

## EXERCISE 15.5

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### 1. State Pythagoras theorem and its converse.

#### Solution:

The Pythagoras Theorem:

In a right triangle, the square of the hypotenuse is always equal to the sum of the squares of the other two sides.

Converse of the Pythagoras Theorem:

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle, with the angle opposite to the first side as right angle.

### 2. In right $\triangle ABC$ , the lengths of the legs are given. Find the length of the hypotenuse

(i)  $a = 6$  cm,  $b = 8$  cm

(ii)  $a = 8$  cm,  $b = 15$  cm

(iii)  $a = 3$  cm,  $b = 4$  cm

(iv)  $a = 2$  cm,  $b = 1.5$  cm

#### Solution:

(i) According to the Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$

Let  $c$  be hypotenuse and  $a$  and  $b$  be other two legs of right angled triangle

Then we have

$$c^2 = a^2 + b^2$$

$$c^2 = 6^2 + 8^2$$

$$c^2 = 36 + 64 = 100$$

$$c = 10 \text{ cm}$$

(ii) According to the Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$

Let  $c$  be hypotenuse and  $a$  and  $b$  be other two legs of right angled triangle

Then we have

$$c^2 = a^2 + b^2$$

$$c^2 = 8^2 + 15^2$$

$$c^2 = 64 + 225 = 289$$

$$c = 17\text{cm}$$

(iii) According to the Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$

Let  $c$  be hypotenuse and  $a$  and  $b$  be other two legs of right angled triangle

Then we have

$$c^2 = a^2 + b^2$$

$$c^2 = 3^2 + 4^2$$

$$c^2 = 9 + 16 = 25$$

$$c = 5 \text{ cm}$$

(iv) According to the Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$

Let  $c$  be hypotenuse and  $a$  and  $b$  be other two legs of right angled triangle

Then we have

$$c^2 = a^2 + b^2$$

$$c^2 = 2^2 + 1.5^2$$

$$c^2 = 4 + 2.25 = 6.25$$

$$c = 2.5 \text{ cm}$$

**3. The hypotenuse of a triangle is 2.5 cm. If one of the sides is 1.5 cm. find the length of the other side.**

**Solution:**

Let  $c$  be hypotenuse and the other two sides be  $b$  and  $a$

According to the Pythagoras theorem, we have

$$c^2 = a^2 + b^2$$

$$2.5^2 = 1.5^2 + b^2$$

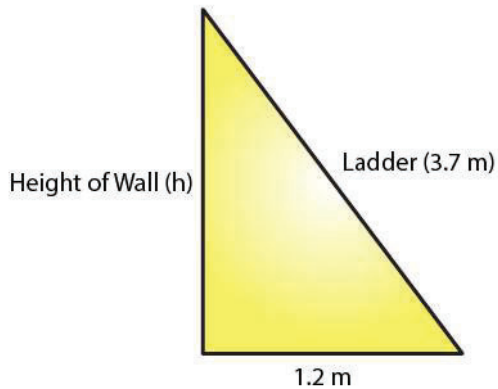
$$b^2 = 6.25 - 2.25 = 4$$

$$b = 2 \text{ cm}$$

Hence, the length of the other side is 2 cm.

**4. A ladder 3.7 m long is placed against a wall in such a way that the foot of the ladder is 1.2 m away from the wall. Find the height of the wall to which the ladder reaches.**

**Solution:**



Let the height of the ladder reaches to the wall be  $h$ .

According to the Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$

$$3.7^2 = 1.2^2 + h^2$$

$$h^2 = 13.69 - 1.44 = 12.25$$

$$h = 3.5 \text{ m}$$

Hence, the height of the wall is 3.5 m.

5. If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is right-angled triangle.

**Solution:**

In the given triangle, the largest side is 6 cm.

We know that in a right angled triangle, the sum of the squares of the smaller sides should be equal to the square of the largest side.

Therefore,

$$3^2 + 4^2 = 9 + 16 = 25$$

$$\text{But, } 6^2 = 36$$

$$3^2 + 4^2 = 25 \text{ which is not equal to } 6^2$$

Hence, the given triangle is not a right angled triangle.

6. The sides of certain triangles are given below. Determine which of them are right triangles.

(i)  $a = 7 \text{ cm}$ ,  $b = 24 \text{ cm}$  and  $c = 25 \text{ cm}$

(ii)  $a = 9 \text{ cm}$ ,  $b = 16 \text{ cm}$  and  $c = 18 \text{ cm}$

**Solution:**

(i) We know that in a right angled triangle, the square of the largest side is equal to the

sum of the squares of the smaller sides.  
Here, the larger side is  $c$ , which is 25 cm.

$$c^2 = 625$$

Given that,

$$a^2 + b^2 = 7^2 + 24^2$$

$$= 49 + 576$$

$$= 625$$

$$= c^2$$

Thus, the given triangle is a right triangle.

(ii) We know that in a right angled triangle, the square of the largest side is equal to the sum of the squares of the smaller sides.

Here, the larger side is  $c$ , which is 18 cm.

$$c^2 = 324$$

Given that

$$a^2 + b^2 = 9^2 + 16^2$$

$$= 81 + 256$$

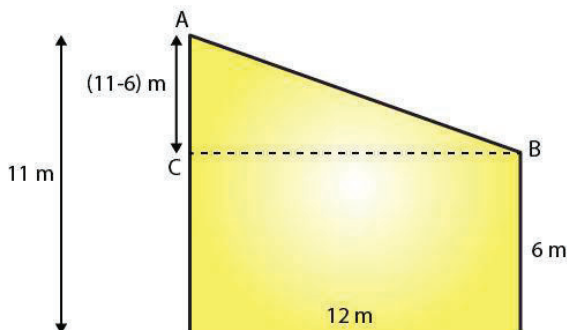
$$= 337 \text{ which is not equal to } c^2$$

Thus, the given triangle is not a right triangle.

**7. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m. Find the distance between their tops.**

**(Hint: Find the hypotenuse of a right triangle having the sides  $(11 - 6)$  m = 5 m and 12 m)**

**Solution:**



Let the distance between the tops of the poles is the distance between points A and B. We can see from the given figure that points A, B and C form a right triangle, with AB as the hypotenuse.

By using the Pythagoras Theorem in  $\triangle ABC$ , we get

$$(11-6)^2 + 12^2 = AB^2$$

$$AB^2 = 25 + 144$$

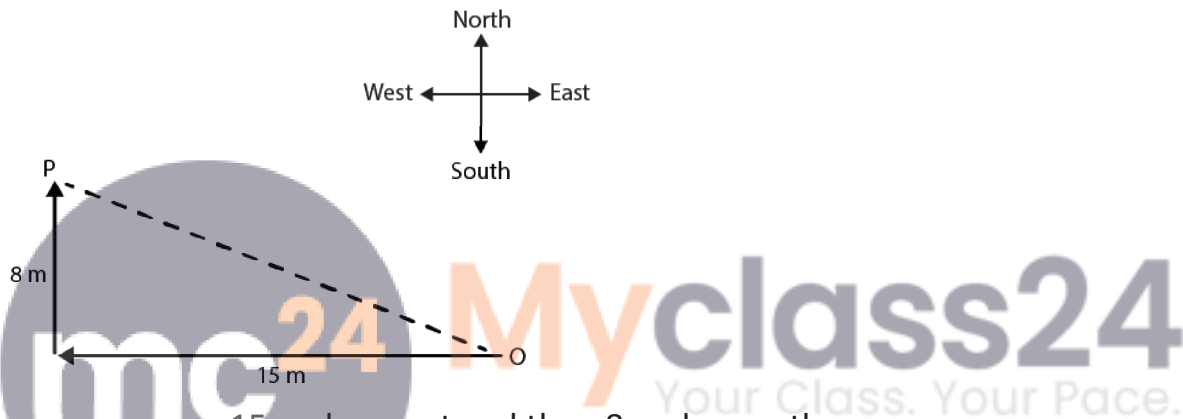
$$AB^2 = 169$$

$$AB = 13$$

Hence, the distance between the tops of the poles is 13 m.

**8. A man goes 15 m due west and then 8 m due north. How far is he from the starting point?**

**Solution:**



Given a man goes 15 m due west and then 8 m due north

Let O be the starting point and P be the final point.

Then OP becomes the hypotenuse in the triangle.

So by using the Pythagoras theorem, we can find the distance OP.

$$OP^2 = 15^2 + 8^2$$

$$OP^2 = 225 + 64$$

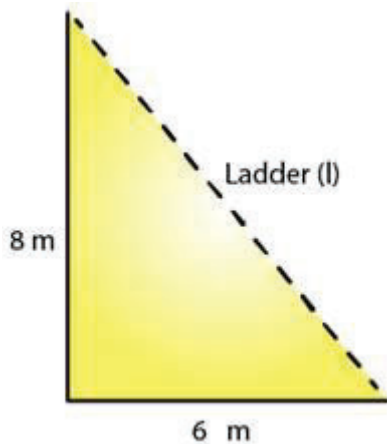
$$OP^2 = 289$$

$$OP = 17$$

Hence, the required distance is 17 m.

**9. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its top reach?**

**Solution:**



Given Let the length of the ladder be  $L$  m.

By using the Pythagoras theorem, we can find the length of the ladder.

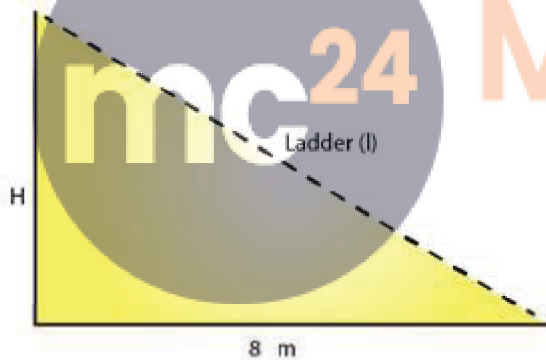
$$6^2 + 8^2 = L^2$$

$$L^2 = 36 + 64 = 100$$

$$L = 10$$

Thus, the length of the ladder is 10 m.

When ladder is shifted,



Let the height of the ladder after it is shifted be  $H$  m.

By using the Pythagoras theorem, we can find the height of the ladder after it is shifted.

$$8^2 + H^2 = 10^2$$

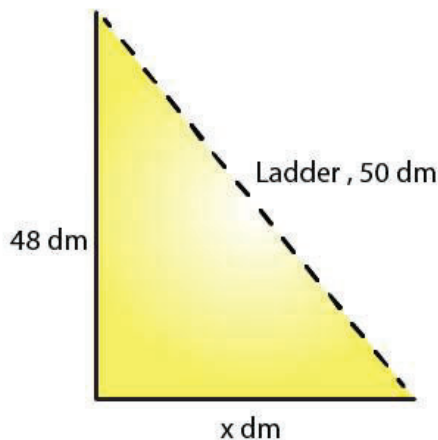
$$H^2 = 100 - 64 = 36$$

$$H = 6$$

Thus, the height of the ladder is 6 m.

**10. A ladder 50 dm long when set against the wall of a house just reaches a window at a height of 48 dm. How far is the lower end of the ladder from the base of the wall?**

**Solution:**



Given that length of a ladder is 50dm

Let the distance of the lower end of the ladder from the wall be x dm.

By using the Pythagoras theorem, we get

$$x^2 + 48^2 = 50^2$$

$$x^2 = 50^2 - 48^2$$

$$= 2500 - 2304$$

$$= 196$$

$$H = 14 \text{ dm}$$

Hence, the distance of the lower end of the ladder from the wall is 14 dm.

**11. The two legs of a right triangle are equal and the square of the hypotenuse is 50. Find the length of each leg.**

**Solution:**

According to the Pythagoras theorem, we have

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$

Given that the two legs of a right triangle are equal and the square of the hypotenuse, which is 50

Let the length of each leg of the given triangle be x units.

Using the Pythagoras theorem, we get

$$x^2 + x^2 = (\text{Hypotenuse})^2$$

$$x^2 + x^2 = 50$$

$$2x^2 = 50$$

$$x^2 = 25$$

$$x = 5$$

Hence, the length of each leg is 5 units.

**12. Verity that the following numbers represent Pythagorean triplet:**

**(i) 12, 35, 37**

**(ii) 7, 24, 25**

**(iii) 27, 36, 45**

**(iv) 15, 36, 39**

**Solution:**

(i) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$$37^2 = 1369$$

$$12^2 + 35^2 = 144 + 1225 = 1369$$

$$12^2 + 35^2 = 37^2$$

Yes, they represent a Pythagorean triplet.

(ii) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$$25^2 = 625$$

$$7^2 + 24^2 = 49 + 576 = 625$$

$$7^2 + 24^2 = 25^2$$

Yes, they represent a Pythagorean triplet.

(iii) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$$45^2 = 2025$$

$$27^2 + 36^2 = 729 + 1296 = 2025$$

$$27^2 + 36^2 = 45^2$$

Yes, they represent a Pythagorean triplet.

(iv) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$$39^2 = 1521$$

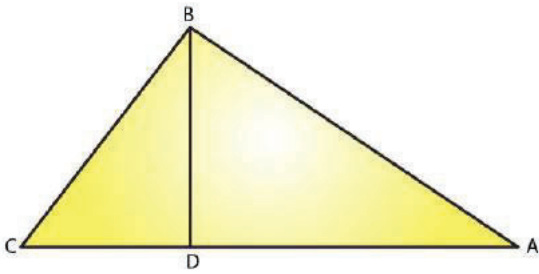
$$15^2 + 36^2 = 225 + 1296 = 1521$$

$$15^2 + 36^2 = 39^2$$

Yes, they represent a Pythagorean triplet.

**13. In  $\triangle ABC$ ,  $\angle ABC = 100^\circ$ ,  $\angle BAC = 35^\circ$  and  $BD \perp AC$  meets side  $AC$  in  $D$ . If  $BD = 2$  cm, find  $\angle C$ , and length  $DC$ .**

**Solution:**



We know that the sum of all angles of a triangle is  $180^\circ$

Therefore, for the given  $\triangle ABC$ , we can say that:

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$100^\circ + 35^\circ + \angle ACB = 180^\circ$$

$$\angle ACB = 180^\circ - 135^\circ$$

$$\angle ACB = 45^\circ$$

$$\angle C = 45^\circ$$

On applying same steps for the  $\triangle BCD$ , we get

$$\angle BCD + \angle BDC + \angle CBD = 180^\circ$$

$$45^\circ + 90^\circ + \angle CBD = 180^\circ \quad (\angle ACB = \angle BCD \text{ and } BD \text{ is perpendicular to } AC)$$

$$\angle CBD = 180^\circ - 135^\circ$$

$$\angle CBD = 45^\circ$$

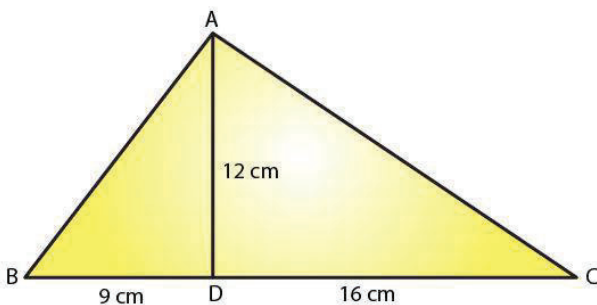
We know that the sides opposite to equal angles have equal length.

Thus,  $BD = DC$

$$DC = 2 \text{ cm}$$

**14. In a  $\triangle ABC$ , AD is the altitude from A such that  $AD = 12 \text{ cm}$ ,  $BD = 9 \text{ cm}$  and  $DC = 16 \text{ cm}$ . Examine if  $\triangle ABC$  is right angled at A.**

**Solution:**



Consider  $\triangle ADC$ ,

$$\angle ADC = 90^\circ \quad (\text{AD is an altitude on BC})$$

Using the Pythagoras theorem, we get

$$12^2 + 16^2 = AC^2$$

$$AC^2 = 144 + 256$$

$$= 400$$

$$AC = 20 \text{ cm}$$

Again consider  $\triangle ADB$ ,

$\angle ADB = 90^\circ$  (AD is an altitude on BC)

Using the Pythagoras theorem, we get

$$12^2 + 9^2 = AB^2$$

$$AB^2 = 144 + 81 = 225$$

$$AB = 15 \text{ cm}$$

Consider  $\triangle ABC$ ,

$$BC^2 = 25^2 = 625$$

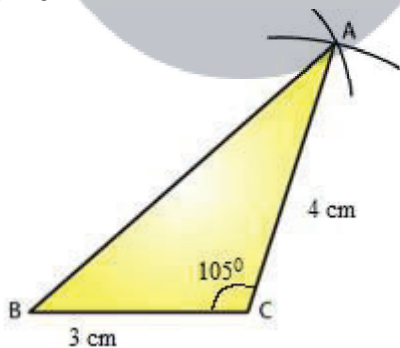
$$AB^2 + AC^2 = 15^2 + 20^2 = 625$$

$$AB^2 + AC^2 = BC^2$$

Because it satisfies the Pythagoras theorem, therefore  $\triangle ABC$  is right angled at A.

**15. Draw a triangle ABC, with  $AC = 4 \text{ cm}$ ,  $BC = 3 \text{ cm}$  and  $\angle C = 105^\circ$ . Measure AB. Is  $(AB)^2 = (AC)^2 + (BC)^2$ ? If not which one of the following is true:  $(AB)^2 > (AC)^2 + (BC)^2$  or  $(AB)^2 < (AC)^2 + (BC)^2$ ?**

**Solution:**



Draw  $\triangle ABC$  as shown in the figure with following steps.

Draw a line  $BC = 3 \text{ cm}$ .

At point C, draw a line at  $105^\circ$  angle with BC.

Take an arc of  $4 \text{ cm}$  from point C, which will cut the line at point A.

Now, join AB, which will be approximately  $5.5 \text{ cm}$ .

$$AC^2 + BC^2 = 4^2 + 3^2$$

$$= 9 + 16$$

$$= 25$$

$$AB^2 = 5.5^2 = 30.25$$

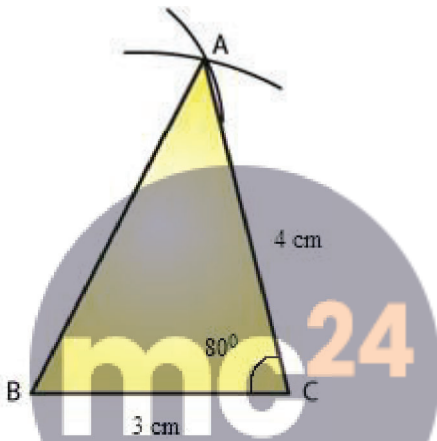
$AB^2$  is not equal to  $AC^2 + BC^2$

Therefore we have

$$AB^2 > AC^2 + BC^2$$

**16. Draw a triangle ABC, with AC = 4 cm, BC = 3 cm and  $\angle C = 80^\circ$ . Measure AB. Is  $(AB)^2 = (AC)^2 + (BC)^2$ ? If not which one of the following is true:  $(AB)^2 > (AC)^2 + (BC)^2$  or  $(AB)^2 < (AC)^2 + (BC)^2$ ?**

**Solution:**



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Draw  $\triangle ABC$  as shown in the figure with following steps.

Draw a line  $BC = 3$  cm.

At point C, draw a line at  $80^\circ$  angle with BC.

Take an arc of 4 cm from point C, which will cut the line at point A.

Now, join AB, it will be approximately 4.5 cm.

$$AC^2 + BC^2 = 4^2 + 3^2$$

$$= 9 + 16$$

$$= 25$$

$$AB^2 = (4.5)^2$$

$$= 20.25$$

$AB^2$  not equal to  $AC^2 + BC^2$

Therefore here  $AB^2 < AC^2 + BC^2$