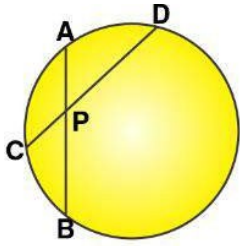


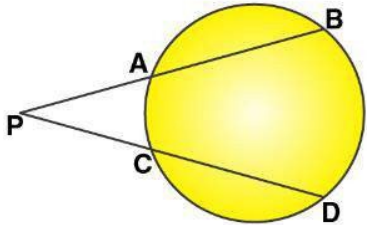
Chapter 18: Tangents and Intersecting Chords

Exercise 18(B)

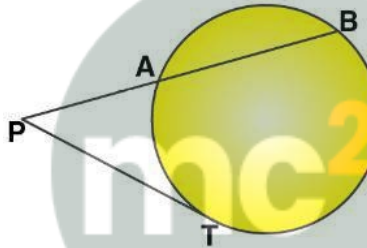
(i) In the given figure, $3 \times CP = PD = 9$ cm and $AP = 4.5$ cm. Find BP .



(ii) In the given figure, $5 \times PA = 3 \times AB = 30$ cm and $PC = 4$ cm. Find CD .



(iii) In the given figure, tangent $PT = 12.5$ cm and $PA = 10$ cm; find AB .



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Solution:

(i) As the two chords AB and CD intersect each other at P , we have
 $AP \times PB = CP \times PD$
 $4.5 \times PB = 3 \times 9$ [$3CP = 9$ cm so, $CP = 3$ cm]
 $PB = (3 \times 9) / 4.5 = 6$ cm

(ii) As the two chords AB and CD intersect each other at P , we have
 $AP \times PB = CP \times PD$
But, $5 \times PA = 3 \times AB = 30$ cm
So, $PA = 30/5 = 6$ cm and $AB = 30/3 = 10$ cm
And, $BP = PA + AB = 6 + 10 = 16$ cm
Now, as
 $AP \times PB = CP \times PD$
 $6 \times 16 = 4 \times PD$
 $PD = (6 \times 16) / 4 = 24$ cm
 $CD = PD - PC = 24 - 4 = 20$ cm

(iii) As PAB is the secant and PT is the tangent, we have
 $PT^2 = PA \times PB$

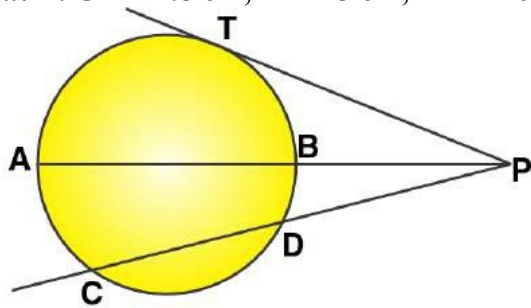
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$$12.5^2 = 10 \times PB$$

$$PB = (12.5 \times 12.5) / 10 = 15.625 \text{ cm}$$

$$AB = PB - PA = 15.625 - 10 = 5.625 \text{ cm}$$

1. In the given figure, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8 cm, PD = 5 cm, PB = 4 cm.



Find

(i) AB.

(ii) the length of tangent PT.

Solution:

(i) $PA = AB + BP = (AB + 4) \text{ cm}$
 $PC = PD + CD = 5 + 7.8 = 12.8 \text{ cm}$

As $PA \times PB = PC \times PD$

$$(AB + 4) \times 4 = 12.8 \times 5$$

$$AB + 4 = (12.8 \times 5) / 4$$

$$AB + 4 = 16$$

Hence, $AB = 12 \text{ cm}$

(ii) As we know,

$$PT^2 = PC \times PD$$

$$PT^2 = 12.8 \times 5 = 64$$

Thus, $PT = 8 \text{ cm}$

2. In the following figure, PQ is the tangent to the circle at A, DB is a diameter and O is the centre of the circle. If $\angle ADB = 30^\circ$ and $\angle CBD = 60^\circ$; calculate:

i) $\angle QAB$

ii) $\angle PAD$

iii) $\angle CDB$

Solution:

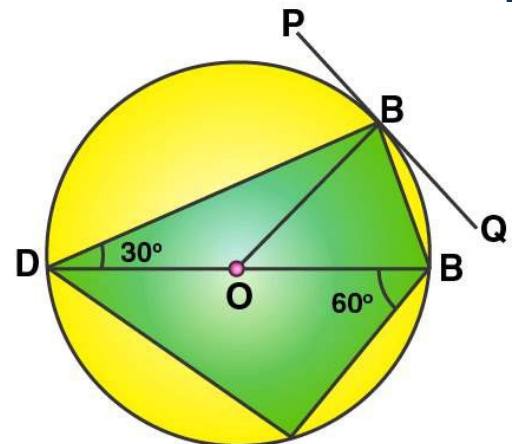
(i) Given, PAQ is a tangent and AB is the chord
 $\angle QAB = \angle ADB = 30^\circ$ [Angles in the alternate segment]

(ii) $OA = OD$ [radii of the same circle]

So, $\angle OAD = \angle ODA = 30^\circ$

But, as $OA \perp PQ$

$$\angle PAD = \angle OAP - \angle OAD = 90^\circ - 30^\circ = 60^\circ$$



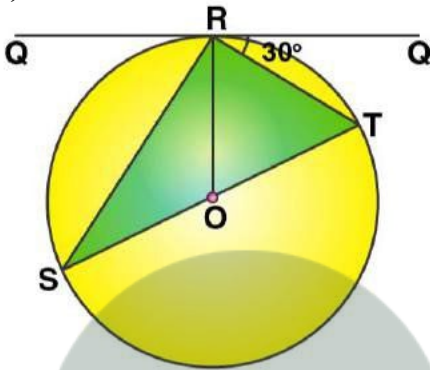
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- (iii) As BD is the diameter, we have
 $\angle BCD = 90^\circ$ [Angle in a semi-circle]
 Now in $\triangle BCD$,
 $\angle CDB + \angle CBD + \angle BCD = 180^\circ$
 $\angle CDB + 60^\circ + 90^\circ = 180^\circ$
 Thus, $\angle CDB = 180^\circ - 150^\circ = 30^\circ$

3. If PQ is a tangent to the circle at R; calculate:

i) $\angle PRS$

ii) $\angle ROT$



Given: O is the centre of the circle and $\angle TRQ = 30^\circ$

Solution:

- (i) As PQ is the tangent and OR is the radius.
 So, $OR \perp PQ$
 $\angle ORT = 90^\circ$
 $\angle TRQ = 90^\circ - 30^\circ = 60^\circ$
 But in $\triangle OTR$, we have
 $OT = OR$ [Radii of same circle]
 $\angle OTR = 60^\circ$ or $\angle STR = 60^\circ$
 But,
 $\angle PRS = \angle STR = 60^\circ$ [Angles in the alternate segment]
- (ii) In $\triangle OTR$,
 $\angle ORT = 90^\circ$
 $\angle OTR = 60^\circ$
 Thus,
 $\angle ROT = 180^\circ - (60^\circ + 60^\circ) = 180^\circ - 120^\circ = 60^\circ$

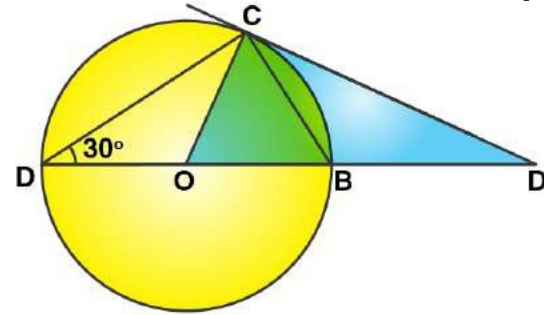
4. AB is diameter and AC is a chord of a circle with centre O such that angle BAC=30°. The tangent to the circle at C intersects AB produced in D. Show that BC = BD.

Solution:

Join OC.

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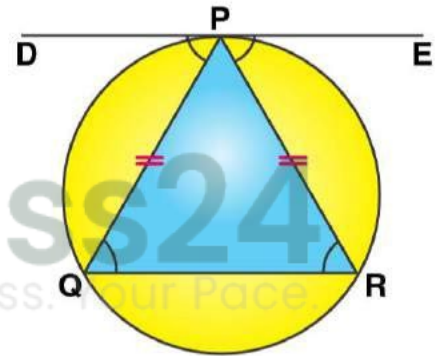
$\angle BCD = \angle BAC = 30^\circ$ [Angles in the alternate segment]
 It's seen that, arc BC subtends $\angle DOC$ at the center of the circle and $\angle BAC$ at the remaining part of the circle.
 So, $\angle BOC = 2\angle BAC = 2 \times 30^\circ = 60^\circ$
 Now, in $\triangle OCD$
 $\angle BOC$ or $\angle DOC = 60^\circ$
 $\angle OCD = 90^\circ$ [$OC \perp CD$]
 $\angle DOC + \angle ODC = 90^\circ$
 $\angle ODC = 90^\circ - 60^\circ = 30^\circ$
 Now, in $\triangle BCD$
 As $\angle ODC$ or $\angle BDC = \angle BCD = 30^\circ$
 Therefore, $BC = BD$



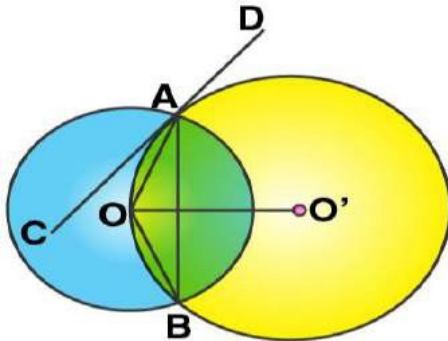
5. Tangent at P to the circumcircle of triangle PQR is drawn. If this tangent is parallel to side QR, show that triangle PQR is isosceles.

Solution:

Let DE be the tangent to the circle at P.
 And, $DE \parallel QR$ [Given]
 $\angle DPR = \angle PRQ$ [Alternate angles are equal]
 $\angle DPQ = \angle PQR$ [Alternate angles are equal] (i)
 Let $\angle DPQ = x$ and $\angle DPR = y$
 As the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment, we have
 $\angle DPQ = \angle PRQ$ (ii) [DE is tangent and PQ is chord]
 So, from (i) and (ii),
 $\angle PQR = \angle PRQ$
 $PQ = PR$
 Therefore, triangle PQR is an isosceles triangle.



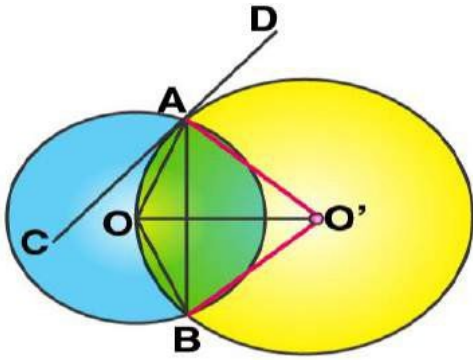
6. Two circles with centres O and O' are drawn to intersect each other at points A and B. Centre O of one circle lies on the circumference of the other circle and CD is drawn tangent to the circle with centre O' at A. Prove that OA bisects angle BAC.



Solution:

Join OA, OB, O'A, O'B and O'O.

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CD is the tangent and AO is the chord.

$$\angle OAC = \angle OBA \dots (i) \quad [\text{Angles in alternate segment}]$$

In $\triangle OAB$,

$$OA = OB \quad [\text{Radii of the same circle}]$$

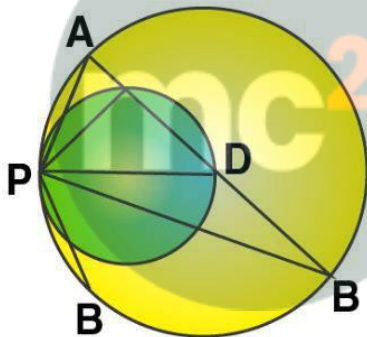
$$\angle OAB = \angle OBA \dots (ii)$$

From (i) and (ii), we have

$$\angle OAC = \angle OAB$$

Thus, OA is the bisector of $\angle BAC$.

7. Two circles touch each other internally at a point P. A chord AB of the bigger circle intersects the other circle in C and D. Prove that: $\angle CPA = \angle DPB$



Solution:

Let's draw a tangent TS at P to the circles given.

As TPS is the tangent and PD is the chord, we have

$$\angle PAB = \angle BPS \dots (i) \quad [\text{Angles in alternate segment}]$$

Similarly,

$$\angle PCD = \angle DPS \dots (ii)$$

Now, subtracting (i) from (ii) we have

$$\angle PCD - \angle PAB = \angle DPS - \angle BPS$$

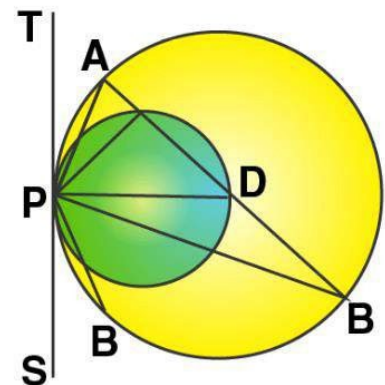
But in $\triangle PAC$,

$$\text{Ext. } \angle PCD = \angle PAB + \angle CPA$$

$$\angle PAB + \angle CPA - \angle PAB = \angle DPS - \angle BPS$$

Thus,

$$\angle CPA = \angle DPB$$



Exercise 18(C)

1. Prove that, of any two chords of a circle, the greater chord is nearer to the center.

Solution:

Given: A circle with center O and radius r. AB and CD are two chords such that $AB > CD$. Also, $OM \perp AB$ and $ON \perp CD$.

Required to prove: $OM < ON$

Proof:

Join OA and OC.

Then in right $\triangle AOM$, we have

$$AO^2 = AM^2 + OM^2$$

$$r^2 = (\frac{1}{2}AB)^2 + OM^2$$

$$r^2 = \frac{1}{4} AB^2 + OM^2 \dots (i)$$

Again, in right $\triangle ONC$, we have

$$OC^2 = NC^2 + ON^2$$

$$r^2 = (\frac{1}{2}CD)^2 + ON^2$$

$$r^2 = \frac{1}{4} CD^2 + ON^2 \dots (ii)$$

On equating (i) and (ii), we get

$$\frac{1}{4} AB^2 + OM^2 = \frac{1}{4} CD^2 + ON^2$$

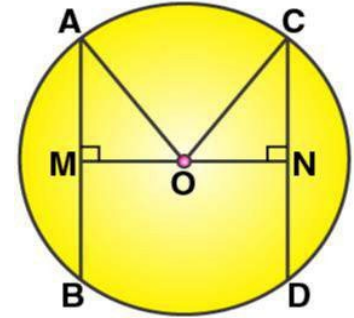
But, $AB > CD$ [Given]

So, ON will be greater than OM to be equal on both sides.

Thus,

$$OM < ON$$

Hence, AB is nearer to the centre than CD.



2. OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O.

i) If the radius of the circle is 10 cm, find the area of the rhombus.

ii) If the area of the rhombus is $32\sqrt{3}$ cm², find the radius of the circle.

Solution:

(i) Given, radius = 10 cm

In rhombus OABC,

$$OC = 10 \text{ cm}$$

So,

$$OE = \frac{1}{2} \times OB = \frac{1}{2} \times 10 = 5 \text{ cm}$$

Now, in right $\triangle OCE$

$$OC^2 = OE^2 + EC^2$$

$$10^2 = 5^2 + EC^2$$

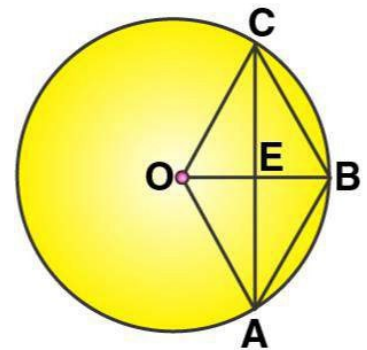
$$EC^2 = 100 - 25 = 75$$

$$EC = \sqrt{75} = 5\sqrt{3}$$

$$\text{Hence, } AC = 2 \times EC = 2 \times 5\sqrt{3} = 10\sqrt{3}$$

We know that,

$$\begin{aligned} \text{Area of rhombus} &= \frac{1}{2} \times OB \times AC \\ &= \frac{1}{2} \times 10 \times 10\sqrt{3} \\ &= 50\sqrt{3} \text{ cm}^2 \approx 86.6 \text{ cm}^2 \end{aligned}$$



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- (ii) We have the area of rhombus = $32\sqrt{3} \text{ cm}^2$
 But area of rhombus OABC = 2 x area of ΔOAB
 Area of rhombus OABC = $2 \times (\sqrt{3}/4) r^2$
 Where r is the side of the equilateral triangle OAB.
 $2 \times (\sqrt{3}/4) r^2 = 32\sqrt{3}$
 $\sqrt{3}/2 r^2 = 32\sqrt{3}$
 $r^2 = 64$
 $r = 8$
 Therefore, the radius of the circle is 8 cm.

3. Two circles with centers A and B, and radii 5 cm and 3 cm, touch each other internally. If the perpendicular bisector of the segment AB meets the bigger circle in P and Q; find the length of PQ.

Solution:

We know that,

If two circles touch internally, then distance between their centres is equal to the difference of their radii. So, $AB = (5 - 3) \text{ cm} = 2 \text{ cm}$.

Also, the common chord PQ is the perpendicular bisector of AB.

Thus, $AC = CB = \frac{1}{2} AB = 1 \text{ cm}$

In right ΔACP , we have

$$AP^2 = AC^2 + CP^2 \quad [\text{Pythagoras Theorem}]$$

$$5^2 = 1^2 + CP^2$$

$$CP^2 = 25 - 1 = 24$$

$$CP = \sqrt{24} \text{ cm} = 2\sqrt{6} \text{ cm}$$

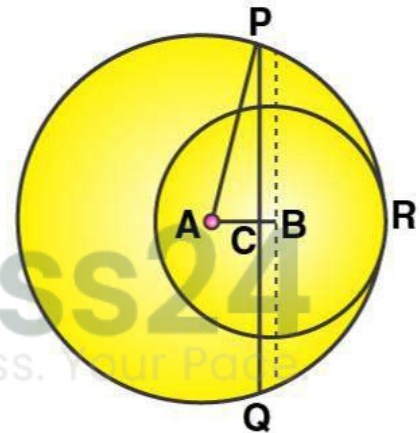
Now,

$$PQ = 2 CP$$

$$= 2 \times 2\sqrt{6} \text{ cm}$$

$$= 4\sqrt{6} \text{ cm}$$

Therefore, the length of PQ is $4\sqrt{6} \text{ cm}$.



4. Two chords AB and AC of a circle are equal. Prove that the center of the circle, lies on the bisector of the angle BAC.

Solution:

Given: AB and AC are two equal chords of C (O, r).

Required to prove: Centre, O lies on the bisector of $\angle BAC$.

Construction: Join BC. Let the bisector of $\angle BAC$ intersects BC in P.

Proof:

In ΔAPB and ΔAPC ,

$$AB = AC \quad [\text{Given}]$$

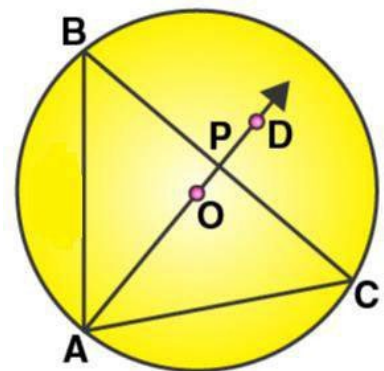
$$\angle BAP = \angle CAP \quad [\text{Given}]$$

$$AP = AP \quad [\text{Common}]$$

Hence, $\Delta APB \cong \Delta APC$ by SAA congruence criterion

So, by CPCT we have

$$BP = CP \text{ and } \angle APB = \angle APC$$



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And,

$$\angle APB + \angle APC = 180^\circ \quad [\text{Linear pair}]$$

$$2\angle APB = 180^\circ \quad [\angle APB = \angle APC]$$

$$\angle APB = 90^\circ$$

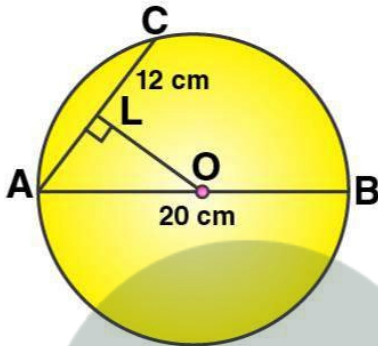
Now, $BP = CP$ and $\angle APB = 90^\circ$

Therefore, AP is the perpendicular bisector of chord BC .

Hence, AP passes through the centre, O of the circle.

5. The diameter and a chord of circle have a common end-point. If the length of the diameter is 20 cm and the length of the chord is 12 cm, how far is the chord from the center of the circle?

Solution:



We have, AB as the diameter and AC as the chord.

Now, draw $OL \perp AC$

Since $OL \perp AC$ and hence it bisects AC , O is the centre of the circle.

Therefore, $OA = 10$ cm and $AL = 6$ cm

Now, in right $\triangle OLA$

$$AO^2 = AL^2 + OL^2 \quad [\text{By Pythagoras Theorem}]$$

$$10^2 = 6^2 + OL^2$$

$$OL^2 = 100 - 36 = 64$$

$$OL = 8 \text{ cm}$$

Therefore, the chord is at a distance of 8 cm from the centre of the circle.

6. ABCD is a cyclic quadrilateral in which BC is parallel to AD , angle $ADC = 110^\circ$ and angle $BAC = 50^\circ$. Find angle DAC and angle DCA .

Solution:

Given, $ABCD$ is a cyclic quadrilateral in which $AD \parallel BC$

And, $\angle ADC = 110^\circ$, $\angle BAC = 50^\circ$

We know that,

$$\angle B + \angle D = 180^\circ \quad [\text{Sum of opposite angles of a quadrilateral}]$$

$$\angle B + 110^\circ = 180^\circ$$

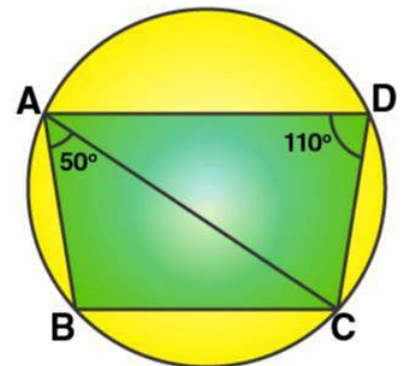
$$\text{So, } \angle B = 70^\circ$$

Now in $\triangle ADC$, we have

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

$$50^\circ + 70^\circ + \angle ACB = 180^\circ$$

$$\angle ACB = 180^\circ - 120^\circ = 60^\circ$$



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And, as $AD \parallel BC$ we have

$$\angle DAC = \angle ACB = 60^\circ \quad [\text{Alternate angles}]$$

Now in $\triangle ADC$,

$$\angle DAC + \angle ADC + \angle DCA = 180^\circ$$

$$60^\circ + 110^\circ + \angle DCA = 180^\circ$$

Thus,

$$\angle DCA = 180^\circ - 170^\circ = 10^\circ$$

7. In the given figure, C and D are points on the semi-circle described on AB as diameter. Given angle $\angle BAD = 70^\circ$ and angle $\angle DBC = 30^\circ$, calculate angle BDC.

Solution:

As ABCD is a cyclic quadrilateral, we have

$$\angle BCD + \angle BAD = 180^\circ \quad [\text{Opposite angles of a cyclic quadrilateral are supplementary}]$$

$$\angle BCD + 70^\circ = 180^\circ$$

$$\angle BCD = 180^\circ - 70^\circ = 110^\circ$$

And, by angle sum property of $\triangle BCD$ we have

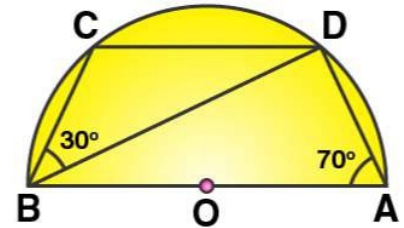
$$\angle CBD + \angle BCD + \angle BDC = 180^\circ$$

$$30^\circ + 110^\circ + \angle BDC = 180^\circ$$

$$\angle BDC = 180^\circ - 140^\circ$$

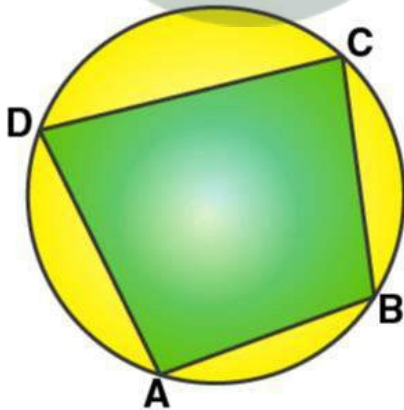
Thus,

$$\angle BDC = 40^\circ$$



8. In cyclic quadrilateral ABCD, $\angle A = 3 \angle C$ and $\angle D = 5 \angle B$. Find the measure of each angle of the quadrilateral.

Solution:



Given, cyclic quadrilateral ABCD

$$\text{So, } \angle A + \angle C = 180^\circ \quad [\text{Opposite angles in a cyclic quadrilateral is supplementary}]$$

$$3\angle C + \angle C = 180^\circ \quad [\text{As } \angle A = 3 \angle C]$$

$$\angle C = 45^\circ$$

Now,

$$\angle A = 3 \angle C = 3 \times 45^\circ$$

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$$\angle A = 135^\circ$$

Similarly,

$$\angle B + \angle D = 180^\circ \quad [\text{As } \angle D = 5 \angle B]$$

$$\angle B + 5\angle B = 180^\circ$$

$$6\angle B = 180^\circ$$

$$\angle B = 30^\circ$$

Now,

$$\angle D = 5\angle B = 5 \times 30^\circ$$

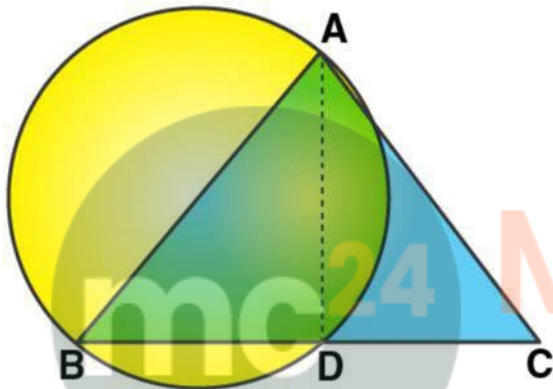
$$\angle D = 150^\circ$$

Therefore,

$$\angle A = 135^\circ, \angle B = 30^\circ, \angle C = 45^\circ, \angle D = 150^\circ$$

9. Show that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.

Solution:



Let's join AD.

And, AB is the diameter.

We have $\angle ADB = 90^\circ$ [Angle in a semi-circle]

But,

$\angle ADB + \angle ADC = 180^\circ$ [Linear pair]

So, $\angle ADC = 90^\circ$

Now, in $\triangle ABD$ and $\triangle ACD$ we have

$\angle ADB = \angle ADC$ [each 90°]

$AB = AC$ [Given]

$AD = AD$ [Common]

Hence, $\triangle ABD \cong \triangle ACD$ by RHS congruence criterion

So, by C.P.C.T

$BD = DC$

Therefore, the circle bisects base BC at D.

10. Bisectors of vertex angles A, B and C of a triangle ABC intersect its circumcircle at points D, E and F respectively. Prove that angle EDF = $90^\circ - \frac{1}{2} \angle A$

Solution:

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Join ED, EF and DF. Also join BF, FA, AE and EC.

$$\angle EBF = \angle ECF = \angle EDF \dots (i) \quad [\text{Angle in the same segment}]$$

In cyclic quadrilateral AFBE,
 $\angle EBF + \angle EAF = 180^\circ \dots (ii)$

[Sum of opposite angles in a cyclic quadrilateral is supplementary]

Similarly in cyclic quadrilateral CEAF,

$$\angle EAF + \angle ECF = 180^\circ \dots (iii)$$

Adding (ii) and (iii) we get,

$$\angle EBF + \angle ECF + 2\angle EAF = 360^\circ$$

$$\angle EDF + \angle EDF + 2\angle EAF = 360^\circ \quad [\text{From (i)}]$$

$$\angle EDF + \angle EAF = 180^\circ$$

$$\angle EDF + \angle 1 + \angle BAC + \angle 2 = 180^\circ$$

But, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ [Angles in the same segment]

$$\angle EDF + \angle 3 + \angle BAC + \angle 4 = 180^\circ$$

$$\text{But, } \angle 4 = \frac{1}{2} \angle C, \angle 3 = \frac{1}{2} \angle B$$

$$\text{Thus, } \angle EDF + \frac{1}{2} \angle B + \angle BAC + \frac{1}{2} \angle C = 180^\circ$$

$$\angle EDF + \frac{1}{2} \angle B + 2 \times \frac{1}{2} \angle A + \frac{1}{2} \angle C = 180^\circ$$

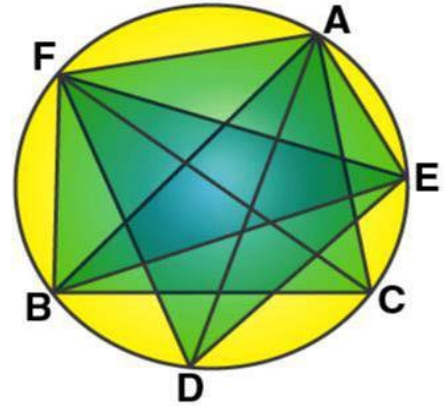
$$\angle EDF + \frac{1}{2} (\angle A + \angle B + \angle C) + \frac{1}{2} \angle A = 180^\circ$$

$$\angle EDF + \frac{1}{2} (180^\circ) + \frac{1}{2} \angle A = 180^\circ$$

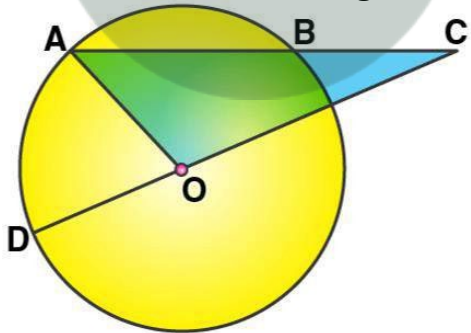
$$\angle EDF + 90^\circ + \frac{1}{2} \angle A = 180^\circ$$

$$\angle EDF = 180^\circ - (90^\circ + \frac{1}{2} \angle A)$$

$$\angle EDF = 90^\circ - \frac{1}{2} \angle A$$



11. In the figure, AB is the chord of a circle with centre O and DOC is a line segment such that BC = DO. If $\angle C = 20^\circ$, find angle AOD.



Solution:

Join OB.

In $\triangle OBC$, we have

$$BC = OD = OB \quad [\text{Radii of the same circle}]$$

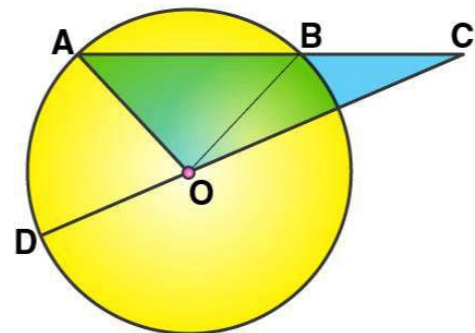
$$\angle BOC = \angle BCO = 20^\circ$$

$$\text{And ext. } \angle ABO = \angle BCO + \angle BOC$$

$$\text{Ext. } \angle ABO = 20^\circ + 20^\circ = 40^\circ \dots (1)$$

Now in $\triangle OAB$,

$$OA = OB \quad [\text{Radii of the same circle}]$$



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$$\angle OAB = \angle OBA = 40^\circ \quad [\text{from (1)}]$$

$$\angle AOB = 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

As DOC is a straight line,

$$\angle AOD + \angle AOB + \angle BOC = 180^\circ$$

$$\angle AOD + 100^\circ + 20^\circ = 180^\circ$$

$$\angle AOD = 180^\circ - 120^\circ$$

$$\text{Thus, } \angle AOD = 60^\circ$$

12. Prove that the perimeter of a right triangle is equal to the sum of the diameter of its incircle and twice the diameter of its circumcircle.

Solution:

Let's join OL, OM and ON.

And, let D and d be the diameter of the circumcircle and incircle.

Also, let R and r be the radius of the circumcircle and incircle.

Now, in circumcircle of $\triangle ABC$,

$$\angle B = 90^\circ$$

Thus, AC is the diameter of the circumcircle i.e. $AC = D$

Let the radius of the incircle be 'r'

$$OL = OM = ON = r$$

Now, from B, BL and BM are the tangents to the incircle.

$$\text{So, } BL = BM = r$$

Similarly,

$$AM = AN \text{ and } CL = CN = R$$

[Tangents from the point outside the circle]

Now,

$$AB + BC + CA = AM + BM + BL + CL + CA$$

$$= AN + r + r + CN + CA$$

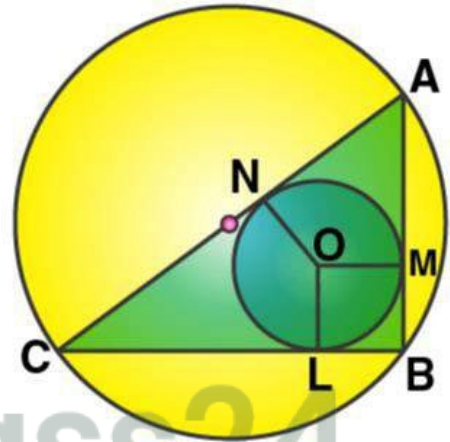
$$= AN + CN + 2r + CA$$

$$= AC + AC + 2r$$

$$= 2AC + 2r$$

$$= 2D + d$$

- Hence Proved



13. P is the midpoint of an arc APB of a circle. Prove that the tangent drawn at P will be parallel to the chord AB.

Solution:

First join AP and BP.

As TPS is a tangent and PA is the chord of the circle.

$$\angle BPT = \angle PAB \quad [\text{Angles in alternate segments}]$$

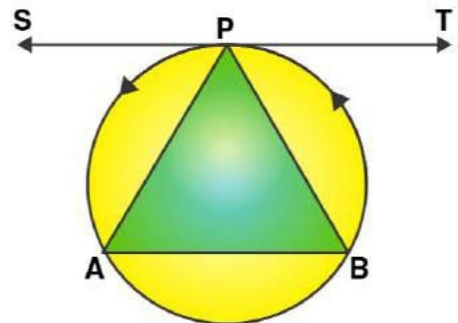
But,

$$\angle PBA = \angle PAB \quad [\text{Since } PA = PB]$$

$$\text{Thus, } \angle BPT = \angle PBA$$

But these are alternate angles,

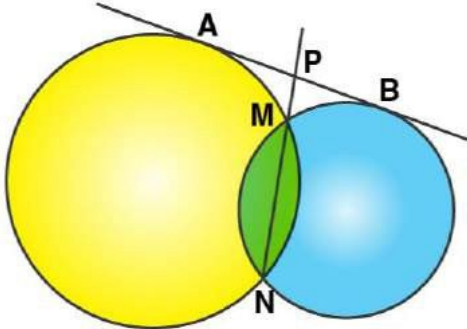
$$\text{Hence, } TPS \parallel AB$$



Chapter 18: Tangents and Intersecting Chords

14. In the given figure, MN is the common chord of two intersecting circles and AB is their common tangent.

Prove that the line NM produced bisects AB at P.



Solution:

From P, AP is the tangent and PMN is the secant for first circle.

$$AP^2 = PM \times PN \dots (1)$$

Again from P, PB is the tangent and PMN is the secant for second circle.

$$PB^2 = PM \times PN \dots (2)$$

From (i) and (ii), we have

$$AP^2 = PB^2$$

$$AP = PB$$

Thus, P is the midpoint of AB.

15. In the given figure, ABCD is a cyclic quadrilateral, PQ is tangent to the circle at point C and BD is its diameter. If $\angle DCQ = 40^\circ$ and $\angle ABD = 60^\circ$, find:

i) $\angle DBC$

ii) $\angle BCP$

iii) $\angle ADB$

Solution:

PQ is a tangent and CD is a chord.

$$\angle DCQ = \angle DBC \quad [\text{Angles in the alternate segment}]$$

$$\angle DBC = 40^\circ \quad [\text{As } \angle DCQ = 40^\circ]$$

$$(ii) \angle DCQ + \angle DCB + \angle BCP = 180^\circ$$

$$40^\circ + 90^\circ + \angle BCP = 180^\circ \quad [\text{As } \angle DCB = 90^\circ]$$

$$\angle BCP = 180^\circ - 130^\circ = 50^\circ$$

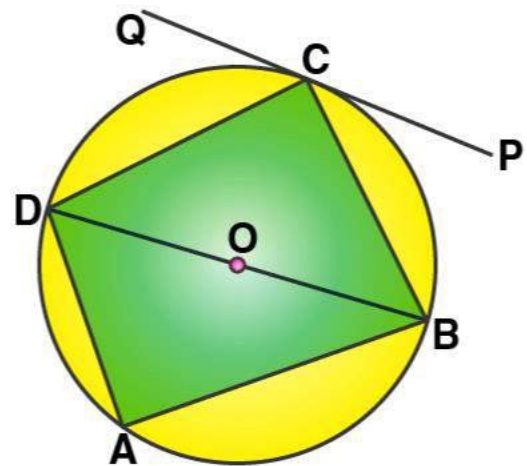
(iii) In $\triangle ABD$,

$$\angle BAD = 90^\circ \quad [\text{Angle in a semi-circle}], \angle ABD = 60^\circ$$

[Given]

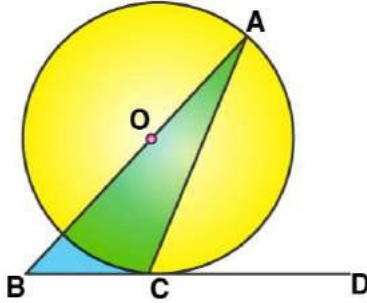
$$\angle ADB = 180^\circ - (90^\circ + 60^\circ)$$

$$\angle ADB = 180^\circ - 150^\circ = 30^\circ$$



Chapter 18: Tangents and Intersecting Chords

16. The given figure shows a circle with centre O and BCD is a tangent to it at C. Show that: $\angle ACD + \angle BAC = 90^\circ$



Solution:

Let's join OC.

BCD is the tangent and OC is the radius.

As, $OC \perp BD$

$\angle OCD = 90^\circ$

$\angle OCD + \angle ACD = 90^\circ \dots (i)$

But, in $\triangle OCA$

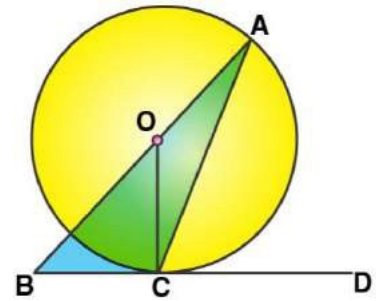
$OA = OC$ [Radii of the same circle]

Thus, $\angle OCA = \angle OAC$

Substituting in (i), we get

$\angle OAC + \angle ACD = 90^\circ$

Hence, $\angle BAC + \angle ACD = 90^\circ$



17. ABC is a right triangle with angle $B = 90^\circ$. A circle with BC as diameter meets by hypotenuse AC at point D. Prove that:

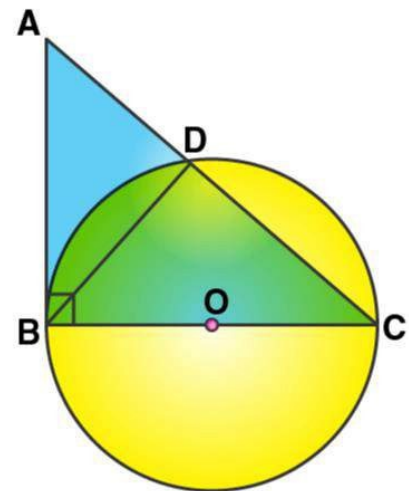
i) $AC \times AD = AB^2$

ii) $BD^2 = AD \times DC$.

Solution:

i) In $\triangle ABC$, we have
 $\angle B = 90^\circ$ and BC is the diameter of the circle.
 Hence, AB is the tangent to the circle at B.
 Now, as AB is tangent and ADC is the secant we have
 $AB^2 = AD \times AC$

ii) In $\triangle ADB$,
 $\angle D = 90^\circ$
 So, $\angle A + \angle ABD = 90^\circ \dots (i)$
 But in $\triangle ABC$, $\angle B = 90^\circ$
 $\angle A + \angle C = 90^\circ \dots (ii)$
 From (i) and (ii),
 $\angle C = \angle ABD$
 Now in $\triangle ABD$ and $\triangle CBD$, we have
 $\angle BDA = \angle BDC = 90^\circ$



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$$\angle ABD = \angle BCD$$

Hence, $\triangle ABD \sim \triangle CBD$ by AA postulate

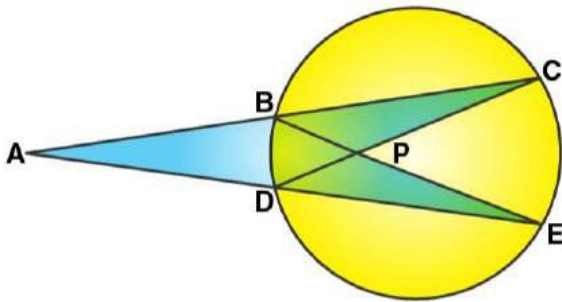
So, we have

$$BD/DC = AD/BD$$

Therefore,

$$BD^2 = AD \times DC$$

18. In the given figure, $AC = AE$.



Show that:

i) $CP = EP$

ii) $BP = DP$

Solution:

In $\triangle ADC$ and $\triangle ABE$,

$\angle ACD = \angle AEB$ [Angles in the same segment]

$AC = AE$ [Given]

$\angle A = \angle A$ [Common]

Hence, $\triangle ADC \cong \triangle ABE$ by ASA postulate

So, by C.P.C.T we have

$AB = AD$

But, $AC = AE$ [Given]

So, $AC - AB = AE - AD$

$BC = DE$

In $\triangle BPC$ and $\triangle DPE$,

$\angle C = \angle E$ [Angles in the same segment]

$BC = DE$

$\angle CBP = \angle CDE$ [Angles in the same segment]

Hence, $\triangle BPC \cong \triangle DPE$ by ASA postulate

So, by C.P.C.T we have

$BP = DP$ and $CP = PE$

19. $ABCDE$ is a cyclic pentagon with centre of its circumcircle at point O such that $AB = BC = CD$ and angle $ABC = 120^\circ$

Calculate:

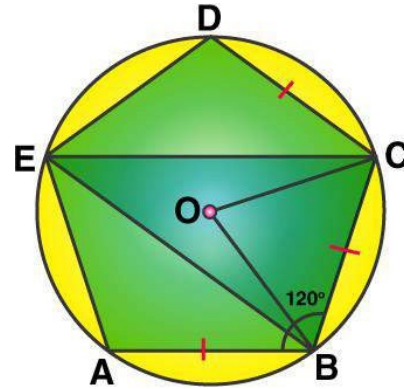
i) $\angle BEC$

ii) $\angle BED$

Solution:

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- i) Join OC and OB.
 $AB = BC = CD$ and $\angle ABC = 120^\circ$ [Given]
 So, $\angle BCD = \angle ABC = 120^\circ$
 OB and OC are the bisectors of $\angle ABC$ and $\angle BCD$ and respectively.
 So, $\angle OBC = \angle BCO = 60^\circ$
 In $\triangle BOC$,
 $\angle BOC = 180^\circ - (\angle OBC + \angle BCO)$
 $\angle BOC = 180^\circ - (60^\circ + 60^\circ) = 180^\circ - 120^\circ$
 $\angle BOC = 60^\circ$
 Arc BC subtends $\angle BOC$ at the centre and $\angle BEC$ at the remaining part of the circle.
 $\angle BEC = \frac{1}{2} \angle BOC = \frac{1}{2} \times 60^\circ = 30^\circ$



- ii) In cyclic quadrilateral BCDE, we have
 $\angle BED + \angle BCD = 180^\circ$
 $\angle BED + 120^\circ = 180^\circ$
 Thus, $\angle BED = 60^\circ$

20. In the given figure, O is the centre of the circle. Tangents at A and B meet at C. If angle ACO = 30° , find:

- (i) angle BCO
 (ii) angle AOB
 (iii) angle APB

Solution:

In the given fig, O is the centre of the circle and, CA and CB are the tangents to the circle from C.

Also, $\angle ACO = 30^\circ$

P is any point on the circle. P and B are joined.

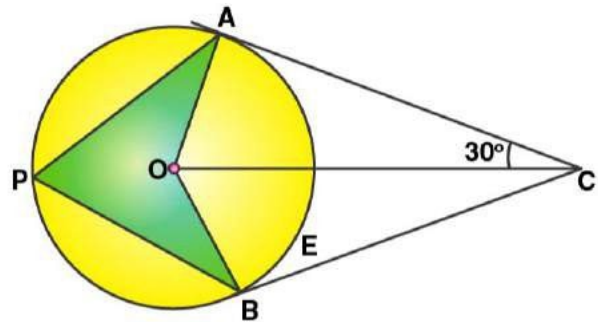
To find:

- (i) $\angle BCO$
 (ii) $\angle AOB$
 (iii) $\angle APB$

Proof:

- (i) In $\triangle OAC$ and $\triangle OBC$, we have
 $OC = OC$ [Common]
 $OA = OB$ [Radii of the same circle]
 $CA = CB$ [Tangents to the circle]
 Hence, $\triangle OAC \cong \triangle OBC$ by SSS congruence criterion
 Thus, $\angle ACO = \angle BCO = 30^\circ$

- (ii) As $\angle ACB = 30^\circ + 30^\circ = 60^\circ$



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And, $\angle AOB + \angle ACB = 180^\circ$

$$\angle AOB + 60^\circ = 180^\circ$$

$$\angle AOB = 180^\circ - 60^\circ$$

$$\angle AOB = 120^\circ$$

(iii) Arc AB subtends $\angle AOB$ at the center and $\angle APB$ is the remaining part of the circle.

$$\angle APB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 120^\circ = 60^\circ$$

21. ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centers. Find the radii of the three circles.

Solution:

Given: ABC is a triangle with AB = 10 cm, BC = 8 cm, AC = 6 cm. Three circles are drawn with centre A, B and C touch each other at P, Q and R respectively.

So, we need to find the radii of the three circles.

Let,

$$PA = AQ = x$$

$$QC = CR = y$$

$$RB = BP = z$$

So, we have

$$x + z = 10 \dots\dots (i)$$

$$z + y = 8 \dots\dots (ii)$$

$$y + x = 6 \dots\dots (iii)$$

Adding all the three equations, we have

$$2(x + y + z) = 24$$

$$x + y + z = 24/2 = 12 \dots\dots (iv)$$

Subtracting (i), (ii) and (iii) from (iv) we get

$$y = 12 - 10 = 2$$

$$x = 12 - 8 = 4$$

$$z = 12 - 6 = 6$$

Thus, radii of the three circles are 2 cm, 4 cm and 6 cm.

