \frac{\pi}{3}$ divides the interval $\left(0, \frac{\pi}{2}\right)$ into two distinct intervals.

i.e. $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Now, in interval, $\left(0, \frac{\pi}{3}\right)$,

$f_3'(x) = -3\sin 3x < 0$ as $(0 < x < \frac{\pi}{2} \Rightarrow 0 < 3x < \pi)$

Therefore, f_3 is strictly decreasing in interval $\left(0, \frac{\pi}{3}\right)$.

Now, in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$f_3'(x) = -3\sin 3x > 0$ as $\frac{\pi}{3} < x < \frac{\pi}{2} \Rightarrow \pi < 3x < \frac{3\pi}{2}$

Therefore, f_3 is strictly increasing in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

(D) Let $f_4 = \tan x$

$\therefore f_3'(x) = sec^2 x$

In interval $\left(0, \frac{\pi}{2}\right)$,

$f_3'(x) = sec^2 x > 0$

Therefore, f4 is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$.

13. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ strictly decreasing?

A. (0,1) B. $\left(\frac{\pi}{2}, \pi\right)$

C. $\left(0, \frac{\pi}{2}\right)$ D. None of these

13. The correct option is (D).

Explanation: It is given that $f(x) = x^{100} + \sin x - 1$

Then, $f'(x) = 100x^{99} + \cos x$

In interval (0,1), cos x >0 and $100x^{99} > 0$

$\Rightarrow$ f'(x)>0

Therefore, function f is strictly increasing in interval (0,1).

In interval $\left(\frac{\pi}{2}, \pi\right)$, cos x < 0 and $100x^{99} > 0$.

Also, $100x^{99}$ > cos x

$\Rightarrow$ f'(x) > 0 in $\left(\frac{\pi}{2}, \pi\right)$

Therefore, function f is strictly increasing in interval $\left(\frac{\pi}{2}, \pi\right)$.

In interval $\left(0, \frac{\pi}{2}\right)$, cos x < 0 and $100x^{99} > 0$.

Also, $100x^{99}$ > cos x

$\Rightarrow$ f'(x) > 0 on $\left(0, \frac{\pi}{2}\right)$

Therefore, function f is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$.

Hence, function f is strictly decreasing on none of the intervals.

14. Find the least value of a that the function f given by f (x) = x^2 + ax + 1 is strictly increasing on [1, 2].

14. It is given that function f(x) = x^2 + ax + 1

f'(x) = 2x + a

Now, function f will be increasing in (1,2) ,

if f'(x) >0 in (1,2)

$\Rightarrow$ 2x +a > 0

$\Rightarrow$ 2x > -a

$\Rightarrow x > -\frac{a}{2}$

Therefore, we have to find the least value of a such that

$\Rightarrow x > -\frac{a}{2}$ when x ϵ (1,2)

$\Rightarrow x > -\frac{a}{2}$ (when 1 < x < 2)

Therefore, the least value of a for f to be increasing on (1,2) is given by

-a/2 = 1

$\Rightarrow$ a = -2

Therefore, the least value of a is -2.

15. Let I be any interval disjoint from [–1, 1]. Prove that the function f given by $f(x)=x+\frac{1}{x}$ is strictly increasing on I.

15. We have,

$f(x)=x+\frac{1}{x}$

$\therefore f'(x)=1-\frac{1}{x^2}$

Now,

$f'(x)=0 \Rightarrow \frac{1}{x^2}=1 \Rightarrow x=\pm 1$

The points $x=1$ and $x=-1$ divide the real line in three disjoint intervals i.e., $(-\infty,-1),(-1,1)$, and $(1,\infty)$

In interval $(-1,1)$, it is observed that:

$-1<x<1$

$\Rightarrow x^2<1$

$\Rightarrow 1<\frac{1}{x^2}, x\neq 0$

$\Rightarrow 1-\frac{1}{x^2}<0, x\neq 0$

$\therefore f'(x)=1-\frac{1}{x^2}<0$ on $(-1,1)\sim\{0\}$

$\therefore f$ is strictly decreasing on $(-1,1)\sim\{0\}$.

In intervals $(-\infty,-1)$ and $(1,\infty)$, it is observed that:

$x<-1$ or $1<x$

$\Rightarrow x^2>1$

$\Rightarrow 1>\frac{1}{x^2}$

$\Rightarrow 1-\frac{1}{x^2}>0$

$\therefore f'(x)=1-\frac{1}{x^2}>0$ on $(-\infty,-1)$ and $(1,\infty)$

$\therefore f$ is strictly increasing on $(-\infty,1)$ and $(1,\infty)$.

Hence, function f is strictly increasing in interval I disjoint from $(-1,1)$. Hence, the given result is proved.

16. Prove that the function f given by f (x) = log sin x is strictly increasing on $\left(0,\frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2},\pi\right)$.

16. It is given that f (x) = log sin x

$\Rightarrow f'(x)=\frac{1}{sinx}cosx=cotx$

In interval $\left(0,\frac{\pi}{2}\right)$, f'(x) = cot x >0

Therefore, f is strictly increasing in $\left(0,\frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2},\pi\right)$, f'(x) = cot x < 0

Therefore, f is strictly decreasing in $\left(\frac{\pi}{2},\pi\right)$.

17. Prove that the function f given by f (x) = log |cos x| is strictly decreasing on $\left(0,\frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{3\pi}{2},2\pi\right)$.

17. We have,

$f(x)=\log\cos x$

$\therefore f'(x)=\frac{1}{\cos x}(-\sin x)=-\tan x$

In interval $\left(0,\frac{\pi}{2}\right)$, $\tan x>0\Rightarrow-\tan x<0$.

$\therefore f'(x)<0$ on $\left(0,\frac{\pi}{2}\right)$

$\therefore f$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\tan x \langle 0 \Rightarrow -\tan x \rangle 0$.

$\therefore f'(x) > 0$ on $\left(\frac{\pi}{2}, \pi\right)$

$\therefore f$ is strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$

18. Prove that the function given by f (x) = $x^3 - 3x^2 + 3x - 100$ is increasing in R.

18. We have, f (x) = $x^3 - 3x^2 + 3x - 100$

$\Rightarrow$ f'(x) = $3x^2 - 6x + 3$

= $3(x^2 - 2x + 1)$

= 3(x-1)2

For any x ϵ R, (x -1)2 > 0

Thus, f'(x) is always positive in R.

Therefore, the given function (f) is increasing in R.

19. The interval in which y = $x^2 e^{-x}$ is increasing is

A. (– ∞, ∞) B. (– 2, 0)

C. (2, ∞) D. (0, 2)

19. We have

$y = x^2 e^{-x}$

$\therefore \frac{dy}{dx} = 2xe^{-x} - x^2 e^{-x} = xe^{-x}(2-x)$

Now, $\frac{dy}{dx} = 0. \Rightarrow x = 0$ and $x = 2$

The points $x = 0$ and $x = 2$ divide the real line into three disjoint intervals i.e., $(-\infty, 0), (0, 2)$, and $(2, \infty)$.

In intervals $(-\infty, 0)$ and $(2, \infty)$, $f'(x) < 0$ as e^{-x} is always positive.

$\therefore f$ is decreasing on $(-\infty, 0)$ and $(2, \infty)$.

In interval $(0, 2)$, $f'(x) > 0$

$\therefore f$ is strictly increasing on $(0, 2)$.

Hence, f is strictly increasing in interval $(0, 2)$.

The correct answer is D.