EXERCISE 23.9

Reduce the equation √3x + y + 2 = 0 to:
(i) slope - intercept form and find slope and y - intercept;
(ii) Intercept form and find intercept on the axes
(iii) The normal form and find p and α.
Solution:
(i) Given:
√3x + y + 2 = 0
y = - √3x – 2
This is the slope intercept form of the given line.
∴ The slope = - √3 and y - intercept = -2

(ii) Given:
√3x + y + 2 = 0
√3x + y = -2
Divide both sides by -2, we get
√3x/-2 + y/-2 = 1
∴ The intercept form of the given line. Here, x - intercept = - 2/√3 and y - intercept = -2

(iii) Given:
√3x + y + 2 = 0
-√3x - y = 2

$$-\frac{\sqrt{3}x}{\sqrt{(-\sqrt{3})^2+(-1)^2}}-\frac{y}{\sqrt{(-\sqrt{3})^2+(-1)^2}}=\frac{2}{\sqrt{(-\sqrt{3})^2+(-1)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$$-\frac{\sqrt{3}x}{2}-\frac{y}{2}=1$$

This is the normal form of the given line.
So, p = 1 cos α = -√3/2 and sin α = - 1/2
∴ p = 1 and α = 210

2. Reduce the following equations to the normal form and find p and α in each case:
(i) x + √3y – 4 = 0
(ii) x + y + √2 = 0
Solution:
(i) x + √3y – 4 = 0

$x + \sqrt{3}y = 4$

$$\frac{x}{\sqrt{1^2 + (\sqrt{3})^2}} + \frac{\sqrt{3}y}{\sqrt{1^2 + (\sqrt{3})^2}} = \frac{4}{\sqrt{1^2 + (\sqrt{3})^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$$\frac{x}{2} + \frac{\sqrt{3}y}{2} = 2$$

The normal form of the given line, where $p = 2$, $\cos\alpha = 1/2$ and $\sin\alpha = \sqrt{3}/2$

$\therefore p = 2$ and $\alpha = \pi/3$

(ii) $x + y + \sqrt{2} = 0$

$-x - y = \sqrt{2}$

$$\frac{-x}{\sqrt{(-1)^2 + (-1)^2}} + \frac{y}{\sqrt{(-1)^2 + (-1)^2}} = \frac{\sqrt{2}}{\sqrt{(-1)^2 + (-1)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$$-\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 1$$

The normal form of the given line, where $p = 1$, $\cos\alpha = -1/\sqrt{2}$ and $\sin\alpha = -1/\sqrt{2}$

$\therefore p = 1$ and $\alpha = 225^\circ$

3. Put the equation x/a + y/b = 1 the slope intercept form and find its slope and y - intercept.

Solution:

Given: the equation is $x/a + y/b = 1$

We know that,

General equation of line $y = mx + c$.

$bx + ay = ab$

$ay = - bx + ab$

$y = -bx/a + b$

The slope intercept form of the given line.

$\therefore$ Slope = $- b/a$ and y - intercept = b

4. Reduce the lines 3x – 4y + 4 = 0 and 2x + 4y – 5 = 0 to the normal form and hence find which line is nearer to the origin.

Solution:

Given:

The normal forms of the lines 3x − 4y + 4 = 0 and 2x + 4y − 5 = 0.
Let us find, in given normal form of a line, which is nearer to the origin.
-3x + 4y = 4

$$-\frac{3\,x}{\sqrt{(-3)^2 + (4)^2}} + 4\frac{y}{\sqrt{(-3)^2 + (4)^2}} = \frac{4}{\sqrt{(-3)^2 + (4)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$-\frac{3}{5}x + \frac{4}{5}y = \frac{4}{5} \;\ldots\ldots (1)$$

Now 2x + 4y = - 5
-2x – 4y = 5

$$-\frac{2\,x}{\sqrt{(-2)^2 + (-4)^2}} - 4\frac{y}{\sqrt{(-2)^2 + (-4)^2}} = \frac{5}{\sqrt{(-2)^2 + (-4)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$-\frac{2}{2\sqrt{5}}x - \frac{4}{2\sqrt{5}}y = \frac{5}{2\sqrt{5}} \;\ldots\ldots (2)$$

From equations (1) and (2):
45 < 525
∴ The line 3x − 4y + 4 = 0 is nearer to the origin.

5. Show that the origin is equidistant from the lines 4x + 3y + 10 = 0; 5x – 12y + 26 = 0 and 7x + 24y = 50.

Solution:

Given:
The lines 4x + 3y + 10 = 0; 5x – 12y + 26 = 0 and 7x + 24y = 50.
We need to prove that, the origin is equidistant from the lines 4x + 3y + 10 = 0; 5x – 12y + 26 = 0 and 7x + 24y = 50.
Let us write down the normal forms of the given lines.
First line: 4x + 3y + 10 = 0
-4x - 3y = 10

$$-\frac{4x}{\sqrt{(-4)^2 + (-3)^2}} - 3\frac{y}{\sqrt{(-4)^2 + (-3)^2}} = \frac{10}{\sqrt{(-4)^2 + (-3)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$-\frac{4}{5}x - \frac{3}{5}y = 2$$

So, p = 2

Second line: $5x - 12y + 26 = 0$

$-5x + 12y = 26$

$$-\frac{5x}{\sqrt{(-5)^2 + (12)^2}} + 12\frac{y}{\sqrt{(-5)^2 + (12)^2}} = \frac{26}{\sqrt{(-5)^2 + (12)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$-\frac{5}{13}x + \frac{12}{13}y = 2$$

So, $p = 2$

Third line: $7x + 24y = 50$

$$\frac{7x}{\sqrt{(7)^2 + (24)^2}} + 24\frac{y}{\sqrt{(7)^2 + (24)^2}} = \frac{50}{\sqrt{(7)^2 + (24)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$\frac{7}{25}x + \frac{24}{25}y = 2$$

So, $p = 2$

∴ The origin is equidistant from the given lines.