

Exercise 5(A)

Factorise by taking out the common factors:

1. $2(2x - 5y) (3x + 4y) - 6(2x - 5y) (x - y)$

Solution:

$$\begin{aligned} &\text{Identifying and taking } (2x - 5y) \text{ common from both the terms, we have} \\ &= (2x - 5y) [2(3x + 4y) - 6(x - y)] \\ &= (2x - 5y) (6x + 8y - 6x + 6y) \\ &= (2x - 5y) (8y + 6y) \\ &= (2x - 5y) (14y) \\ &= (2x - 5y)14y \end{aligned}$$

2. $xy(3x^2 - 2y^2) - yz(2y^2 - 3x^2) + zx(15x^2 - 10y^2)$

Solution:

$$\begin{aligned} &\text{We have, } xy(3x^2 - 2y^2) - yz(2y^2 - 3x^2) + zx(15x^2 - 10y^2) \\ &\text{Changing signs to arrive at a common term} \\ &\text{So,} \\ &= xy(3x^2 - 2y^2) + yz(3x^2 - 2y^2) + zx(15x^2 - 10y^2) \\ &= xy(3x^2 - 2y^2) + yz(3x^2 - 2y^2) + 5zx(3x^2 - 2y^2) \\ &= (3x^2 - 2y^2) (xy + yz + 5zx) \end{aligned}$$

3. $ab(a^2 + b^2 - c^2) - bc(c^2 - a^2 - b^2) + ca(a^2 + b^2 - c^2)$

Solution:

$$\begin{aligned} &\text{We have, } ab(a^2 + b^2 - c^2) - bc(c^2 - a^2 - b^2) + ca(a^2 + b^2 - c^2) \\ &\text{Changing signs to arrive at a common term} \\ &\text{So,} \\ &= ab(a^2 + b^2 - c^2) + bc(a^2 + b^2 - c^2) + ca(a^2 + b^2 - c^2) \\ &= (a^2 + b^2 - c^2) (ab + bc + ca) \end{aligned}$$

4. $2x(a - b) + 3y(5a - 5b) + 4z(2b - 2a)$

Solution:

$$\begin{aligned} &\text{We have, } 2x(a - b) + 3y(5a - 5b) + 4z(2b - 2a) \\ &\text{Taking common factors, we get} \\ &= 2x(a - b) + 15y(a - b) - 8z(a - b) \\ &= (a - b) (2x + 15y - 8z) \end{aligned}$$

Factorize by the grouping method:

5. $a^3 + a - 3a^2 - 3$

Solution:

$$\begin{aligned} &\text{We have, } a^3 + a - 3a^2 - 3 \\ &\text{Grouping to arrive at a common term} \\ &= a(a^2 + 1) - 3(a^2 + 1) \end{aligned}$$

Taking common, we get
 $= (a^2 + 1) (a - 3)$

6. $16(a + b)^2 - 4a - 4b$

Solution:

We have, $16(a + b)^2 - 4a - 4b$
Grouping to arrive at a common term
 $= 16(a + b)^2 - 4(a + b)$
Taking common, we get
 $= 4(a + b) [4(a + b) - 1]$
 $= 4(a + b) (4a + 4b - 1)$

7. Factorize by the grouping method:

$$a^4 - 2a^3 - 4a + 8$$

Solution:

We have, $a^4 - 2a^3 - 4a + 8$
Grouping to arrive at a common term
 $= a^3(a - 2) - 4(a - 2)$
Taking common, we get
 $= (a^3 - 4) (a - 2)$

8. $ab - 2b + a^2 - 2a$

Solution:

We have, $ab - 2b + a^2 - 2a$
Grouping to arrive at a common term
 $= b(a - 2) + a(a - 2)$
Taking common, we get
 $= (b + a) (a - 2)$

9. $ab(x^2 + 1) + x(a^2 + b^2)$

Solution:

We have, $ab(x^2 + 1) + x(a^2 + b^2)$
On expanding,
 $= abx^2 + ab + a^2x + b^2x$
Now, grouping to arrive at a common term
 $= abx^2 + a^2x + b^2x + ab$
 $= ax(bx + a) + b(bx + a)$
Taking common, we get
 $= (ax + b) (bx + a)$

10. $a^2 + b - ab - a$

Solution:

We have, $a^2 + b - ab - a$
Grouping to arrive at a common term
 $= a^2 - a + b - ab$
 $= a(a - 1) - b(-1 + a)$
 $= a(a - 1) - b(a - 1)$
Taking common, we get
 $= (a - b)(a - 1)$

11. $(ax + by)^2 + (bx - ay)^2$

Solution:

We have, $(ax + by)^2 + (bx - ay)^2$
On expanding,
 $= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$
 $= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$
Rearranging terms, we get
 $= a^2x^2 + b^2x^2 + a^2y^2 + b^2y^2$
Taking common, we get
 $= x^2(a^2 + b^2) + y^2(a^2 + b^2)$
 $= (x^2 + y^2)(a^2 + b^2)$

12. $a^2x^2 + (ax^2 + 1)x + a$

Solution:

We have, $a^2x^2 + (ax^2 + 1)x + a$
Regrouping the terms, we have
 $= a^2x^2 + a + (ax^2 + 1)x$
 $= a(ax^2 + 1) + x(ax^2 + 1)$
Taking common, we get
 $= (ax^2 + 1)(a + x)$

13. $(2a - b)^2 - 10a + 5b$

Solution:

We have, $(2a - b)^2 - 10a + 5b$
Taking common,
 $= (2a - b)^2 - 5(2a - b)$
Now,
 $= (2a - b)[(2a - b) - 5]$
 $= (2a - b)(2a - b - 5)$

14. $a(a - 4) - a + 4$

Solution:

We have, $a(a - 4) - a + 4$

By grouping, we get
 $= a(a - 4) - 1(a - 4)$
 Now, taking the common term
 $= (a - 4)(a - 1)$

15. $y^2 - (a + b)y + ab$

Solution:

We have, $y^2 - (a + b)y + ab$
 On expanding,
 $= y^2 - ay - by + ab$
 $= (y^2 - ay) - by + ab$
 Taking 'y' and 'b' common from the group, we get
 $= y(y - a) - b(y - a)$
 $= (y - a)(y - b)$

16. $a^2 + 1/a^2 - 2 - 3a + 3/a$

Solution:

We have, $a^2 + 1/a^2 - 2 - 3a + 3/a$
 On grouping terms, we get
 $= (a^2 - 2 + 1/a^2) - 3a + 3/a$
 $= [a^2 - (2 \times a \times 1/a) + 1/a^2] - 3(a - 1/a)$
 $= (a - 1/a)^2 - 3(a - 1/a)$ {Since, $(x - y)^2 = x^2 - 2xy + y^2$ }
 Taking $(a - 1/a)$ as common, we get
 $= (a - 1/a)[(a - 1/a) - 3]$
 $= (a - 1/a)(a - 1/a - 3)$

17. $x^2 + y^2 + x + y + 2xy$

Solution:

We have, $x^2 + y^2 + x + y + 2xy$
 On rearranging terms, we get
 $= (x^2 + y^2 + 2xy) + (x + y)$ {Since, $(x + y)^2 = x^2 + 2xy + y^2$ }
 Now,
 $= (x + y)^2 + (x + y)$
 $= (x + y)(x + y + 1)$

18. $a^2 + 4b^2 - 3a + 6b - 4ab$

Solution:

We have, $a^2 + 4b^2 - 3a + 6b - 4ab$
 On rearranging terms, we get
 $= a^2 + 4b^2 - 4ab - 3a + 6b$
 Now,
 $= a^2 + (2b)^2 - 2 \times a \times (2b) - 3(a - 2b)$ {Since, $(a - b)^2 = a^2 - 2ab + b^2$ }

$$\begin{aligned} &= (a - 2b)^2 - 3(a - 2b) \\ &= (a - 2b) [(a - 2b) - 3] \\ &= (a - 2b) (a - 2b - 3) \end{aligned}$$

19. $m(x - 3y)^2 + n(3y - x) + 5x - 15y$

Solution:

We have, $m(x - 3y)^2 + n(3y - x) + 5x - 15y$

Now,

Taking $(x - 3y)$ common from all the three terms, we get

$$\begin{aligned} &= m(x - 3y)^2 - n(x - 3y) + 5(x - 3y) \\ &= (x - 3y) [m(x - 3y) - n + 5] \\ &= (x - 3y) (mx - 3my - n + 5) \end{aligned}$$

20. $x(6x - 5y) - 4(6x - 5y)^2$

Solution:

We have, $x(6x - 5y) - 4(6x - 5y)^2$

Now,

Taking $(6x - 5y)$ common from the three terms, we get

$$\begin{aligned} &= (6x - 5y) [x - 4(6x - 5y)] \\ &= (6x - 5y) (x - 24x + 20y) \\ &= (6x - 5y) (-23x + 20y) \\ &= (6x - 5y) (20y - 23x) \end{aligned}$$