

EXERCISE 15A

State, if the triangles are possible with the following angles :

(i) 20° , 70° and 90°

(ii) 40° , 130° and 20°

(iii) 60° , 60° and 50°

(iv) 125° , 40° and 15°

Solution:

In a triangle, the sum of three angles is 180°

(i) 20° , 70° and 90°

$$\text{Sum} = 20^\circ + 70^\circ + 90^\circ = 180^\circ$$

Here the sum is 180° and therefore it is possible.

(ii) 40° , 130° and 20°

$$\text{Sum} = 40^\circ + 130^\circ + 20^\circ = 290^\circ$$

Here the sum is not 180° and therefore it is not possible.

(iii) 60° , 60° and 50°

$$\text{Sum} = 60^\circ + 60^\circ + 50^\circ = 170^\circ$$

Here the sum is not 180° and therefore it is not possible.

(iv) 125° , 40° and 15°

$$\text{Sum} = 125^\circ + 40^\circ + 15^\circ = 180^\circ$$

Here the sum is 180° and therefore it is possible.

2. If the angles of a triangle are equal, find its angles.

Solution:

In a triangle, the sum of three angles is 180°

$$\text{So each angle} = 180^\circ / 3 = 60^\circ$$

3. In a triangle ABC, $\angle A = 45^\circ$ and $\angle B = 75^\circ$, find $\angle C$.

Solution:

In a triangle, the sum of three angles is 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$45^\circ + 75^\circ + \angle C = 180^\circ$$

By further calculation

$$120^\circ + \angle C = 180^\circ$$

So we get

$$\angle C = 180^\circ - 120^\circ = 60^\circ$$

4. In a triangle PQR, $\angle P = 60^\circ$ and $\angle Q = \angle R$, find $\angle R$.

Solution:

Consider $\angle Q = \angle R = x$

$$\angle P = 60^\circ$$

We can write it as

$$\angle P + \angle Q + \angle R = 180^\circ$$

Substituting the values

$$60^\circ + x + x = 180^\circ$$

By further calculation

$$60^\circ + 2x = 180^\circ$$

$$2x = 180^\circ - 60^\circ = 120^\circ$$

So we get

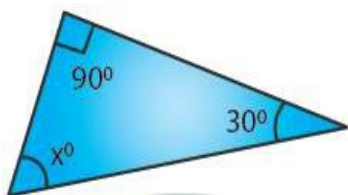
$$x = 120^\circ / 2 = 60^\circ$$

$$\angle Q = \angle R = 60^\circ$$

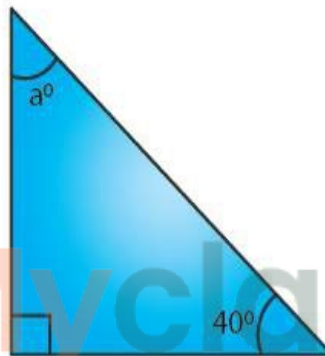
Therefore, $\angle R = 60^\circ$.

5. Calculate the unknown marked angles in each figure:

(i)



(iii)



(ii)



Solution:

In a triangle, the sum of three angles is 180°

(i) From figure (i)

$$90^\circ + 30^\circ + x = 180^\circ$$

By further calculation

$$120^\circ + x = 180^\circ$$

So we get

$$x = 180^\circ - 120^\circ = 60^\circ$$

Therefore, $x = 60^\circ$.

(ii) From figure (ii)

$$y + 80^\circ + 20^\circ = 180^\circ$$

By further calculation

$$y + 100^\circ = 180^\circ$$

So we get

$$y = 180^\circ - 100^\circ = 80^\circ$$

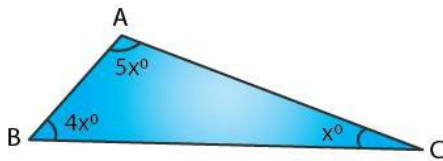
Therefore, $y = 80^\circ$.

(iii) From figure (iii)

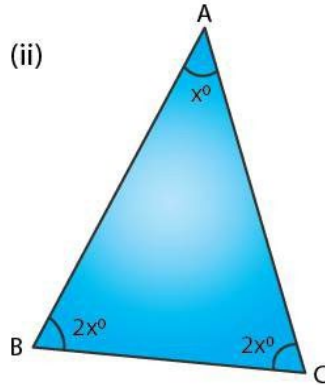
$a + 90^\circ + 40^\circ = 180^\circ$
By further calculation
 $a + 130^\circ = 180^\circ$
So we get
 $a = 180^\circ - 130^\circ = 50^\circ$
Therefore, $a = 50^\circ$.

6. Find the value of each angle in the given figures:

(i)



(ii)



Solution:

(i) From the figure (i)

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$5x^\circ + 4x^\circ + x^\circ = 180^\circ$$

By further calculation

$$10x^\circ = 180^\circ$$

$$x = 180/10 = 18^\circ$$

So we get

$$\angle A = 5x^\circ = 5 \times 18^\circ = 90^\circ$$

$$\angle B = 4x^\circ = 4 \times 18^\circ = 72^\circ$$

$$\angle C = x = 18^\circ$$

(ii) From the figure (ii)

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$x^\circ + 2x^\circ + 2x^\circ = 180^\circ$$

By further calculation

$$5x^\circ = 180^\circ$$

$$x^\circ = 180^\circ/5 = 36^\circ$$

So we get

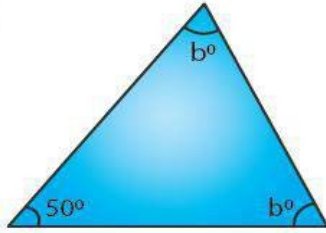
$$\angle A = x^\circ = 36^\circ$$

$$\angle B = 2x^\circ = 2 \times 36^\circ = 72^\circ$$

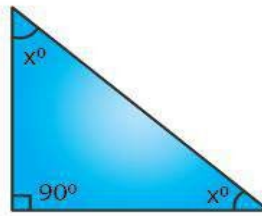
$$\angle C = 2x^\circ = 2 \times 36^\circ = 72^\circ$$

7. Find the unknown marked angles in the given figure:

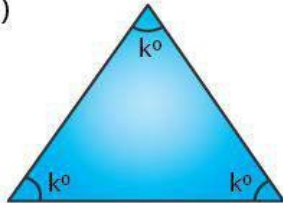
(i)



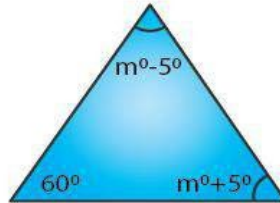
(ii)



(iii)



(iv)



Solution:

(i) From the figure (i)

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$b^\circ + 50^\circ + b^\circ = 180^\circ$$

By further calculation

$$2b^\circ = 180^\circ - 50^\circ = 130^\circ$$

$$b^\circ = 130^\circ / 2 = 65^\circ$$

Therefore, $\angle A = \angle C = b^\circ = 65^\circ$.

(ii) From the figure (ii)

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$x^\circ + 90^\circ + x^\circ = 180^\circ$$

By further calculation

$$2x^\circ = 180^\circ - 90^\circ = 90^\circ$$

$$x^\circ = 90^\circ / 2 = 45^\circ$$

Therefore, $\angle A = \angle C = x^\circ = 45^\circ$.

(iii) From the figure (iii)

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$k^\circ + k^\circ + k^\circ = 180^\circ$$

By further calculation

$$3k^\circ = 180^\circ$$

$$k^\circ = 180^\circ / 3 = 60^\circ$$

Therefore, $\angle A = \angle B = \angle C = 60^\circ$.

(iv) From the figure (iv)

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$(m^\circ - 5^\circ) + 60^\circ + (m^\circ + 5^\circ) = 180^\circ$$

By further calculation

$$m^{\circ} - 5^{\circ} + 60^{\circ} + m^{\circ} + 5^{\circ} = 180^{\circ}$$

$$2m^{\circ} = 180^{\circ} - 60^{\circ} + 5^{\circ} = 120^{\circ}$$

$$m^{\circ} = 120^{\circ}/2 = 60^{\circ}$$

$$\text{Therefore, } \angle A = m^{\circ} - 5^{\circ} = 60^{\circ} - 5^{\circ} = 55^{\circ}$$

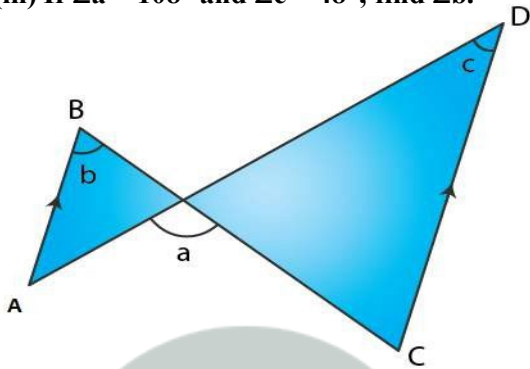
$$\angle C = m^{\circ} + 5^{\circ} = 60^{\circ} + 5^{\circ} = 65^{\circ}$$

8. In the given figure, show that: $\angle a = \angle b + \angle c$

(i) If $\angle b = 60^{\circ}$ and $\angle c = 50^{\circ}$; find $\angle a$.

(ii) If $\angle a = 100^{\circ}$ and $\angle b = 55^{\circ}$; find $\angle c$.

(iii) If $\angle a = 108^{\circ}$ and $\angle c = 48^{\circ}$; find $\angle b$.



Solution:

From the figure

$AB \parallel CD$

$b = c$ and $\angle A = \angle C$ are alternate angles

In triangle PCD

Exterior $\angle APC = \angle C + \angle D$

$a = b + c$

(i) If $\angle b = 60^{\circ}$ and $\angle c = 50^{\circ}$

$$\angle a = \angle b + \angle c$$

Substituting the values

$$\angle a = 60 + 50 = 110^{\circ}$$

(ii) If $\angle a = 100^{\circ}$ and $\angle b = 55^{\circ}$

$$\angle a = \angle b + \angle c$$

Substituting the values

$$\angle c = 100 - 55 = 45^{\circ}$$

(iii) If $\angle a = 108^{\circ}$ and $\angle c = 48^{\circ}$

$$\angle a = \angle b + \angle c$$

Substituting the values

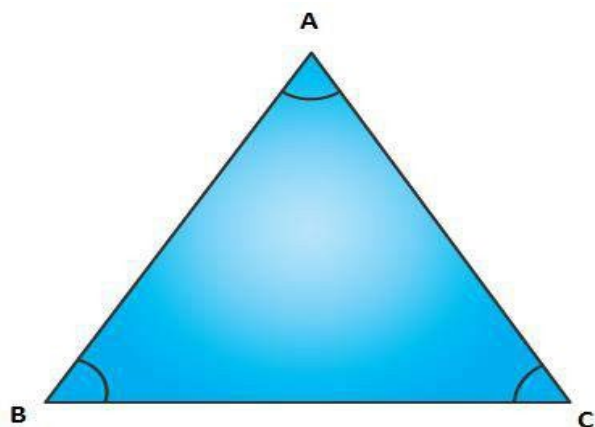
$$\angle b = 108 - 48 = 60^{\circ}$$

9. Calculate the angles of a triangle if they are in the ratio 4 : 5 : 6.

Solution:

In a triangle, the sum of angles of a triangle is 180°

$$\angle A + \angle B + \angle C = 180^{\circ}$$



It is given that

$$\angle A : \angle B : \angle C = 4 : 5 : 6$$

Consider $\angle A = 4x$, $\angle B = 5x$ and $\angle C = 6x$

Substituting the values

$$4x + 5x + 6x = 180^\circ$$

By further calculation

$$15x = 180^\circ$$

$$x = 180^\circ / 15 = 12^\circ$$

So we get

$$\angle A = 4x = 4 \times 12^\circ = 48^\circ$$

$$\angle B = 5x = 5 \times 12^\circ = 60^\circ$$

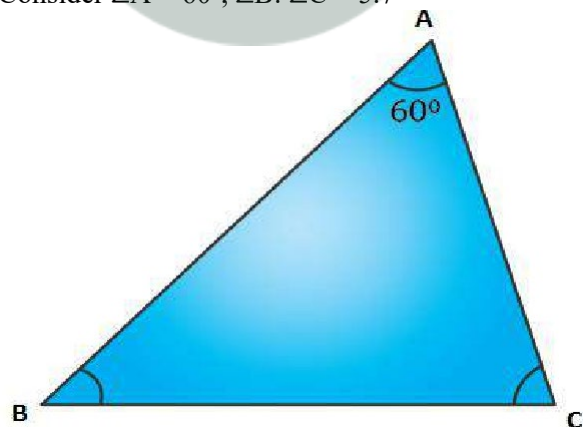
$$\angle C = 6x = 6 \times 12^\circ = 72^\circ$$

10. One angle of a triangle is 60° . The other two angles are in the ratio of 5 : 7. Find the two angles.

Solution:

From the triangle ABC

Consider $\angle A = 60^\circ$, $\angle B : \angle C = 5 : 7$



In a triangle

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$60^\circ + \angle B + \angle C = 180^\circ$$

By further calculation

$$\angle B + \angle C = 180^\circ - 60^\circ = 120^\circ$$

Take $\angle B = 5x$ and $\angle C = 7x$

Substituting the values

$$5x + 7x = 120^\circ$$

$$12x = 120^\circ$$

$$x = 120^\circ / 12 = 10^\circ$$

So we get

$$\angle B = 5x = 5 \times 10^\circ = 50^\circ$$

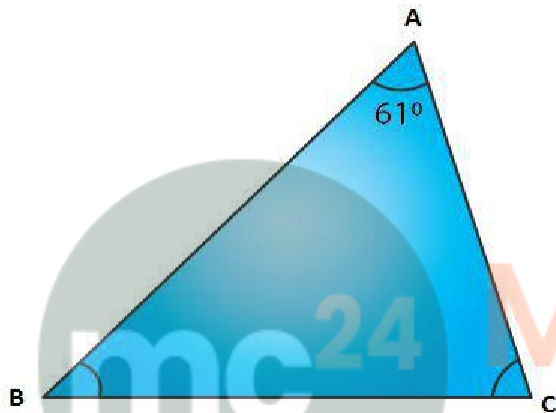
$$\angle C = 7x = 7 \times 10^\circ = 70^\circ$$

11. One angle of a triangle is 61° and the other two angles are in the ratio $1\frac{1}{2} : 1\frac{1}{3}$. Find these angles.

Solution:

From the triangle ABC

Consider $\angle A = 61^\circ$



In a triangle

$$\angle A + \angle B + \angle C = 180^\circ$$

Substituting the values

$$61^\circ + \angle B + \angle C = 180^\circ$$

By further calculation

$$\angle B + \angle C = 180^\circ - 61^\circ = 119^\circ$$

$$\angle B : \angle C = 1\frac{1}{2} : 1\frac{1}{3} = \frac{3}{2} : \frac{4}{3}$$

Taking LCM

$$\angle B : \angle C = 9 : 8$$

$$\angle B : \angle C = 9 : 8$$

Consider $\angle B = 9x$ and $\angle C = 8x$

Substituting the values

$$9x + 8x = 119^\circ$$

$$17x = 119^\circ$$

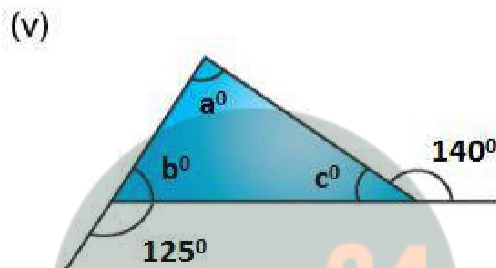
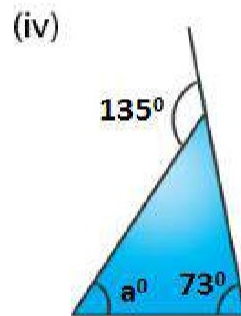
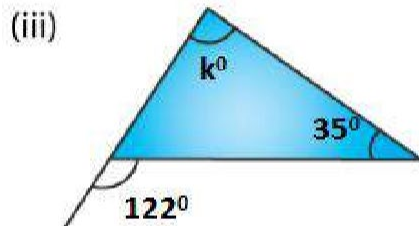
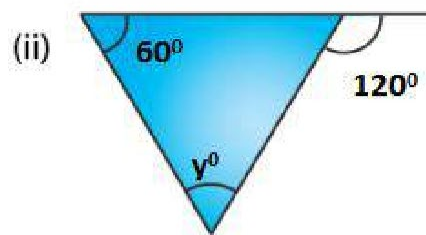
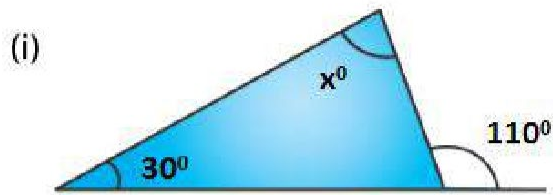
$$x = 119^\circ / 17 = 7^\circ$$

So we get

$$\angle B = 9x = 9 \times 7^\circ = 63^\circ$$

$$\angle C = 8x = 8 \times 7^\circ = 56^\circ$$

12. Find the unknown marked angles in the given figures:



Solution:

In a triangle, if one side is produced
Exterior angle is the sum of opposite interior angles

(i) From the figure (i)

$$110^\circ = x^\circ + 30^\circ$$

By further calculation

$$x^\circ = 110^\circ - 30^\circ = 80^\circ$$

(ii) From the figure (ii)

$$120^\circ = y^\circ + 60^\circ$$

By further calculation

$$y^\circ = 120^\circ - 60^\circ = 60^\circ$$

(iii) From the figure (iii)

$$122^\circ = k^\circ + 35^\circ$$

By further calculation

$$k^\circ = 122^\circ - 35^\circ = 87^\circ$$

(iv) From the figure (iv)

$$135^\circ = a^\circ + 73^\circ$$

By further calculation

$$a^\circ = 135^\circ - 73^\circ = 62^\circ$$

(v) From the figure (v)

$$125^\circ = a + c \dots\dots (1)$$

$$140^\circ = a + b \dots\dots (2)$$

By adding both the equations

$$a + c + a + b = 125^\circ + 140^\circ$$

On further calculation

$$a + a + b + c = 265^\circ$$

We know that $a + b + c = 180^\circ$

Substituting it in the equation

$$a + 180^\circ = 265^\circ$$

So we get

$$a = 265 - 180 = 85^\circ$$

If $a + b = 140^\circ$

Substituting it in the equation

$$85^\circ + b = 140^\circ$$

So we get

$$b = 140 - 85 = 55^\circ$$

If $a + c = 125^\circ$

Substituting it in the equation

$$85^\circ + c = 125^\circ$$

So we get

$$c = 125 - 85 = 40^\circ$$

Therefore, $a = 85^\circ$, $b = 55^\circ$ and $c = 40^\circ$.

