$\Rightarrow sec^2 x\, dx = dt$

At x = 0, t = 0

At x $= \frac{\pi}{4}$, t = 1

$$y = \int_0^1 e^t \, dt$$

$= e^t\big|_0^1$

$= e^1 - e^0$

= e - 1

15. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\cos x}{\left(1+\sin^2 x\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. π

D. none of these

Answer

Let, sin x = t

Differentiating both side with respect to t

$$Cos\, x \frac{dx}{dt} = 1$$

$\Rightarrow cos\, x\, dx = dt$

At x = 0, t = 0

At x $= \frac{\pi}{2}$, t = 1

$$y = \int_0^1 \frac{1}{1+t^2} dt$$

$= (tan^{-1} t)_0^1$

$= \tan^{-1}1 - \tan^{-1}0$

= π/4

16. Question

Mark (√) against the correct answer in the following:

$$\int_{1/\pi}^{2/\pi} \frac{\sin\left(1/x\right)}{x^2} dx = ?$$

A. 1

B. $\frac{1}{2}$

C. $\frac{3}{2}$

D. none of these

Answer

Let, 1/x = t

Differentiating both side with respect to t

$$\frac{-1}{x^2}\frac{dx}{dt} = 1$$

$$\Rightarrow \frac{1}{x^2} dx = -dt$$

At x = 1/π, t = π

At x = 2/π, t = π/2

$$y = \int_{\pi}^{\frac{\pi}{2}} \sin t \ dt$$

$$= (-\cos t)_{\pi}^{\frac{\pi}{2}}$$

= 1

17. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi} \frac{dx}{(1+\sin x)} = ?$$

A. $\frac{1}{2}$

B. 1

C. 2

D. 0

Answer

$$y = \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$$

$$= \int_0^{\pi} \frac{1-\sin x}{\cos^2 x} dx$$

$$= \int_0^{\pi} \frac{1}{cos^2 x} - \frac{sin\, x}{cos^2 x} dx$$

$$= \int_0^{\pi} sec^2 x\, dx - \int_0^{\pi} \frac{sin\, x}{cos^2 x} dx$$

Let, cos x = t

Differentiating both side with respect to t

$$-sin\, x \frac{dx}{dt} = 1$$

$$\Rightarrow sin\, x\, dx = -dt$$

At x = 0, t = 1

At x = π, t = -1

$$y = (tan\, x)_0^{\pi} + \int_1^{-1} \frac{1}{t^2} dt$$

$$= (tan\, \pi - tan\, 0) + \left(\frac{t^{-1}}{-1}\right)_1^{-1}$$

=2

18. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \left(\sqrt{\sin x} \cos x\right)^3 dx = ?$$

A. $\frac{2}{9}$

B. $\frac{2}{15}$

C. $\frac{8}{45}$

D. $\frac{5}{2}$

Answer

$$y = \int_0^{\frac{\pi}{2}} sin^{\frac{3}{2}} x cos^3 x\, dx$$

$$y = \int_0^{\frac{\pi}{2}} sin^{\frac{3}{2}} x\, cos\, x\, (1 - sin^2 x)\, dx$$

Let, sin x = t

Differentiating both side with respect to t

$$Cos\, x \frac{dx}{dt} = 1$$

$\Rightarrow \cos x\, dx = dt$

At x = 0, t = 0

At x = π/2, t = 1

$$y = \int_0^1 t^{\frac{3}{2}} - t^{\frac{7}{2}}\, dt$$

$$= \left(\frac{t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{t^{\frac{9}{2}}}{\frac{9}{2}}\right)_0^1$$

$$= \frac{2}{5} - \frac{2}{9}$$

$$= \frac{8}{45}$$

19. Question

Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{xe^x}{(1+x)^2}\, dx = ?$$

A. $\left(\frac{e}{2} - 1\right)$

B. (e – 1)

C. e(e – 1)

D. none of these

Answer

$$y = \int_0^1 \frac{e^x(x+1-1)}{(1+x)^2} dx$$

$$= \int_0^1 e^x\left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right) dx$$

Use formula $\int e^x(f(x) + f'(x))dx = e^x f(x)$

If $f(x) = \frac{1}{1+x}$

then $f'(x) = -\frac{1}{(1+x)^2}$

$$y = \left(\frac{e^x}{1+x}\right)_0^1$$

$$y = \frac{e}{2} - 1$$

20. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} e^x\left(\frac{1+\sin x}{1+\cos x}\right) dx = ?$$

A. 0

B. $\frac{\pi}{4}$

C. $e^{\pi/2}$

D. $\left(e^{\pi/2}-1\right)$

Answer

$y = \int_0^{\frac{\pi}{2}} e^x \left(\frac{1+\sin x}{2\cos^2\frac{x}{2}}\right) dx$

$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{\sin x}{2\cos^2\frac{x}{2}}\right) dx$

$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) dx$

$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right) dx$

Use formula $\int e^x(f(x) + f'(x))dx = e^x f(x)$

If $f(x) = \tan\frac{x}{2}$ then $f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$

$y = \left(e^x \tan\frac{x}{2}\right)_0^{\frac{\pi}{2}}$

$= e^{\frac{\pi}{2}} \tan\frac{\frac{\pi}{2}}{2} - e^0 \tan\frac{0}{2}$

$= e^{\frac{\pi}{2}}$

21. Question

Mark (√) against the correct answer in the following:

$\int_0^{\pi/4} \sqrt{1+\sin 2x}\, dx = ?$

A. 0

B. 1

C. 2

D. $\sqrt{2}$

Answer

$y = \int_0^{\frac{\pi}{4}} \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}\, dx$

$$= \int_0^{\frac{\pi}{4}} \sin x + \cos x \, dx$$

$$= (-\cos x + \sin x)_0^{\frac{\pi}{4}}$$

$$= \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (-1 + 0)$$

y = 1

22. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \sqrt{1 + \cos 2x} \, dx = ?$$

A. $\sqrt{2}$

B. $\frac{3}{2}$

C. $\sqrt{3}$

D. 2

Answer

$$y = \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2 x} \, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2} \cos x \, dx$$

$$= \sqrt{2}(\sin x)_0^{\frac{\pi}{2}}$$

=√2

23. Question

Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{(1-x)}{(1+x)} dx = ?$$

A. $\frac{1}{2}\log 2$

B. (2 log 2 + 1)

C. (2 log 2 - 1)

D. $\left(\frac{1}{2}\log 2 - 1\right)$

Answer

$$y = \int_0^1 \frac{1-x-1+1}{1+x} dx$$

$$= \int_0^1 \frac{2}{1+x} - 1\, dx$$

$$= \left(2\ln(1+x) - x\right)_0^1$$

= 2 ln 2 - 1

24. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \sin^2 x\, dx = ?$$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer

$$y = \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$$

$$= \left(\frac{x}{2} - \frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{4} - \frac{\sin \pi}{4}$$

$$= \frac{\pi}{4}$$

25. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/6} \cos x \cos 2x\, dx = ?$$

A. $\frac{1}{4}$

B. $\frac{5}{12}$

C. $\frac{1}{3}$

D. $\frac{7}{12}$

Answer

$y = \int_0^{\frac{\pi}{6}} \cos x\,(1 - 2\sin^2 x)\,dx$

$= \int_0^{\frac{\pi}{6}} \cos x - 2\cos x \sin^2 x\,dx$

$= (\sin x)_0^{\frac{\pi}{6}} - 2\int_0^{\frac{\pi}{6}} \cos x \sin^2 x\,dx$

Let, $\sin x = t$

Differentiating both side with respect to t

$\cos x \frac{dx}{dt} = 1$

$\Rightarrow \cos x\,dx = dt$

At $x = 0$, $t = 0$

At $x = \pi/6$, $t = 1/2$

$y = \sin\frac{\pi}{6} - \sin 0 - 2\int_0^{\frac{1}{2}} t^2\,dt$

$= \frac{1}{2} - 2\left(\frac{t^3}{3}\right)_0^{\frac{1}{2}}$

$= \frac{1}{2} - \frac{1}{12}$

$= \frac{5}{12}$

26. Question

Mark (√) against the correct answer in the following:

$\int_0^{\pi/2} \sin x \sin 2x\,dx = ?$

A. $\frac{2}{3}$

B. $\frac{3}{4}$

C. $\frac{5}{6}$

D. $\frac{3}{5}$

Answer

$y = \int_0^{\frac{\pi}{2}} \sin x\,(2\sin x \cos x)\,dx$

$$= 2\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$$

Let, sin x = t

Differentiating both side with respect to t

$$\text{Cos}\, x\frac{dx}{dt} = 1$$

$$\Rightarrow \cos x \, dx = dt$$

At x = 0, t = 0

At x = π/2, t = 1

$$y = 2\int_0^1 t^2 \, dt$$

$$= 2\left(\frac{t^3}{3}\right)_0^1$$

$$= \frac{2}{3}$$

27. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi} (\sin 2x \cos 3x) dx = ?$$

A. $\frac{4}{5}$

B. $-\frac{4}{5}$

C. $\frac{5}{12}$

D. $-\frac{12}{5}$

Answer

$$y = \int_0^{\pi} (2\sin x \cos x)(4\cos^3 x - 3\cos x)\, dx$$

Let, cos x = t

Differentiating both side with respect to t

$$-\sin x\frac{dx}{dt} = 1$$

$$\Rightarrow \sin x \, dx = -dt$$

At x = 0, t = 1

At x = π, t = -1

$y = -\int_{1}^{-1} 8t^4 - 6t^2\,dt$

$= -\left(8\frac{t^5}{5} - 6\frac{t^3}{3}\right)_1^{-1}$

$= -\left[\left(\frac{-8}{5}+2\right) - \left(\frac{8}{5}-2\right)\right]$

$= -\frac{4}{5}$

28. Question

Mark (√) against the correct answer in the following:

$\int_0^1 \frac{dx}{\left(e^x + e^{-x}\right)} = ?$

A. $\left(1-\frac{\pi}{4}\right)$

B. $\tan^{-1} e$

C. $\tan^{-1} e + \frac{\pi}{4}$

D. $\tan^{-1} e - \frac{\pi}{4}$

Answer

$y = \int_0^1 \frac{e^x}{1+e^{2x}} dx$

Let $e^x = t$

Differentiating both side with respect to t

$e^x \frac{dx}{dt} = 1$

$\Rightarrow e^x dx = dt$

At x = 0, t = 1

At x = 1, t = e

$y = \int_1^e \frac{1}{1+t^2} dt$

$= (\tan^{-1} t)_1^e$

$= \tan^{-1}e - \tan^{-1}1$

$= \tan^{-1}e - \pi/4$

29. Question

Mark (√) against the correct answer in the following:

$$\int_0^9 \frac{dx}{\left(1+\sqrt{x}\right)} = ?$$

A. (3 – 2 log 2)

B. (3 + 2 log 2)

C. (6 – 2 log 4)

D. (6 + 2 log 4)

Answer

Let, $x = t^2$

Differentiating both side with respect to t

$$\frac{dx}{dt} = 2t$$

$$\Rightarrow dx = 2t\,dt$$

At x = 0, t = 0

At x = 9, t = 3

$$y = \int_0^3 \frac{2t}{1+t} dt$$

$$= 2\int_0^3 \frac{t+1-1}{1+t} dt$$

$$= 2\int_0^3 1 - \frac{1}{1+t} dt$$

$$= 2(t - \ln(1+t))_0^3$$

y = 2[(3 – ln 4) – (0 – ln 1)]

= 6 – 2 log 4

30. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} x \cos x \, dx = ?$$

A. $\frac{\pi}{2}$

B. $\left(\frac{\pi}{2} - 1\right)$

C. $\left(\frac{\pi}{2} + 1\right)$

D. none of these

Answer

Use integration by parts

$$\int \mathrm{I}\times \mathrm{II}\,dx = \mathrm{I}\times \int \mathrm{II}\ dx - \int \frac{d}{dx}\mathrm{I}\left(\int \mathrm{II}\,dx\right)dx$$

$$y = x\int_0^{\frac{\pi}{2}} \cos x\,dx - \int_0^{\frac{\pi}{2}} \frac{d}{dx}x\left(\int \cos x\,dx\right)dx$$

$$= (x\sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\,dx$$

$$= \frac{\pi}{2} - (-\cos x)_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} + (0-1)$$

$$= \frac{\pi}{2} - 1$$

31. Question

Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{dx}{\left(1+x+x^2\right)} = ?$$

A. $\frac{\pi}{\sqrt{3}}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{3\sqrt{3}}$

D. none of these

Answer

We have to convert denominator into perfect square

$$1 + x + x2 = x^2 + 2(x)\left(\frac{1}{2}\right) + \frac{1}{4} - \frac{1}{4} + 1$$

$$= \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$$

$$= \left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$y = \int_0^1 \frac{1}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dx$$

Use formula $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$$y = \left(\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)_0^1$$

$$= \frac{2}{\sqrt{3}}\left(\tan^{-1}\frac{2}{\sqrt{3}}\left(\frac{3}{2}\right) - \tan^{-1}\frac{2}{\sqrt{3}}\left(\frac{1}{2}\right)\right)$$

$$= \frac{2}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{\pi}{3\sqrt{3}}$$

32. Question

Mark (√) against the correct answer in the following:

$$\int_0^1 \sqrt{\frac{1-x}{1+x}}dx = ?$$

A. $\frac{\pi}{2}$

B. $\left(\frac{\pi}{2} - 1\right)$

C. $\left(\frac{\pi}{2} + 1\right)$

D. none of these

Answer

Let, x = sin t

Differentiating both side with respect to t

$$\frac{dx}{dt} = \cos t \Rightarrow dx = \cos t\ dt$$

At x = 0, t = 0

At x = 1, t = π/2

$$y = \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\sin t}{1+\sin t}}\cos t\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\sin t}{1+\sin t} \times \frac{1-\sin t}{1-\sin t}}\cos t\, dt$$

$$= \int_0^{\frac{\pi}{2}} \frac{1-\sin t}{\cos t}\cos t\, dt$$

$$= \int_0^{\frac{\pi}{2}} 1 - \sin t\, dt$$

$= (t + \cos t)_0^{\frac{\pi}{2}}$

$= \left(\frac{\pi}{2} + 0\right) - (0 + 1)$

$= \frac{\pi}{2} - 1$

33. Question

Mark (√) against the correct answer in the following:

$\int_0^1 \frac{(1-x)}{(1+x)} dx = ?$

A. (log 2 + 1)

B. (log 2 - 1)

C. (2 log 2 - 1)

D. (2 log 2 + 1)

Answer

$y = \int_0^1 \frac{1-x+1-1}{1+x} dx$

$= \int_0^1 \frac{2}{1+x} - 1\, dx$

$= (2\ln(1+x) - x)_0^1$

= 2 log 2 - 1

34. Question

Mark (√) against the correct answer in the following:

$\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} dx = ?$

A. aπ

B. $\frac{a\pi}{2}$

C. 2 aπ

D. none of these

Answer

Let, x = a sin t

Differentiating both side with respect to t

$\frac{dx}{dt} = a\cos t \Rightarrow dx = a\cos t\ dt$

At x = -a, t = - π/2

At x = a, t = π/2

$$y = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{a - a\sin t}{a + a\sin t}}\, a\cos t\, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{1 - \sin t}{1 + \sin t} \times \frac{1 - \sin t}{1 - \sin t}}\cos t\, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 - \sin t}{\cos t}\cos t\, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 - \sin t\, dt$$

$$= a(t + \cos t)_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= a\left[\left(\frac{\pi}{2} + 0\right) - \frac{\pi}{2} + 0\right)$$

$= a\pi$

35. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\sqrt{2}} \sqrt{2 - x^2}\, dx = ?$$

A. π

B. 2π

C. $\frac{\pi}{2}$

D. none of these

Answer

Use formula $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

$$y = \int_0^{\sqrt{2}} \sqrt{\left(\sqrt{2}\right)^2 - x^2}\, dx$$

$$= \left(\frac{x}{2}\sqrt{2 - x^2} + \frac{2}{2}\sin^{-1}\frac{x}{\sqrt{2}}\right)_0^{\sqrt{2}}$$

$$= \left(\frac{\sqrt{2}}{2}\sqrt{2 - 2} + \sin^{-1}\frac{\sqrt{2}}{\sqrt{2}}\right) - (0 + \sin^{-1} 0)$$

$$= \frac{\pi}{2}$$

36. Question

Mark (√) against the correct answer in the following:

$$\int_{-2}^{2} |x|\,dx = ?$$

A. 4

B. 3.5

C. 2

D. 0

Answer

We know that

$|x| = -x$ in $[-2, 0)$

$|x| = x$ in $[0, 2]$

$$y = \int_{-2}^{0} |x|\,dx + \int_{0}^{2} |x|\,dx$$

$$= \int_{-2}^{0} -x\,dx + \int_{0}^{2} x\,dx$$

$$= \left(-\frac{x^2}{2}\right)_{-2}^{0} + \left(\frac{x^2}{2}\right)_{0}^{2}$$

y = 0 - (-2) + 2 - 0

= 4

37. Question

Mark (√) against the correct answer in the following:

$$\int_{0}^{1} |2x-1|\,dx = ?$$

A. 2

B. $\frac{1}{2}$

C. 1

D. 0

Answer

We know that

$|2x - 1| = -(2x - 1)$ in $[0, 1/2)$

$|2x - 1| = (2x - 1)$ in $[1/2, 1]$

$$y = \int_{0}^{\frac{1}{2}} |2x-1|\,dx + \int_{\frac{1}{2}}^{1} |2x-1|\,dx$$

$$= \int_0^{\frac{1}{2}} -(2x-1)\,dx + \int_{\frac{1}{2}}^{1} 2x-1\,dx$$

$$= -(x^2-x)_0^{\frac{1}{2}} + (x^2-x)_{\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{1}{4}-\frac{1}{2}\right)-(0-0)\right]+\left[(1-1)-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$y = \frac{1}{2}$$

38. Question

Mark (√) against the correct answer in the following:

$$\int_{-2}^{1} |2x+1|\,dx = ?$$

A. $\frac{5}{2}$

B. $\frac{7}{2}$

C. $\frac{9}{2}$

D. 0

Answer

We know that

|2x + 1| = -(2x + 1) in [-2, -1/2)

|2x + 1| = (2x + 1) in [-1/2, 1]

$$y = \int_{-2}^{-\frac{1}{2}} |2x+1|\,dx + \int_{-\frac{1}{2}}^{1} |2x+1|\,dx$$

$$= \int_{-2}^{-\frac{1}{2}} -(2x+1)\,dx + \int_{-\frac{1}{2}}^{1} 2x+1\,dx$$

$$= -(x^2+x)_{-2}^{-\frac{1}{2}} + (x^2+x)_{-\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{1}{4}-\frac{1}{2}\right)-(4-2)\right]+\left[(1+1)-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$y = \frac{9}{2}$$

39. Question

Mark (√) against the correct answer in the following:

$$\int_{-2}^{1} \frac{|x|}{x} dx = ?$$

A. 3

B. 2.5

C. 1.5

D. none of these

Answer

We know that

|x| = -x in [-2, 0)

|x| = x in [0, 1]

$$y = \int_{-2}^{0} \frac{|x|}{x} dx + \int_{0}^{1} \frac{|x|}{x} dx$$

$$= \int_{-2}^{0} \frac{-x}{x} dx + \int_{0}^{1} \frac{x}{x} dx$$

$$= \int_{-2}^{0} -1\, dx + \int_{0}^{1} 1\, dx$$

$$= (-x)_{-2}^{0} + (x)_{0}^{1}$$

= -(0 - (-2)) + (1 - 0)

= -1

40. Question

Mark (√) against the correct answer in the following:

$$\int_{-a}^{a} x\,|x|\,dx = ?$$

A. 0

B. 2a

C. $\frac{2a^3}{3}$

D. none of these

Answer

We know that

|x| = -x in [-a, 0) where a > 0

|x| = x in [0, a] where a > 0

$$y = \int_{-a}^{0} x|x|\, dx + \int_{0}^{a} x|x|\, dx$$

$$= \int_{-a}^{0} x(-x)\,dx + \int_{0}^{a} x(x)\,dx$$

$$= -\int_{-a}^{0} x^2\,dx + \int_{0}^{a} x^2\,dx$$

$$= -\left(\frac{x^3}{3}\right)_{-a}^{0} + \left(\frac{x^3}{3}\right)_{0}^{a}$$

$$= -\left(0 - \left(\frac{-a^3}{3}\right)\right) + \left(\frac{a^3}{3} - 0\right)$$

= 0

41. Question

Mark (√) against the correct answer in the following:

$$\int_{0}^{\pi} |\cos x|\,dx = ?$$

A. 2

B. $\frac{3}{2}$

C. 1

D. 0

Answer

Find the equivalent expression to |cos x| at $0 \leq x \leq \pi$

In $0 \leq x \leq \frac{\pi}{2}$

=cos x

In $\frac{\pi}{2} \leq x \leq \pi$

=-cos x

$$\Rightarrow \int_{0}^{\frac{\pi}{2}} \cos x\,dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x\,dx$$

$$\Rightarrow \sin\frac{\pi}{2} - \sin 0 - \cos\pi + \cos\frac{\pi}{2}$$

⇒1-0-(-1) +0=2

42. Question

Mark (√) against the correct answer in the following:

$$\int_{0}^{2\pi} |\sin x|\,dx = ?$$

A. 2

B. 4

C. 1

D. none of these

Answer

Find the equivalent expression to |sin x| at $0 \leq x \leq 2\pi$

In $0 \leq x \leq \pi$

|sin x| = sin x

In $\pi \leq x \leq 2\pi$

|sin x| = -sin x

$\Rightarrow \int_0^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} -\sin x dx$ =-cos π-(-cos 0)+cos 2π-cos π

=-(-1)+1+1-(-1)

=2+2

=4

43. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sin x}{(\sin x + \cos x)} dx = ?$$

A. π

B. $\frac{\pi}{2}$

C. 0

D. $\frac{\pi}{4}$

Answer

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$\therefore$ Here,a $= \frac{\pi}{2}$

$f(x) = \frac{\sin x}{(\sin x + \cos x)}$

$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$\frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos(\frac{\pi}{2} - x)} = \frac{\cos x}{\cos x + \sin x}$

$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$

$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$

$= \int_0^{\frac{\pi}{2}} 1dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

44. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\left(\sqrt{\cos x} + \sqrt{\sin x}\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. π

D. 0

Answer

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

∴ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$$\frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos(\frac{\pi}{2} - x)} + \sqrt{\sin(\frac{\pi}{2} - x)}} = \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$

$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$

$= \int_0^{\frac{\pi}{2}} 1dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

45. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sin^4 x}{\left(\sin^4 x + \cos^4 x\right)} dx = ?$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 1

D. 0

Answer

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

∴ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sin^4 x}{\sin^4 x + \cos^4 x}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$$\frac{\sin^4\left(\frac{\pi}{2} - x\right)}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)} = \frac{\cos^4 x}{\sin^4 x + \cos^4 x}$$

$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$

$= \int_0^{\frac{\pi}{2}} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx$

$= \int_0^{\frac{\pi}{2}} 1 dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

46. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\cos^{1/4} x}{\left(\sin^{1/4} x + \cos^{1/4} x\right)} dx = ?$$

A. 0

B. 1

C. $\frac{\pi}{4}$

D. none of these

Answer

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$\therefore$ Here,

$a = \frac{\pi}{2}$;

$$f(x) = \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$\frac{\cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{1}{4}}} \left(\frac{\pi}{2} - x\right) \cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right) = \sin^{\frac{1}{4}} x \sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

47. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sin^n x}{\left(\sin^n x + \cos^n x\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. 1

D. 0

Answer

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$\therefore$ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sin^n x}{\cos^n x + \sin^n x}$

$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$= \frac{\cos^n x}{\cos^n x + \sin^n x}$

$\therefore 2I = \int_0^{\frac{\pi}{2}} 1 \, dx$

$\therefore 2I = \frac{\pi}{2}$

$\therefore I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

48. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx = ?$$

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. none of these

Answer

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$\therefore$ Here,

$a = \frac{\pi}{2}$;

$$f(x) = \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}$$

$$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sqrt{\tan x}}{\sqrt{\cot x} + \sqrt{\tan x}}$$

$$\therefore\ 2I = \int_0^{\frac{\pi}{2}} 1\, dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

49. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sqrt[3]{\tan x}}{\left(\sqrt[3]{\tan x} + \sqrt[3]{\cot x}\right)} dx = ?$$

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. π

Answer

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$= \frac{\sqrt[3]{\tan x}}{\sqrt[3]{\cot x} + \sqrt[3]{\tan x}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}}}{\sqrt[3]{\frac{\sin x}{\cos x}} + \sqrt[3]{\frac{\cos x}{\sin x}}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}} * \left(\sqrt[3]{\sin x}\ \sqrt[3]{\cos x}\right)}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$= \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$\therefore$ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$= \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$

$\therefore\ 2I = \int_0^{\frac{\pi}{2}} 1 dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

50. Question

Mark (√) against the correct answer in the following:

$\int_0^{\pi/2} \frac{1}{(1 + \tan x)} dx = ?$

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. π

Answer

$\frac{1}{1 + \tan x} = \frac{1}{1 + \frac{\sin x}{\cos x}}$

$= \frac{1}{(\cos x + \sin x)\frac{1}{\cos x}}$

$= \frac{\cos x}{\cos x + \sin x}$

So our integral becomes, $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

∴ Here,

$a = \frac{\pi}{2}$

$f(x) = \frac{\sin x}{(\sin x + \cos x)}$

$\therefore\ f(a - x) = f\left(\frac{\pi}{2} - x\right)$

$= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}$

$= \frac{\cos x}{\cos x + \sin x}$

$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a - x)$

$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$

$= \int_0^{\frac{\pi}{2}} 1 dx$

$= \int_0^{\frac{\pi}{2}} 1 dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

51. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{1}{\left(1 + \sqrt{\cot x}\right)} dx = ?$$

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

Answer

So our integral becomes

$\frac{1}{\sqrt[1]{\cot x} + 1} = \frac{1}{\sqrt{\frac{\cos x}{\sin x}} + 1}$

$= \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$

∴ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$= \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}}$

$= \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$

$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$

$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$

$= \int_0^{\frac{\pi}{2}} 1 dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

52. Question

Mark (√) against the correct answer in the following:

$\int_0^{\pi/2} \frac{1}{\left(1 + \tan^3 x\right)} dx = ?$

A. $\frac{\pi}{4}$

B. 0

C. $\frac{\pi}{2}$

D. none of these

Answer

$\frac{1}{1 + \tan^3 x} = \frac{\cos^3 x}{\sin^3 x + \cos^3 x}$

∴ Here,

$a = \frac{\pi}{2}$;

$$f(x) = \frac{\cos^3 x}{\sin^3 x + \cos^3 x}$$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$f(a-x) = \frac{\sin^3 x}{\sin^3 x + \cos^3 x}$$

$$\therefore 2I = \int_0^{\frac{\pi}{2}} 1dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

53. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sec^5 x}{\left(\sec^5 x + \operatorname{cosec}^5 x\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. 0

C. $\frac{\pi}{4}$

D. π

Answer

so our integral becomes,

$$\frac{\sec^5 x}{\sec^5 x + \operatorname{cosec}^5 x} = \frac{\frac{1}{\cos^5 x}}{\frac{1}{\cos^5 x} + \frac{1}{\sin^5 x}}$$

$$= \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$$

Here $a = \frac{\pi}{2}$ and $f(x) = \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$

$$f(a-x) = \frac{\cos^5 x}{\sin^5 x + \cos^5 x}$$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$\therefore 2I == \int_0^{\frac{\pi}{2}} 1dx$$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

54. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\left(1+\sqrt{\cot x}\right)} dx = ?$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 0

D. 1

Answer

So our integral becomes,

$$\frac{\sqrt{\cot x}}{1+\sqrt{\cot x}} = \frac{\sqrt{\frac{\cos x}{\sin x}}}{1+\sqrt{\frac{\cos x}{\sin x}}}$$

$$= \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}$$

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

so, we know that,

∴ Here,

$a = \frac{\pi}{2}$;

$$f(a-x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}$$

$$\therefore\ f(x) = \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

55. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\tan x}{(1+\tan x)} dx = ?$$

A. 0

B. 1

C. $\frac{\pi}{4}$

D. π

Answer

So our integral becomes,

$$\frac{\tan x}{1+\tan x} = \frac{\sin x}{\cos x}\left(\frac{1}{1+\frac{\sin x}{\cos x}}\right)$$

$$= \frac{\sin x}{\sin x + \cos x}$$

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

∴ Here,

$a = \frac{\pi}{2}$

$f(x) = \frac{\sin x}{(\sin x + \cos x)}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$$= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$= \int_0^{\frac{\pi}{2}} 1dx$

$\therefore\ 2I = \frac{\pi}{2}$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

56. Question

Mark (√) against the correct answer in the following:

$$\int_{-\pi}^{\pi} x^4 \sin x\, dx = ?$$

A. 2π

B. π

C. 0

D. none of these

Answer

If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)\,dx = -\int_{-a}^0 f(x)dx$

here $f(x)=x^4\sin x$

we will see $f(-x)=(-x)^4\sin(-x)$

$=- x^4\sin x$

Therefore, f(x) is a odd function,

$$\int_{-\pi}^{\pi} x^4 \sin x dx = 0$$

57. Question

Mark (√) against the correct answer in the following:

$$\int_{-\pi}^{\pi} x^3 \cos^3 x\, dx = ?$$

A. π

B. $\frac{\pi}{4}$

C. 2π

D. 0

Answer

If f is an odd function,

$\int_{-a}^{a} f(x)dx = 0$

as, $\int_0^a f(x)dx = -\int_{-a}^0 f(x)dx$

here $f(x)=x^3 \cos^3 x$

we will see $f(-x)= (-x)^3 \cos^3(-x)$

$=-x^3 \cos^3 x$

Therefore, f(x) is a odd function,

$\int_{-\pi}^{\pi} x^3 \cos^3 x = 0$

58. Question

Mark (√) against the correct answer in the following:

$\int_{-\pi}^{\pi} \sin^5 x \, dx = ?$

A. $\frac{3\pi}{4}$

B. 2π

C. $\frac{5\pi}{16}$

D. 0

Answer

If f is an odd function,

$\int_{-a}^{a} f(x)dx = 0$

as, $\int_0^a f(x)dx = -\int_{-a}^0 f(x)dx$

$f(x)=\sin^5 x$

$f(-x)=\sin^5(-x)$

$=-\sin^5 x$

Therefore, f(x) is a odd function,

$\int_{-\pi}^{\pi} \sin^5 x \, dx = 0$

59. Question

Mark (√) against the correct answer in the following:

$\int_{-1}^{-2} x^3 \left(1-x^2\right) dx = ?$

A. $-\frac{40}{3}$

B. $\frac{40}{3}$

C. $\frac{5}{6}$

D. 0

Answer

$$\int_{-1}^{-2} x^3(1-x^2)dx = \int_{-1}^{-2}(x^3-x^5)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^6}{6}\right]$$

$$= \left[\frac{2^4}{4} - \frac{1^6}{4} - \frac{2^6}{6} + \frac{1^6}{6}\right]$$

$$= -\frac{27}{4}$$

60. Question

Mark (√) against the correct answer in the following:

$$\int_{-a}^{a} \log\left(\frac{a-x}{a+x}\right)dx = ?$$

A. 2a

B. a

C. 0

D. 1

Answer

If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)\,dx = -\int_{-a}^0 f(x)dx$

$$f(x) = \log\left(\frac{a-x}{a+x}\right)$$

$$f(-x) = \log\frac{a-(-x)}{a-x}$$

$$= \log\frac{a+x}{a-x}$$

$$= -\log\frac{a-x}{a+x}$$

Hence it is a odd function

$$\int_{-a}^{a} \log\frac{a-x}{a+x} = 0$$

61. Question

Mark (√) against the correct answer in the following:

$$\int_{-\pi}^{\pi}\left(\sin^{61}x + x^{123}\right)dx = ?$$

A. 2π

B. 0

C. $\frac{\pi}{2}$

D. 125π

Answer

If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)\,dx = -\int_{-a}^0 f(x)dx$

$\sin^{61}x$ and $x^{123}$is an odd function,

so there integral is zero.

62. Question

Mark (√) against the correct answer in the following:

$$\int_{-\pi}^{\pi} \tan x \, dx = ?$$

A. 2

B. $\frac{1}{2}$

C. -2

D. 0

Answer

f(x)=tan x

f(-x) =tan(-x)

=-tan x

hence the function is odd,

therefore, I=0

63. Question

Mark (√) against the correct answer in the following:

$$\int_{-1}^{1} \log\left(x + \sqrt{x^2+1}\right)dx = ?$$

A. $\log\frac{1}{2}$

B. log 2

C. $\frac{1}{2}\log 2$

D. 0

Answer

By by parts,

$$\int \log\left(x+\sqrt{x^2+1}\right) = x\log\left(x+\sqrt{x^2+1}\right) - \int \frac{x}{\left(x+\sqrt{x^2+1}\right)\left(1+\frac{x}{\sqrt{x^2+1}}\right)}$$

$$= x\log(x+\sqrt{x^2+1}) - \int \frac{x}{\sqrt{x^2+1}} = x\log(x+\sqrt{x^2+1}) - \sqrt{x^2+1}$$

64. Question

Mark (√) against the correct answer in the following:

$$\int_{-\pi/2}^{\pi/2} \cos x\, dx = ?$$

A. 0

B. 2

C. -1

D. none of these

Answer

cosx is an even function so,

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2\int_{0}^{\frac{\pi}{2}} \cos x dx$$

=2(1-0)

=2

65. Question

Mark (√) against the correct answer in the following:

$$\int_{0}^{a} \frac{\sqrt{x}}{\left(\sqrt{x}+\sqrt{a-x}\right)} dx = ?$$

A. $\frac{a}{2}$

B. 2a

C. $\frac{2a}{3}$

D. $\frac{\sqrt{a}}{2}$

Answer

Here,

$$f(x) = \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}}$$

$$f(a-x) = \frac{\sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}}$$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx$$

$$= \int_0^a dx$$

$$I = \frac{a}{2}$$

66. Question

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/4} \log(1 + \tan x) dx = ?$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{4} \log 2$

C. $\frac{\pi}{8} \log 2$

D. 0

Answer

let $I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$$

$$\therefore f(a-x) = \log(1 + \tan(\frac{\pi}{4} - x))$$

$$= \log\left(1 + \frac{\left(\tan\frac{\pi}{4} - \tan x\right)}{1 + \tan\frac{\pi}{4}\tan x}\right) = \log(1 + 1(1 - \tan x)\frac{1}{1 + \tan x}$$

$$= \log\frac{2}{1 + \tan x}$$

$$\therefore \int_0^a f(a-x) = I$$

$$= \int_0^{\frac{\pi}{4}} \log\frac{2}{1+\tan x}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2dx - \int_0^{\frac{\pi}{4}}(1+\tan x)dx$$

$$\therefore\ I = \int_0^{\frac{\pi}{4}} \log 2\,dx - I$$

$$\therefore\ 2I = \frac{\pi}{4}\log 2$$

$$\therefore\ I = \frac{\pi}{8}\log 2$$

67. Question

Mark (√) against the correct answer in the following:

$$\int_{-a}^{a} f(x)dx = ?$$

A. $2\int_0^a \{f(x)+f(-x)\}dx$

B. $2\int_0^a \{f(x)-f(-x)\}dx$

C. $\int_0^a \{f(x)+f(-x)\}dx$

D. none of these

Answer

$$\therefore\ \int_{-a}^{a} f(x)dx$$

$$\therefore\ \int_{-a}^{0} f(x)dx + \int_0^a f(x)dx$$

$$\therefore\ \int_0^a f(-x)dx = \int_{-a}^{0} f(x)dx$$

$$\therefore\ \int_0^a f(-x)dx + \int_0^a f(x)dx$$

68. Question

Mark (√) against the correct answer in the following:

Let [x] denote the greatest integer less than or equal to x.

Then, $\int_0^{1.5} [x]dx = ?$

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. 2

D. 3

Answer

$\therefore \int_0^{1.5} [x]dx$

$= \int_0^1 [x]dx + \int_1^{1.5} [x]dx$

$= \int_0^1 0dx + \int_1^{1.5} 1.dx$

$= \frac{3}{2} - 1$

$= \frac{1}{2}$

69. Question

Mark (√) against the correct answer in the following:

Let [x] denote the greatest integer less than or equal to x.

Then, $\int_{-1}^{1} [x]dx = ?$

A. -1

B. 0

C. $\frac{1}{2}$

D. 2

Answer

$\int_{-1}^1 [x]dx = \int_{-1}^0 [x]dx + \int_0^1 [x]dx$

$= \int_{-1}^0 -1dx + \int_0^1 0dx$

=-1-0+0

=-1

70. Question

Mark (√) against the correct answer in the following:

$\int_1^2 \left|x^2 - 3x + 2\right| dx = ?$

A. $\frac{-1}{6}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer

$$\int_1^2 |x^2 - 3x + 2| dx$$

$\therefore x^2-3x+2=0$

(x-2)(x-1)=0

so, 2, and 1 itself are the limits so no breaking points for the integral,

$$\therefore \int_1^2 (-x^2 + 3x - 2) dx$$

$$= \left[\frac{-x^3}{3} + \frac{3x^2}{2} - 2x\right] (1 to 2)$$

$$\therefore = \frac{1}{6}$$

71. Question

Mark (√) against the correct answer in the following:

$$\int_{\pi}^{2\pi} |\sin x| dx = ?$$

A. 0

B. 1

C. 2

D. none of these

Answer

$\therefore \sin x=0$

$\therefore x=0, \pi, 2\pi....$

So $\pi, 2\pi$ are the limits so no breaking points for the integral,

$$\therefore \int_{\pi}^{2\pi} -\sin x dx = -\cos x (\pi \text{ to } 2\pi)$$

=2

72. Question

Mark (√) against the correct answer in the following:

$$\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{\left(1-x^2\right)^{3/2}} dx = ?$$

A. $\frac{1}{2}(\pi - \log 2)$

B. $\left(\frac{\pi}{2} - 2\log 2\right)$

C. $\left(\frac{\pi}{4} - \frac{1}{2}\log 2\right)$

D. none of these

Answer

put $\sin^{-1} x = t;$

$dt = \frac{dx}{\sqrt{1-x^2}};$

x=sin t

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

=t;

and $\sin^{-1} 0=0$

=t

Limit changes to,

$\int_0^{\frac{\pi}{4}} \frac{tdt}{1-\sin^2 t} = \int_0^{\frac{\pi}{4}} t\sec^2 tdt$

$= t\tan t - \int_0^{\frac{\pi}{4}} \tan tdt$

$= [t\tan t + \log \cos t]\left(0 \text{ to } \frac{\pi}{4}\right)$

$= \frac{\pi}{4} - \frac{1}{2}\log 2$

73. Question

Mark (√) against the correct answer in the following:

$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = ?$

A. $\frac{1}{2}(\pi - \log 2)$

B. $\left(\frac{\pi}{2} - \log 2\right)$

C. $(\pi - 2\log 2)$

D. none of these

Answer

put x=tan y

dx=sec^{2}ydy

$$\int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2y)\sec^2 y dy$$

$$= 2\int_0^{\frac{\pi}{4}} y\sec^2 y dy$$

$$= 2[y\tan y - \int_0^{\frac{\pi}{4}} \tan y dy]$$

$$= 2[y\tan y + \log\cos y]\left(0 \text{ to } \frac{\pi}{4}\right)$$

$$= 2[\frac{\pi}{4} - \frac{1}{2}\log 2]$$

$$= \frac{\pi}{2} - \log 2$$