

14. Some Special Integrals

Exercise 14A

1. Question

Evaluate:

$$\int \frac{dx}{(1-9x)^2}$$

Answer

To find: $\int \frac{dx}{(1-9x)^2}$

Formula Used: $\int x^n = \frac{x^{n+1}}{n+1} + C$

Let $y = (1 - 9x) \dots (1)$

Differentiating with respect to x ,

$$\frac{dy}{dx} = -9$$

i.e., $dy = -9 dx$

Substituting in the equation to evaluate,

$$\Rightarrow \int \frac{dy}{y^2}$$

$$\Rightarrow \frac{-1}{9} \int \frac{dy}{y^2}$$

$$\Rightarrow \frac{-1}{9} \times \int y^{-2} dy$$

$$\Rightarrow \frac{-1}{9} \times \frac{y^{-2+1}}{-2+1} + C$$

Simplifying and substituting the value of y from (1),

$$\Rightarrow \frac{-1}{9} \times \frac{-1}{(1-9x)} + C$$

$$\Rightarrow \frac{1}{9(1-9x)} + C$$

Therefore,

$$\int \frac{dx}{(1-9x)^2} = \frac{1}{9(1-9x)} + C$$

2. Question

Evaluate:

$$\int \frac{dx}{(25-4x^2)}$$

Answer



To find: $\int \frac{dx}{(25-4x^2)}$

Formula Used: $\frac{dx}{(a^2-x^2)} = \frac{1}{2a} \times \log \left| \frac{a+x}{a-x} \right| + C$

Given equation = $\int \frac{dx}{4\left(\frac{25}{4}-x^2\right)}$

$$\Rightarrow \frac{1}{4} \int \frac{dx}{\left(\left(\frac{5}{2}\right)^2-x^2\right)} \dots (1)$$

Here $a = \frac{5}{2}$

Therefore, (1) becomes

$$\Rightarrow \frac{1}{4} \times \frac{1}{5} \times \log \left| \frac{\frac{5}{2}+x}{\frac{5}{2}-x} \right| + C$$

$$\Rightarrow \frac{1}{20} \times \log \left| \frac{5+2x}{5-2x} \right| + C$$

Therefore,

$$\int \frac{dx}{(25-4x^2)} = \frac{1}{20} \times \log \left| \frac{5+2x}{5-2x} \right| + C$$

3. Question

Evaluate:

$$\int \frac{dx}{(x^2+16)}$$



Answer

To find: $\int \frac{dx}{(x^2+16)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{4^2+x^2}$$

Here $a = 4$

$$\Rightarrow \frac{1}{4} \times \tan^{-1} \left(\frac{x}{4} \right) + C$$

Therefore,

$$\int \frac{dx}{(x^2+16)} = \frac{1}{4} \times \tan^{-1} \left(\frac{x}{4} \right) + C$$

4. Question

Evaluate:

$$\int \frac{dx}{(4+9x^2)}$$

Answer

To find: $\int \frac{dx}{(4+9x^2)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{9} \int \frac{dx}{\left(\frac{4}{9}\right) + x^2}$$

$$\Rightarrow \frac{1}{9} \int \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2}$$

Here $a = \frac{2}{3}$

$$\Rightarrow \frac{1}{9} \times \frac{3}{2} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

$$\Rightarrow \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

Therefore,

$$\int \frac{dx}{(4+9x^2)} = \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

5. Question

Evaluate:

$$\int \frac{dx}{(50+2x^2)}$$

**Answer**

To find: $\int \frac{dx}{(50+2x^2)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{2} \int \frac{dx}{25 + x^2}$$

$$\Rightarrow \frac{1}{2} \int \frac{dx}{5^2 + x^2}$$

Here $a = 5$

$$\Rightarrow \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2 + 16)} = \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

6. Question

Evaluate:

$$\int \frac{dx}{(16x^2 - 25)}$$

Answer

To find: $\int \frac{dx}{(16x^2 - 25)}$

Formula Used: $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{16} \int \frac{dx}{x^2 - \left(\frac{25}{16}\right)}$$

$$\Rightarrow \frac{1}{16} \int \frac{dx}{x^2 - \left(\frac{5}{4}\right)^2}$$

Here $a = \frac{5}{4}$

$$\Rightarrow \frac{1}{16} \times \frac{2}{5} \times \ln \left| \frac{x - \frac{5}{4}}{x + \frac{5}{4}} \right| + C$$

$$\Rightarrow \frac{1}{40} \times \ln \left| \frac{4x - 5}{4x + 5} \right| + C$$

Therefore,

$$\int \frac{dx}{(16x^2 - 25)} = \frac{1}{40} \times \log \left| \frac{4x - 5}{4x + 5} \right| + C$$



7. Question

Evaluate:

$$\int \frac{(x^2 - 1)}{(x^2 + 4)} dx$$

Answer

To find: $\int \frac{(x^2 - 1)}{(x^2 + 4)} dx$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \int \frac{(x^2 + 4 - 5)}{(x^2 + 4)} dx$$

$$\Rightarrow \int \frac{(x^2 + 4)}{(x^2 + 4)} dx - \int \frac{5}{(x^2 + 4)} dx$$

$$\Rightarrow \int dx - 5 \int \frac{1}{(x^2 + 2^2)} dx$$

Here $a = 2$,

$$\Rightarrow x - \frac{5}{2} \tan^{-1} \frac{x}{2} + C$$

Therefore,

$$\int \frac{(x^2 - 1)}{(x^2 + 4)} dx = x - \frac{5}{2} \tan^{-1} \frac{x}{2} + C$$

8. Question

Evaluate:

$$\int \frac{x^2}{(9 + 4x^2)} dx$$

Answer

To find: $\int \frac{x^2}{(9 + 4x^2)} dx$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \frac{1}{4} \int \frac{x^2}{(x^2 + \frac{9}{4})} dx$$

$$\Rightarrow \frac{1}{4} \int \frac{x^2 + \frac{9}{4} - \frac{9}{4}}{(x^2 + \frac{9}{4})} dx$$

$$\Rightarrow \frac{1}{4} \int dx - \frac{9}{16} \int \frac{1}{(x^2 + (\frac{3}{2})^2)} dx$$

Here $a = \frac{3}{2}$,

$$\Rightarrow \frac{x}{4} - \left(\frac{9}{16} \times \frac{2}{3} \tan^{-1} \frac{2x}{3} \right) + C$$

$$\Rightarrow \frac{x}{4} - \frac{3}{8} \tan^{-1} \left(\frac{2x}{3} \right) + C$$

Therefore,

$$\int \frac{x^2}{(9 + 4x^2)} dx = \frac{x}{4} - \frac{3}{8} \tan^{-1} \left(\frac{2x}{3} \right) + C$$

9. Question

Evaluate:

$$\int \frac{e^x}{(e^{2x} + 1)} dx$$

Answer

To find: $\int \frac{e^x}{(e^{2x} + 1)} dx$

Formula Used: $\int \frac{dx}{1 + x^2} = \tan^{-1} x$

Let $y = e^x \dots (1)$

Differentiating both sides, we get

$$dy = e^x dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{dy}{y^2 + 1}$$

$$\Rightarrow \tan^{-1} y$$

From (1),

$$\Rightarrow \tan^{-1}(e^x)$$

Therefore,

$$\int \frac{e^x}{(e^{2x} + 1)} dx = \tan^{-1}(e^x) + C$$

10. Question

Evaluate:

$$\int \frac{\sin x}{(1 + \cos^2 x)} dx$$

Answer

To find: $\int \frac{\sin x}{(1 + \cos^2 x)} dx$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Let $y = \cos x \dots (1)$

Differentiating both sides, we get

$$dy = -\sin x dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{-dy}{1 + y^2}$$

$$\Rightarrow -\tan^{-1} y$$

From (1),

$$\Rightarrow -\tan^{-1}(\cos x)$$

Therefore,

$$\int \frac{\sin x}{(1 + \cos^2 x)} dx = -\tan^{-1}(\cos x) + C$$

11. Question

Evaluate:

$$\int \frac{\cos x}{(1 + \sin^2 x)} dx$$

Answer



To find: $\int \frac{\cos x}{(1+\sin^2 x)} dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Let $y = \sin x \dots (1)$

Differentiating both sides, we get

$$dy = \cos x dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{dy}{1+y^2}$$

$$\Rightarrow \tan^{-1} y$$

From (1),

$$\Rightarrow \tan^{-1}(\sin x)$$

Therefore,

$$\int \frac{\cos x}{(1+\sin^2 x)} dx = \tan^{-1}(\sin x) + C$$

12. Question

Evaluate:

$$\int \frac{3x^5}{(1+x^{12})} dx$$

Answer

To find: $\int \frac{3x^5}{(1+x^{12})} dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Let $y = x^6 \dots (1)$

Differentiating both sides, we get

$$dy = 6x^5 dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2} dy}{1+y^2}$$

$$\Rightarrow \frac{1}{2} \tan^{-1} y + C$$

From (1),

$$\Rightarrow \frac{1}{2} \tan^{-1}(x^6) + C$$

Therefore,

$$\int \frac{3x^5}{(1+x^{12})} dx = \frac{1}{2} \tan^{-1}(x^6) + C$$

13. Question



Evaluate:

$$\int \frac{2x^3}{(4+x^8)} dx$$

Answer

To find: $\int \frac{2x^3}{(4+x^8)} dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Let $y = x^4 \dots (1)$

Differentiating both sides, we get

$$dy = 4x^3 dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2} dy}{4+y^2}$$

$$\Rightarrow \frac{1}{2} \int \frac{1}{2^2+y^2} dy$$

$$\Rightarrow \frac{1}{4} \tan^{-1}\left(\frac{y}{2}\right) + C$$

From (1),

$$\Rightarrow \frac{1}{4} \tan^{-1}\left(\frac{x^4}{2}\right) + C$$



Therefore,

$$\int \frac{2x^3}{(4+x^8)} dx = \frac{1}{4} \tan^{-1}\left(\frac{x^4}{2}\right) + C$$

14. Question

Evaluate:

$$\int \frac{dx}{(e^x + e^{-x})}$$

Answer

To find: $\int \frac{dx}{(e^x + e^{-x})}$

Formula Used: $\int \frac{dx}{1+x^2} = \tan^{-1} x$

Given equation is:

$$\int \frac{dx}{(e^x + e^{-x})} = \int \frac{e^x dx}{(e^{2x} + 1)} \dots (1)$$

Let $y = e^x \dots (1)$

Differentiating both sides, we get

$$dy = e^x dx$$

Substituting in (1),

$$\Rightarrow \int \frac{dy}{y^2 + 1}$$

$$\Rightarrow \tan^{-1} y$$

From (1),

$$\Rightarrow \tan^{-1}(e^x)$$

Therefore,

$$\int \frac{dx}{(e^x + e^{-x})} = \tan^{-1}(e^x) + C$$

15. Question

Evaluate:

$$\int \frac{x}{(1-x^4)} dx$$

Answer

To find: $\int \frac{x dx}{(1-x^4)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

Let $y = x^2 \dots (1)$

Differentiating both sides, we get

$$dy = 2x dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2} dy}{1-y^2}$$

Here $a = 1$,

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \log \left| \frac{1+y}{1-y} \right| + C$$

$$\Rightarrow \frac{1}{4} \log \left| \frac{1+y}{1-y} \right| + C$$

From (1),

$$\Rightarrow \frac{1}{4} \log \left| \frac{1+x^2}{1-x^2} \right| + C$$

Therefore,

$$\int \frac{x dx}{(1-x^4)} = \frac{1}{4} \log \left| \frac{1+x^2}{1-x^2} \right| + C$$

16. Question

Evaluate:



$$\int \frac{x^2}{(a^6 - x^6)} dx$$

Answer

To find: $\int \frac{x^2 dx}{(a^6 - x^6)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

Let $y = x^3 \dots (1)$

Differentiating both sides, we get

$$dy = 3x^2 dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{3} dy}{a^6 - y^2}$$

$$\Rightarrow \frac{1}{3} \int \frac{1}{(a^3)^2 - y^2} dy$$

$$\Rightarrow \frac{1}{3} \times \frac{1}{2a^3} \times \log \left| \frac{a^3 + y}{a^3 - y} \right| + C$$

$$\Rightarrow \frac{1}{6a^3} \log \left| \frac{a^3 + y}{a^3 - y} \right| + C$$

From (1),

$$\Rightarrow \frac{1}{6a^3} \log \left| \frac{a^3 + x^3}{a^3 - x^3} \right| + C$$



Therefore,

$$\int \frac{x^2 dx}{(a^6 - x^6)} = \frac{1}{6a^3} \log \left| \frac{a^3 + x^3}{a^3 - x^3} \right| + C$$

17. Question

Evaluate:

$$\int \frac{dx}{(x^2 + 4x + 8)}$$

Answer

To find: $\int \frac{dx}{(x^2 + 4x + 8)}$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{((x + 2)^2 + 4)}$$

$$\Rightarrow \int \frac{dx}{((x+2)^2 + 2^2)} \dots (1)$$

Let $y = x + 2 \dots (2)$

Differentiating both sides,

$$dy = dx$$

Substituting in (1),

$$\Rightarrow \int \frac{dy}{(y^2 + 2^2)}$$

Here $a = 2$,

$$\Rightarrow \frac{1}{2} \tan^{-1} \left(\frac{y}{2} \right) + C$$

From (2),

$$\Rightarrow \frac{1}{2} \tan^{-1} \left(\frac{x+2}{2} \right) + C$$

Therefore,

$$\int \frac{dx}{(x^2 + 4x + 8)} = \frac{1}{2} \tan^{-1} \left(\frac{x+2}{2} \right) + C$$

18. Question

Evaluate:

$$\int \frac{dx}{(4x^2 - 4x + 3)}$$

Answer

To find: $\int \frac{dx}{(4x^2 - 4x + 3)}$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{((2x-1)^2 + 2)} \dots (1)$$

Let $y = 2x - 1 \dots (2)$

Differentiating both sides,

$$dy = 2dx$$

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{2} dy}{(y^2 + (\sqrt{2})^2)}$$

Here $a = \sqrt{2}$,

$$\Rightarrow \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{y}{\sqrt{2}} \right) + C$$

From (2),

$$\Rightarrow \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{2x-1}{\sqrt{2}} \right) + C$$

Therefore,



$$\int \frac{dx}{(4x^2 - 4x + 3)} = \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{2x-1}{\sqrt{2}} \right) + C$$

19. Question

Evaluate:

$$\int \frac{dx}{(2x^2 + x + 3)}$$

Answer

To find: $\int \frac{dx}{(2x^2 + x + 3)}$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x + \frac{1}{2\sqrt{2}} \right)^2 + 3 - \frac{1}{8} \right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x + \frac{1}{2\sqrt{2}} \right)^2 + \frac{23}{8} \right)} \dots (1)$$

Let $y = \sqrt{2}x + \frac{1}{2\sqrt{2}} \dots (2)$

Differentiating both sides,

$$dy = \sqrt{2} dx$$

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}} dy}{\left(y^2 + \left(\frac{\sqrt{23}}{2\sqrt{2}} \right)^2 \right)}$$

Here $a = \frac{\sqrt{23}}{2\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{23}} \tan^{-1} \left(\frac{y \times 2\sqrt{2}}{\sqrt{23}} \right) + C$$

From (2),

$$\Rightarrow \frac{2}{\sqrt{23}} \tan^{-1} \left(\frac{4x+1}{\sqrt{23}} \right) + C$$

Therefore,

$$\int \frac{dx}{(2x^2 + x + 3)} = \frac{2}{\sqrt{23}} \tan^{-1} \left(\frac{4x+1}{\sqrt{23}} \right) + C$$

20. Question

Evaluate:

$$\int \frac{dx}{(2x^2 - x - 1)}$$

Answer



To find: $\int \frac{dx}{(2x^2-x-1)}$

Formula Used: $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}} \right)^2 - 1 - \left(\frac{1}{2\sqrt{2}} \right)^2 \right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}} \right)^2 - 1 - \frac{1}{8} \right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}} \right)^2 - \frac{9}{8} \right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}} \right)^2 - \left(\frac{3}{2\sqrt{2}} \right)^2 \right)} \dots (1)$$

Let $y = \sqrt{2}x - \frac{1}{2\sqrt{2}} \dots (2)$

Differentiating both sides,

$$dy = \sqrt{2} dx$$

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}} dy}{\left(y^2 - \left(\frac{3}{2\sqrt{2}} \right)^2 \right)}$$



Here $a = \frac{3}{2\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{3} \times \log \left| \frac{\frac{3}{2\sqrt{2}} + y}{\frac{3}{2\sqrt{2}} - y} \right| + C$$

$$\Rightarrow \frac{1}{3} \times \log \left| \frac{3 + 2\sqrt{2}y}{3 - 2\sqrt{2}y} \right| + C$$

From (2),

$$\Rightarrow \frac{1}{3} \times \log \left| \frac{3 + 4x - 1}{3 - 4x + 1} \right| + C$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{1 + 2x}{2(1-x)} \right| + C$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{2(x-1)}{2x+1} \right| + C$$

Therefore,

$$\int \frac{dx}{(2x^2-x-1)} = \frac{1}{3} \log \left| \frac{2(x-1)}{2x+1} \right| + C$$

21. Question

Evaluate:

$$\int \frac{dx}{(3-2x-x^2)}$$

Answer

To find: $\int \frac{dx}{(3-2x-x^2)}$

Formula Used: $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{-dx}{(x^2+2x-3)}$$

$$\Rightarrow \int \frac{-dx}{(x+1)^2-4}$$

$$\Rightarrow \int \frac{-dx}{(x+1)^2-2^2} \dots (1)$$

Let $y = x + 1 \dots (2)$

Differentiating both sides wrt x ,

$$dy = dx$$

Substituting in (1),

$$\Rightarrow \int \frac{-dy}{y^2-2^2}$$

$$\Rightarrow \int \frac{dy}{2^2-y^2}$$

Here $a = 2$,

$$\Rightarrow \frac{1}{4} \log \left| \frac{2+y}{2-y} \right| + C$$

From (2),

$$\Rightarrow \frac{1}{4} \log \left| \frac{x+3}{1-x} \right| + C$$

Therefore,

$$\int \frac{dx}{(3-2x-x^2)} = \frac{1}{4} \log \left| \frac{x+3}{1-x} \right| + C$$

22. Question

Evaluate:

$$\int \frac{x}{(x^2+3x+2)} dx$$

Answer

To find: $\int \frac{x dx}{(x^2+3x+2)}$



Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Using partial fractions,

$$x = A \left(\frac{d}{dx} (x^2 + 3x + 2) \right) + B$$

$$x = A(2x + 3) + B$$

Equating the coefficients of x ,

$$1 = 2A$$

$$A = \frac{1}{2}$$

$$\text{Also, } 0 = 3A + B$$

$$B = \frac{-3}{2}$$

Therefore, the given equation becomes,

$$\Rightarrow \int \frac{\frac{1}{2}(2x + 3) - \frac{3}{2}}{(x^2 + 3x + 2)} dx$$

$$\Rightarrow \frac{1}{2} \log|x^2 + 3x + 2| - \frac{3}{2} \int \frac{1}{\left(\left(x + \frac{3}{2} \right)^2 + 2 - \left(\frac{3}{2} \right)^2 \right)} dx$$

$$\Rightarrow \frac{1}{2} \log|x^2 + 3x + 2| - \frac{3}{2} \int \frac{1}{\left(\left(x + \frac{3}{2} \right)^2 - \left(\frac{1}{2} \right)^2 \right)} dx$$

$$\Rightarrow \frac{1}{2} \log|x^2 + 3x + 2| - \frac{3}{2} \times \log \left| \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right| + C$$

$$\Rightarrow \frac{1}{2} \log|x^2 + 3x + 2| - \frac{3}{2} \log \left| \frac{x+1}{x+2} \right| + C$$

Therefore,

$$\int \frac{x dx}{(x^2 + 3x + 2)} = \frac{1}{2} \log|x^2 + 3x + 2| - \frac{3}{2} \log \left| \frac{x+1}{x+2} \right| + C$$

23. Question

Evaluate:

$$\int \frac{(x-3)}{(x^2 + 2x - 4)} dx$$

Answer

$$\text{To find: } \int \frac{(x-3) dx}{(x^2 + 2x - 4)}$$

Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + C$$

Using partial fractions,

$$(x - 3) = A \left(\frac{d}{dx} (x^2 + 2x - 4) \right) + B$$

$$x - 3 = A(2x + 2) + B$$

Equating the coefficients of x,

$$1 = 2A$$

$$\Rightarrow A = \frac{1}{2}$$

$$\text{Also, } -3 = 2A + B$$

$$\Rightarrow B = -4$$

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(2x + 2) - 4}{(x^2 + 2x - 4)} dx$$

$$\Rightarrow \frac{1}{2} \log |x^2 + 2x - 4| - 4 \int \frac{1}{(x + 1)^2 - (\sqrt{5})^2} dx$$

$$\Rightarrow \frac{1}{2} \log |x^2 + 2x - 4| - \left(4 \times \frac{1}{2\sqrt{5}} \times \log \left| \frac{x + 1 - \sqrt{5}}{x + 1 + \sqrt{5}} \right| \right) + C$$

$$\Rightarrow \frac{1}{2} \log |x^2 + 2x - 4| - \frac{2}{\sqrt{5}} \log \left| \frac{x + 1 - \sqrt{5}}{x + 1 + \sqrt{5}} \right| + C$$

Therefore,

$$\int \frac{(x - 3) dx}{(x^2 + 2x - 4)} = \frac{1}{2} \log |x^2 + 2x - 4| - \frac{2}{\sqrt{5}} \log \left| \frac{x + 1 - \sqrt{5}}{x + 1 + \sqrt{5}} \right| + C$$

24. Question

Evaluate:

$$\int \frac{(2x - 3)}{(x^2 + 3x - 18)} dx$$

Answer

$$\text{To find: } \int \frac{(2x - 3)}{(x^2 + 3x - 18)} dx$$

Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + C$$

Using partial fractions,

$$(2x - 3) = A \left(\frac{d}{dx} (x^2 + 3x - 18) \right) + B$$

$$2x - 3 = A(2x + 3) + B$$

Equating the coefficients of x ,

$$2 = 2A$$

$$A = 1$$

$$\text{Also, } -3 = 3A + B$$

$$\Rightarrow B = -6$$

Substituting in the given equation,

$$\Rightarrow \int \frac{(2x + 3) - 6}{(x^2 + 3x - 18)} dx$$

$$\Rightarrow \log|x^2 + 3x - 18| + C_1 - 6 \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - 18 - \left(\frac{3}{2}\right)^2} dx \dots (1)$$

$$\text{Let } I = 6 \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - 18 - \left(\frac{3}{2}\right)^2} dx$$

$$\Rightarrow 6 \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} dx$$

$$\text{Here } a = \frac{9}{2}$$

$$\Rightarrow \frac{6}{9} \times \log \left| \frac{x + \frac{3}{2} - \frac{9}{2}}{x + \frac{3}{2} + \frac{9}{2}} \right| + C_2$$

$$\Rightarrow \frac{2}{3} \times \log \left| \frac{x-3}{x+6} \right| + C_2 \dots (2)$$

Substituting (2) in (1),

$$\Rightarrow \log|x^2 + 3x - 18| - \frac{2}{3} \log \left| \frac{x-3}{x+6} \right| + C$$

Therefore,

$$\int \frac{(2x - 3)}{(x^2 + 3x - 18)} dx = \log|x^2 + 3x - 18| - \frac{2}{3} \log \left| \frac{x-3}{x+6} \right| + C$$

25. Question

Evaluate:

$$\int \frac{x^2}{(x^2 + 6x - 3)} dx$$

Answer

$$\text{To find: } \int \frac{x^2}{(x^2 + 6x - 3)} dx$$

Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$



$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Given equation can be rewritten as following:

$$\Rightarrow \int \frac{x^2 + (6x - 3) - (6x - 3)}{(x^2 + 6x - 3)} dx$$

$$\Rightarrow \int \frac{(x^2 + 6x - 3) - (6x - 3)}{(x^2 + 6x - 3)} dx$$

$$\Rightarrow x - \int \frac{6x - 3}{x^2 + 6x - 3} dx$$

$$\text{Let } I = \int \frac{6x-3}{x^2+6x-3} dx \dots (2)$$

Using partial fractions,

$$(6x - 3) = A \left(\frac{d}{dx}(x^2 + 6x - 3) \right) + B$$

$$6x - 3 = A(2x + 6) + B$$

Equating the coefficients of x ,

$$6 = 2A$$

$$A = 3$$

$$\text{Also, } -3 = 6A + B$$

$$\Rightarrow B = -21$$

Substituting in (1),

$$\Rightarrow \int \frac{3(2x + 6) - 21}{(x^2 + 6x - 3)} dx$$

$$\Rightarrow 3 \times \log|x^2 + 6x - 3| + C_1 - 21 \int \frac{1}{(x + 3)^2 - (\sqrt{12})^2} dx$$

$$\Rightarrow 3 \times \log|x^2 + 6x - 3| + C_1 - 21 \times \frac{1}{2\sqrt{12}} \times \log \left| \frac{x + 3 - \sqrt{12}}{x + 3 + \sqrt{12}} \right| + C_2$$

$$I = 3\log|x^2 + 6x - 3| - \frac{7\sqrt{3}}{4} \times \log \left| \frac{x+3-2\sqrt{3}}{x+3+2\sqrt{3}} \right| + C$$

Therefore,

$$\int \frac{x^2}{(x^2 + 6x - 3)} dx = x - 3\log|x^2 + 6x - 3| + \frac{7\sqrt{3}}{4} \times \log \left| \frac{x + 3 - 2\sqrt{3}}{x + 3 + 2\sqrt{3}} \right| + C$$

26. Question

Evaluate:

$$\int \frac{(2x - 1)}{(2x^2 + 2x + 1)} dx$$

Answer

$$\text{To find: } \int \frac{2x-1}{(2x^2+2x+1)} dx$$

Formula Used:



$$1. \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Using partial fractions,

$$(2x - 1) = A \left(\frac{d}{dx} (2x^2 + 2x + 1) \right) + B$$

$$2x - 1 = A(4x + 2) + B$$

Equating the coefficients of x ,

$$2 = 4A$$

$$A = \frac{1}{2}$$

$$\text{Also, } -1 = 2A + B$$

$$\Rightarrow B = -2$$

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(4x + 2) - 2}{(2x^2 + 2x + 1)} dx$$

$$\Rightarrow \frac{1}{2} \log|2x^2 + 2x + 1| - 2 \int \frac{1}{2(x^2 + x + \frac{1}{2})} dx$$

$$\text{Let } I = 2 \int \frac{1}{2(x^2 + x + \frac{1}{2})} dx \dots (1)$$

$$\Rightarrow \int \frac{1}{(x^2 + x + \frac{1}{2})} dx$$

$$\Rightarrow \int \frac{1}{\left((x + \frac{1}{2})^2 + \frac{1}{2} - (\frac{1}{2})^2 \right)} dx$$

$$\Rightarrow \int \frac{1}{\left((x + \frac{1}{2})^2 + \frac{1}{2} - \frac{1}{4} \right)} dx$$

$$\Rightarrow \int \frac{1}{\left((x + \frac{1}{2})^2 + (\frac{1}{2})^2 \right)} dx$$

$$\text{Here } a = \frac{1}{2}$$

$$\Rightarrow 2 \tan^{-1} \left(\frac{x + \frac{1}{2}}{\frac{1}{2}} \right) + C$$

$$\Rightarrow 2 \tan^{-1}(2x + 1) + C$$

Substituting in (1) and combining with original equation,

$$\Rightarrow \frac{1}{2} \log|2x^2 + 2x + 1| - 2 \tan^{-1}(2x + 1) + C$$



Therefore,

$$\int \frac{2x-1}{(2x^2+2x+1)} dx = \frac{1}{2} \log|2x^2+2x+1| - 2 \tan^{-1}(2x+1) + C$$

27. Question

Evaluate:

$$\int \frac{(1-3x)}{(3x^2+4x+2)} dx$$

Answer

To find: $\int \frac{1-3x}{(3x^2+4x+2)} dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

2. $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$

Rewriting the given equation,

$$\Rightarrow - \int \frac{3x-1}{(3x^2+4x+2)} dx$$

Using partial fractions,

$$(3x-1) = A \left(\frac{d}{dx} (3x^2+4x+2) \right) + B$$

$$3x-1 = A(6x+4) + B$$

Equating the coefficients of x,

$$3 = 6A$$

$$A = \frac{1}{2}$$

Also, $-1 = 4A + B$

$$\Rightarrow B = -3$$

Substituting in the original equation,

$$\Rightarrow - \int \frac{\frac{1}{2}(6x+4) - 3}{(3x^2+4x+2)} dx$$

$$\Rightarrow -\frac{1}{2} \log|3x^2+4x+2| + 3 \int \frac{1}{3(x^2 + \frac{4}{3}x + \frac{2}{3})} dx$$

$$\text{Let } I = 3 \int \frac{1}{3(x^2 + \frac{4}{3}x + \frac{2}{3})} dx$$

$$\Rightarrow \int \frac{1}{(x^2 + \frac{4}{3}x + \frac{2}{3})} dx$$

$$\Rightarrow \int \frac{1}{\left((x + \frac{2}{3})^2 + \frac{2}{3} - \frac{4}{9} \right)} dx$$



$$\Rightarrow \int \frac{1}{\left(\left(x + \frac{2}{3} \right)^2 + \left(\frac{\sqrt{2}}{3} \right)^2 \right)} dx$$

Here $a = \frac{\sqrt{2}}{3}$

$$\Rightarrow \frac{3}{\sqrt{2}} \tan^{-1} \left(\frac{x + \frac{2}{3}}{\frac{\sqrt{2}}{3}} \right) + C$$

$$\Rightarrow \frac{3}{\sqrt{2}} \tan^{-1} \left(\frac{3x + 2}{\sqrt{2}} \right) + C$$

Substituting in (1) and combining with original equation,

$$\Rightarrow -\frac{1}{2} \log|3x^2 + 4x + 2| + \frac{3}{\sqrt{2}} \tan^{-1} \left(\frac{3x + 2}{\sqrt{2}} \right) + C$$

Therefore,

$$\int \frac{1 - 3x}{(3x^2 + 4x + 2)} dx = -\frac{1}{2} \log|3x^2 + 4x + 2| + \frac{3}{\sqrt{2}} \tan^{-1} \left(\frac{3x + 2}{\sqrt{2}} \right) + C$$

28. Question

Evaluate:

$$\int \frac{2x}{(2 + x - x^2)} dx$$

Answer

To find: $\int \frac{2x}{(2+x-x^2)} dx$

Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Rewriting the given equation,

$$\Rightarrow -2 \int \frac{x}{(x^2 - x - 2)} dx$$

Using partial fractions,

$$x = A \left(\frac{d}{dx} (x^2 - x - 2) \right) + B$$

$$x = A(2x - 1) + B$$

Equating the coefficients of x,

$$1 = 2A$$

$$A = \frac{1}{2}$$

Also, $0 = -A + B$



$$B = \frac{1}{2}$$

Substituting in the original equation,

$$\Rightarrow -2 \int \frac{\frac{1}{2}(2x-1) + \frac{1}{2}}{(x^2-x-2)} dx$$

$$\Rightarrow -\log|x^2-x-2| - \int \frac{1}{(x^2-x-2)} dx$$

$$\text{Let } I = \int \frac{1}{(x^2-x-2)} dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2 - 2 - \frac{1}{4}\right)} dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right)} dx$$

$$\text{Here } a = \frac{3}{2}$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{x - \frac{1}{2} - \frac{3}{2}}{x - \frac{1}{2} + \frac{3}{2}} \right| + C$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{x-2}{x+1} \right| + C$$

Substituting for I and combining with the original equation,

$$\Rightarrow -\log|x^2-x-2| + \frac{1}{3} \log \left| \frac{x-2}{x+1} \right| + C$$

Therefore,

$$\int \frac{2x}{(2+x-x^2)} dx = -\log|x^2-x-2| + \frac{1}{3} \log \left| \frac{x-2}{x+1} \right| + C$$

or

$$\int \frac{2x}{(2+x-x^2)} dx = -\log|2+x-x^2| + \frac{1}{3} \log \left| \frac{1+x}{2-x} \right| + C$$

29. Question

Evaluate:

$$\int \frac{dx}{(1+\cos^2 x)}$$

Answer

$$\text{To find: } \int \frac{1}{(1+\cos^2 x)} dx$$

Formula Used:

$$1. \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$2. \sec^2 x = 1 + \tan^2 x$$

Dividing the given equation by $\cos^2 x$ in the numerator and denominator gives us,

$$\Rightarrow \int \frac{\sec^2 x \, dx}{1 + \sec^2 x} \dots (1)$$

Let $y = \tan x$

$$dy = \sec^2 x \, dx \dots (2)$$

$$\text{Also, } y^2 = \tan^2 x$$

$$\text{i.e., } y^2 = \sec^2 x - 1$$

$$\sec^2 x = y^2 + 1 \dots (3)$$

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{1 + y^2 + 1}$$

$$\Rightarrow \int \frac{dy}{y^2 + (\sqrt{2})^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{y}{\sqrt{2}} \right) + C$$

Since $y = \tan x$,

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan x}{\sqrt{2}} \right) + C$$

Therefore,

$$\int \frac{1}{(1 + \cos^2 x)} dx = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan x}{\sqrt{2}} \right) + C$$



30. Question

Evaluate:

$$\int \frac{dx}{(2 + \sin^2 x)}$$

Answer

$$\text{To find: } \int \frac{1}{(2 + \sin^2 x)} dx$$

Formula Used:

$$1. \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$2. \sec^2 x = 1 + \tan^2 x$$

Dividing the given equation by $\cos^2 x$ in the numerator and denominator gives us,

$$\Rightarrow \int \frac{\sec^2 x \, dx}{2 \sec^2 x + \tan^2 x} \dots (1)$$

Let $y = \tan x$

$$dy = \sec^2 x \, dx \dots (2)$$

$$\text{Also, } y^2 = \tan^2 x$$

$$\text{i.e., } y^2 = \sec^2 x - 1$$

$$\sec^2 x = y^2 + 1 \dots (3)$$

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{2y^2 + 2 + y^2}$$

$$\Rightarrow \int \frac{dy}{3y^2 + 2}$$

$$\Rightarrow \frac{1}{3} \int \frac{dy}{y^2 + \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{\sqrt{2}} \tan^{-1} \left(\frac{y\sqrt{3}}{\sqrt{2}} \right) + C$$

Since $y = \tan x$,

$$\Rightarrow \frac{1}{\sqrt{6}} \tan^{-1} \left(\frac{\sqrt{3} \tan x}{\sqrt{2}} \right) + C$$

Therefore,

$$\int \frac{1}{(2 + \sin^2 x)} dx = \frac{1}{\sqrt{6}} \tan^{-1} \left(\frac{\sqrt{3} \tan x}{\sqrt{2}} \right) + C$$

31. Question

Evaluate:

$$\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)}$$



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Answer

To find: $\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x dx}{a^2 + b^2 \tan^2 x}$$

Let $y = \tan x$

$$dy = \sec^2 x dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{a^2 + b^2 y^2}$$

$$\Rightarrow \frac{1}{b^2} \int \frac{dy}{\left(\frac{a}{b}\right)^2 + y^2}$$

$$\Rightarrow \frac{1}{b^2} \times \frac{b}{a} \tan^{-1} \frac{yb}{a} + C$$

Since $y = \tan x$,

$$\Rightarrow \frac{1}{ab} \tan^{-1} \left(\frac{b \tan x}{a} \right) + C$$

Therefore,

$$\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)} = \frac{1}{a^2} \tan^{-1} \left(\frac{b}{a} \tan x \right) + C$$

32. Question

Evaluate:

$$\int \frac{dx}{(\cos^2 x - 3 \sin^2 x)}$$

Answer

To find: $\int \frac{dx}{(\cos^2 x - 3 \sin^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x dx}{1 - 3 \tan^2 x}$$

Let $y = \tan x$

$$dy = \sec^2 x dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{1 - 3y^2}$$

$$\Rightarrow \frac{1}{3} \int \frac{dy}{\left(\frac{1}{\sqrt{3}}\right)^2 - y^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{2} \log \left| \frac{\frac{1}{\sqrt{3}} + y}{\frac{1}{\sqrt{3}} - y} \right| + C$$

$$\Rightarrow \frac{1}{2\sqrt{3}} \log \left| \frac{1 + y\sqrt{3}}{1 - y\sqrt{3}} \right| + C$$

Since $y = \tan x$,

$$\Rightarrow \frac{1}{2\sqrt{3}} \log \left| \frac{1 + \sqrt{3} \tan x}{1 - \sqrt{3} \tan x} \right| + C$$

Therefore,



$$\int \frac{dx}{(\cos^2 x - 3 \sin^2 x)} = \frac{1}{2\sqrt{3}} \log \left| \frac{1 + \sqrt{3} \tan x}{1 - \sqrt{3} \tan x} \right| + C$$

33. Question

Evaluate:

$$\int \frac{dx}{(\sin^2 x - 4 \cos^2 x)}$$

Answer

To find: $\int \frac{dx}{(\sin^2 x - 4 \cos^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x dx}{\tan^2 x - 4}$$

Let $y = \tan x$

$$dy = \sec^2 x dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{y^2 - 2^2}$$

$$\Rightarrow \frac{1}{4} \log \left| \frac{y-2}{y+2} \right| + C$$

Since $y = \tan x$,

$$\Rightarrow \frac{1}{4} \log \left| \frac{\tan x - 2}{\tan x + 2} \right| + C$$

Therefore,

$$\int \frac{dx}{(\sin^2 x - 4 \cos^2 x)} = \frac{1}{4} \log \left| \frac{\tan x - 2}{\tan x + 2} \right| + C$$

34. Question

Evaluate:

$$\int \frac{dx}{(\sin x \cos x + 2 \cos^2 x)}$$

Answer

To find: $\int \frac{dx}{(\sin x \cos x + 2 \cos^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x} dx = \log x + C$



Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \, dx}{\tan x + 2}$$

Let $y = \tan x$

$$dy = \sec^2 x \, dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{y + 2}$$

$$\Rightarrow \log |y + 2| + C$$

Since $y = \tan x$,

$$\Rightarrow \log |\tan x + 2| + C$$

Therefore,

$$\int \frac{dx}{(\sin x \cos x + 2 \cos^2 x)} = \log |\tan x + 2| + C$$

35. Question

Evaluate:

$$\int \frac{\sin 2x}{(\sin^4 x + \cos^4 x)} dx$$

Answer

To find: $\int \frac{\sin 2x \, dx}{(\sin^4 x + \cos^4 x)}$



Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

3. $\sin 2x = 2 \sin x \cos x$

Rewriting the given equation,

$$\Rightarrow \int \frac{2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

Dividing by $\cos^4 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{2 \tan x \sec^2 x \, dx}{\tan^4 x + 1}$$

Let $y = \tan x$

$$dy = \sec^2 x \, dx$$

Therefore,

$$\Rightarrow \int \frac{2y}{y^4 + 1} dy$$

Let $z = y^2$

$$dz = 2y \, dy$$

$$\Rightarrow \int \frac{dz}{1+z^2}$$

$$\Rightarrow \tan^{-1} z + C$$

$$\text{Since } z = y^2,$$

$$\Rightarrow \tan^{-1}(y^2) + C$$

$$\text{Since } y = \tan x,$$

$$\Rightarrow \tan^{-1}(\tan^2 x) + C$$

Therefore,

$$\int \frac{\sin 2x \, dx}{(\sin^4 x + \cos^4 x)} = \tan^{-1}(\tan^2 x) + C$$

36. Question

Evaluate:

$$\int \frac{(2 \sin 2\phi - \cos \phi)}{(6 - \cos^2 \phi - 4 \sin \phi)} d\phi$$

Answer

$$\text{To find: } \int \frac{(2 \sin 2\phi - \cos \phi)}{(6 - \cos^2 \phi - 4 \sin \phi)} d\phi$$

Formula Used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$3. \sin 2x = 2 \sin x \cos x$$

Rewriting the given equation,

$$\Rightarrow \int \frac{4 \sin \phi \cos \phi - \cos \phi}{6 - \cos^2 \phi - 4 \sin \phi} d\phi$$

$$\Rightarrow \int \frac{\cos \phi (4 \sin \phi - 1)}{6 - (1 - \sin^2 \phi) - 4 \sin \phi} d\phi$$

$$\Rightarrow \int \frac{\cos \phi (4 \sin \phi - 1)}{5 + \sin^2 \phi - 4 \sin \phi} d\phi$$

$$\text{Let } y = \sin \phi$$

$$dy = \cos \phi \, d\phi$$

Substituting in the original equation,

$$\Rightarrow \int \frac{4y-1}{y^2-4y+5} dy \dots (1)$$

Using partial fraction,

$$4y - 1 = A \left(\frac{d}{dy} (y^2 - 4y + 5) \right) + B$$

$$4y - 1 = A(2y - 4) + B$$

Equating the coefficients of y,



$$4 = 2A$$

$$A = 2$$

$$\text{Also, } -1 = -4A + B$$

$$B = 7$$

Substituting in (1),

$$\Rightarrow \int \frac{2(2y-4) + 7}{y^2 - 4y + 5} dy$$

$$\Rightarrow 2 \log|y^2 - 4y + 5| + 7 \int \frac{1}{((y-2)^2 + 1)} dy$$

$$\Rightarrow 2 \log|y^2 - 4y + 5| + 7 \tan^{-1}(y - 2) + C$$

But $y = \sin \phi$

$$\Rightarrow 2 \log|\sin^2 \phi - 4 \sin \phi + 5| + 7 \tan^{-1}(\sin \phi - 2) + C$$

Therefore,

$$\begin{aligned} \int \frac{(2 \sin 2\phi - \cos \phi)}{(6 - \cos^2 \phi - 4 \sin \phi)} d\phi \\ = 2 \log|\sin^2 \phi - 4 \sin \phi + 5| + 7 \tan^{-1}(\sin \phi - 2) + C \end{aligned}$$

37. Question

Evaluate:

$$\int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)}$$



Answer

$$\text{To find: } \int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)}$$

Formula Used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{x} dx = \log x + C$$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x dx}{(\tan x - 2)(2 \tan x + 1)}$$

Let $y = \tan x$

$$dy = \sec^2 x dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{(y-2)(2y+1)} \dots (1)$$

Let

$$\frac{1}{(y-2)(2y+1)} = \frac{A}{(y-2)} + \frac{B}{(2y+1)}$$

$$1 = A(2y+1) + B(y-2)$$

When $y = 0$,

$$1 = A - 2B \dots (2)$$

When $y = 1$,

$$1 = 3A - B \Rightarrow 2 = 6A - 2B \dots (3)$$

Solving (2) and (3),

$$1 = 5A$$

$$A = \frac{1}{5}$$

$$\text{So, } B = \frac{-2}{5}$$

(1) becomes,

$$\Rightarrow \int \frac{\frac{1}{5}}{(y-2)} + \frac{\frac{-2}{5}}{(2y+1)}$$

$$\Rightarrow \frac{1}{5} \log|y-2| - \frac{2}{5} \log|2y+1| \times \frac{1}{2} + C$$

Since $y = \tan x$,

$$\Rightarrow \frac{1}{5} \log|\tan x - 2| - \frac{1}{5} \log|2 \tan x + 1| + C$$

$$\Rightarrow \frac{1}{5} \log \left| \frac{\tan x - 2}{2 \tan x + 1} \right| + C$$

Therefore,

$$\int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)} = \frac{1}{5} \log \left| \frac{\tan x - 2}{2 \tan x + 1} \right| + C$$

38. Question

Evaluate:

$$\int \frac{(1-x^2)}{(1+x^4)} dx$$

Answer

$$\text{To find: } \int \frac{(1-x^2)}{(1+x^4)} dx$$

$$\text{Formula used: } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

On dividing by x^2 in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2} dx$$

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2 + 2 - 2} dx$$



$$\Rightarrow \int \frac{-\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 - 2} dx$$

Let $y = x + \frac{1}{x}$

Differentiating wrt x ,

$$dy = \left(1 - \frac{1}{x^2}\right) dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{-dy}{y^2 - (\sqrt{2})^2}$$

$$\Rightarrow \frac{-1}{2\sqrt{2}} \log \left| \frac{y - \sqrt{2}}{y + \sqrt{2}} \right| + C$$

Substituting for $y = x + \frac{1}{x}$ and taking reciprocal of the value within logarithm, we get

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{x + \frac{1}{x} + \sqrt{2}}{x + \frac{1}{x} - \sqrt{2}} \right| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}x + x^2 + 1}{\sqrt{2}x - x^2 + 1} \right| + C$$

Therefore,

$$\int \frac{(1-x^2)}{(1+x^4)} dx = \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}x + x^2 + 1}{\sqrt{2}x - x^2 + 1} \right| + C$$



39. Question

Evaluate:

$$\int \frac{(x^2 + 1)}{(x^4 + x^2 + 1)} dx$$

Answer

To find: $\int \frac{(x^2+1)}{(x^4+x^2+1)} dx$

Formula used: $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

On dividing by x^2 in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$\Rightarrow \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 3} dx$$

Let $y = x - \frac{1}{x}$

Differentiating wrt x ,

$$dy = \left(1 + \frac{1}{x^2}\right) dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{dy}{y^2 + (\sqrt{3})^2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \frac{y}{\sqrt{3}} + C$$

Substituting for $y = x - \frac{1}{x}$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x - \frac{1}{x}}{\sqrt{3}} \right) + C$$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) + C$$

Therefore,

$$\int \frac{(x^2 + 1)}{(x^4 + x^2 + 1)} dx = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) + C$$

40. Question

Evaluate:

$$\int \frac{dx}{(\sin^4 x + \cos^4 x)}$$



Answer

To find: $\int \frac{dx}{(\sin^4 x + \cos^4 x)}$

Formula used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

Dividing by $\cos^4 x$ in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\sec^4 x}{(\tan^4 x + 1)} dx$$

$$\Rightarrow \int \frac{\sec^2 x (1 + \tan^2 x)}{(1 + \tan^4 x)} dx$$

Let $y = \tan x$

$$dy = \sec^2 x dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{1 + y^2}{1 + y^4} dy$$

Dividing by y^2 in the numerator and denominator,

$$\Rightarrow \int \frac{y^{-2} + 1}{y^{-2} + y^2} dy$$

$$\Rightarrow \int \frac{1 + y^{-2}}{y^2 + y^{-2} - 2 + 2} dy$$

$$\Rightarrow \int \frac{1 + y^{-2}}{\left(y - \frac{1}{y}\right)^2 + 2} dy$$

Let $z = y - \frac{1}{y}$

$$dz = \left(1 + \frac{1}{y^2}\right) dy$$

Therefore,

$$\Rightarrow \int \frac{dz}{z^2 + (\sqrt{2})^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + C$$

Substituting for z ,

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{y - \frac{1}{y}}{\sqrt{2}}\right) + C$$

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{y^2 - 1}{y\sqrt{2}}\right) + C$$



Substituting for $y = \tan x$,

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2} \tan x}\right) + C$$

Therefore,

$$\int \frac{dx}{(\sin^4 x + \cos^4 x)} = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2} \tan x}\right) + C$$

Exercise 14B

1. Question

Evaluate:

$$\int \frac{dx}{\sqrt{16 - x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{16 - x^2}}$$

$$= \int \frac{dx}{\sqrt{4^2 - x^2}}$$

$$= \sin^{-1} \frac{x}{4} + c, c \text{ being the integrating constant}$$

2. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1-9x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1-9x^2}}$$

$$= \int \frac{dx}{\sqrt{9\left\{\left(\frac{1}{9}\right) - x^2\right\}}}$$

$$= \frac{1}{3} \int \frac{dx}{\sqrt{1^2 - \left(\frac{x}{3}\right)^2}}$$

$$= \frac{1}{3} \sin^{-1} \frac{x}{1} \frac{1}{3} + c$$

$$= \frac{1}{3} \sin^{-1} 3x + c, c \text{ being the integrating constant}$$



3. Question

Evaluate:

$$\int \frac{dx}{\sqrt{15-8x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{15-8x^2}}$$

$$= \int \frac{dx}{\sqrt{15\left\{1 - \left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2\right\}}}$$

$$= \frac{1}{\sqrt{15}} \int \frac{dx}{\sqrt{1^2 - \left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2}}$$

$$= \frac{1}{\sqrt{15}} \sin^{-1} \frac{x}{\left(\frac{\sqrt{15}}{\sqrt{8}}\right)} + c$$

$$= \frac{1}{\sqrt{15}} \sin^{-1} \frac{\sqrt{8}}{\sqrt{15}} x + c, c \text{ being the integrating constant}$$

4. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 4}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 4}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 2^2}}$$

$$= \log|x + \sqrt{x^2 - 4}| + c, c \text{ being the integrating constant}$$

5. Question

Evaluate:

$$\int \frac{dx}{\sqrt{4x^2 - 1}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{4x^2 - 1}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 - 1^2}}$$

$$= \frac{1}{2} \log|2x + \sqrt{4x^2 - 1}| + c, c \text{ being the integrating constant}$$

6. Question

Evaluate:

$$\int \frac{dx}{\sqrt{9x^2 - 7}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{9x^2 - 7}}$$

$$= \int \frac{dx}{\sqrt{(3x)^2 - \sqrt{7}^2}}$$

$$= \log|3x + \sqrt{9x^2 - 7}| + c, c \text{ being the integrating constant}$$

7. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 9}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 9}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 3^2}}$$

$$= \log|x + \sqrt{x^2 - 9}| + c, c \text{ being the integrating constant}$$

8. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1 + 4x^2}}$$

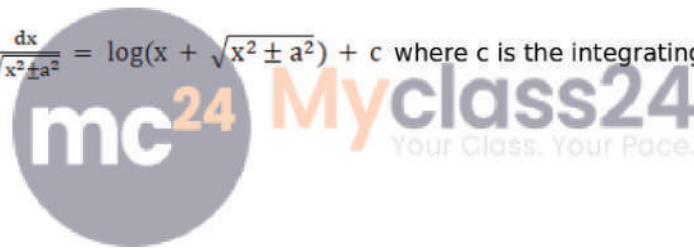
Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 + 4x^2}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 + 1^2}}$$

$$= \frac{1}{2} \log|2x + \sqrt{4x^2 + 1}| + c, c \text{ being the integrating constant}$$



9. Question

Evaluate:

$$\int \frac{dx}{\sqrt{9 + 4x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{9 + 4x^2}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 + 3^2}}$$

$$= \frac{1}{2} \log|2x + \sqrt{4x^2 + 9}| + c, c \text{ being the integrating constant}$$

10. Question

Evaluate: