

EXERCISE 4.4

1. Find the principal value of each of the following:

(i) $\sec^{-1}(-\sqrt{2})$

(ii) $\sec^{-1}(2)$

(iii) $\sec^{-1}(2 \sin(3\pi/4))$

(iv) $\sec^{-1}(2 \tan(3\pi/4))$

Solution:

(i) Given $\sec^{-1}(-\sqrt{2})$

Now let $y = \sec^{-1}(-\sqrt{2})$

$\sec y = -\sqrt{2}$

We know that $\sec \pi/4 = \sqrt{2}$

Therefore, $-\sec(\pi/4) = -\sqrt{2}$

$= \sec(\pi - \pi/4)$

$= \sec(3\pi/4)$

Thus the range of principal value of \sec^{-1} is $[0, \pi] - \{\pi/2\}$

And $\sec(3\pi/4) = -\sqrt{2}$

Hence the principal value of $\sec^{-1}(-\sqrt{2})$ is $3\pi/4$

(ii) Given $\sec^{-1}(2)$

Let $y = \sec^{-1}(2)$

$\sec y = 2$

$= \sec \pi/3$

Therefore the range of principal value of \sec^{-1} is $[0, \pi] - \{\pi/2\}$ and $\sec \pi/3 = 2$

Thus the principal value of $\sec^{-1}(2)$ is $\pi/3$

(iii) Given $\sec^{-1}(2 \sin(3\pi/4))$

But we know that $\sin(3\pi/4) = 1/\sqrt{2}$

Therefore $2 \sin(3\pi/4) = 2 \times 1/\sqrt{2}$

$2 \sin(3\pi/4) = \sqrt{2}$

Therefore by substituting above values in $\sec^{-1}(2 \sin(3\pi/4))$, we get

$\sec^{-1}(\sqrt{2})$

Let $\sec^{-1}(\sqrt{2}) = y$

$\sec y = \sqrt{2}$

$\sec(\pi/4) = \sqrt{2}$

Therefore range of principal value of \sec^{-1} is $[0, \pi] - \{\pi/2\}$ and $\sec(\pi/4) = \sqrt{2}$
Thus the principal value of $\sec^{-1}(2 \sin(3\pi/4))$ is $\pi/4$.

(iv) Given $\sec^{-1}(2 \tan(3\pi/4))$

But we know that $\tan(3\pi/4) = -1$

Therefore, $2 \tan(3\pi/4) = 2 \times -1$

$2 \tan(3\pi/4) = -2$

By substituting these values in $\sec^{-1}(2 \tan(3\pi/4))$, we get

$\sec^{-1}(-2)$

Now let $y = \sec^{-1}(-2)$

$\sec y = -2$

$-\sec(\pi/3) = -2$

$= \sec(\pi - \pi/3)$

$= \sec(2\pi/3)$

Therefore the range of principal value of \sec^{-1} is $[0, \pi] - \{\pi/2\}$ and $\sec(2\pi/3) = -2$

Thus, the principal value of $\sec^{-1}(2 \tan(3\pi/4))$ is $(2\pi/3)$.

