

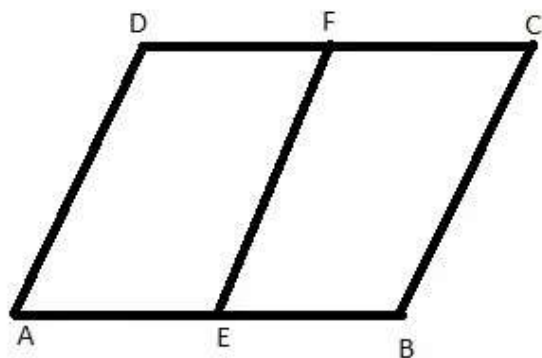
Excise 14C

Solution 1:

Let us draw a parallelogram $ABCD$ Where F is the midpoint

Of side DC of parallelogram $ABCD$

To prove: $AEBF$ is a parallelogram



Proof:

Therefore $ABCD$

$$AB \parallel DC$$

$$BC \parallel AD$$

$$AB = DC$$

$$\frac{1}{2}AB = \frac{1}{2}DC$$

$$AE = DF$$

Also $AD \parallel EF$

therefore $AEBF$ is a parallelogram.

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Solution 2:

GIVEN: $ABCD$ is a parallelogram where the diagonal BD bisects

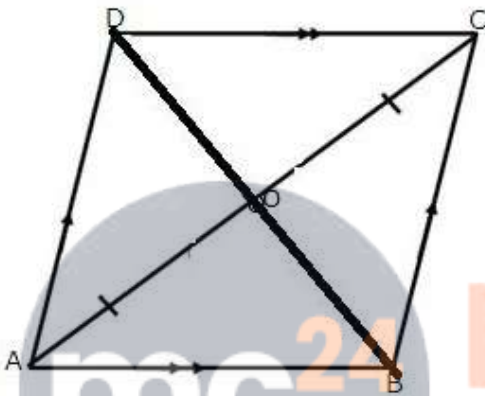
parallelogram $ABCD$ at angle B and D

TO PROVE: $ABCD$ is a rhombus

Proof: Let us draw a parallelogram $ABCD$ where the diagonal BD bisects the parallelogram at angle B and D

Construction: Let us join AC as a diagonal of the parallelogram

$ABCD$



Since $ABCD$ is a parallelogram

Therefore

$$AB = DC$$

$$AD = BC$$

Diagonal BD bisects angle B and D

$$\text{So } \angle COD = \angle DOA$$

Again AC also bisects at A and C

$$\text{Therefore } \angle AOB = \angle BOC$$

Thus $ABCD$ is a rhombus.

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Solution 3:

Given $ABCD$ is a parallelogram and $AE=EF=FC$.

We have to prove at first that $DEBF$ is a parallelogram.

Proof: From $\triangle ADE$ and $\triangle BCF$

$$AE=FC$$

$$AD=BC$$

$$\angle D = \angle B$$

$$\triangle ADE \cong \triangle BCF \text{ [SAS]}$$

Therefore $DE=FB$

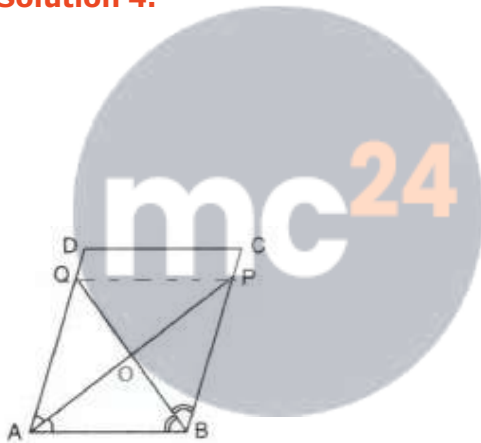
$DC=EF$ [since $AE+EF+FC=AC$ and $AE=EF=FC$]

Therefore $DEBF$ is a parallelogram.

So $DE \parallel FB$

Hence proved

Solution 4:



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Let us join PQ .

Consider the $\triangle AOQ$ and $\triangle BOP$

$$\angle AOQ = \angle BOP \text{ [opposite angles]}$$

$$\angle OAQ = \angle BPO \text{ [alternate angles]}$$

$$\Rightarrow \triangle AOQ \cong \triangle BOP \text{ [AA test]}$$

Hence $AQ = BP$

Consider the $\triangle QOP$ and $\triangle AOB$

$$\angle AOB = \angle QOP \text{ [opposite angles]}$$

$$\angle OAB = \angle OPQ \text{ [alternate angles]}$$

$$\Rightarrow \triangle QOP \cong \triangle AOB \text{ [AA test]}$$

Hence $PQ = AB = CD$

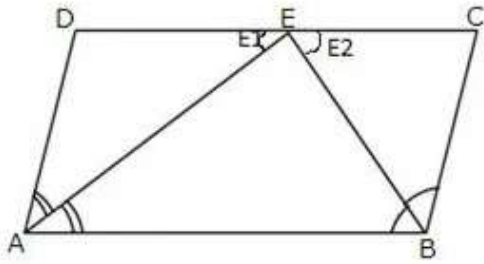
Consider the quadrilateral $QPCD$

$$DQ = CP \text{ and } DQ \parallel CP \text{ [Since } AD = BC \text{ and } AD \parallel BC \text{]}$$

$$\text{Also } QP = DC \text{ and } AB \parallel QP \parallel DC$$

Given $ABCD$ is a parallelogram

To prove: $AB = 2BC$



Proof: $ABCD$ is a parallelogram

$$\angle A + \angle D = \angle B + \angle C = 180^\circ$$

From the $\triangle AEB$ we have

$$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} + \angle E = 180^\circ$$

$$\Rightarrow \angle A - \frac{\angle A}{2} + \angle D + \angle E_1 = 180^\circ \text{ [taking } E_1 \text{ as new angle]}$$

$$\Rightarrow \angle A + \angle D + \angle E_1 = 180^\circ + \frac{\angle A}{2}$$

$$\Rightarrow \angle E_1 = \frac{\angle A}{2} \text{ [Since } \angle A + \angle D = 180^\circ]$$

Again,

similarly,

$$\angle E_2 = \frac{\angle B}{2}$$

NOW

$$AB = DE + EC$$

$$= AD + BC$$

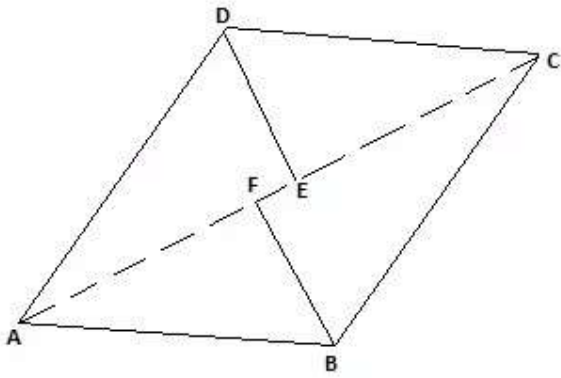
$$= 2BC \text{ [since } AD = BC]$$

Hence proved

Solution 6:

Given $ABCD$ is a parallelogram. The bisectors of $\angle ADC$ and $\angle BCD$ meet at E . The bisectors of $\angle ABC$ and $\angle BCD$ meet at F

From the parallelogram $ABCD$ we have



$$\angle ADC + \angle BCD = 180^\circ \text{ [sum of adjacent angles of a parallelogram]}$$

$$\Rightarrow \frac{\angle ADC}{2} + \frac{\angle BCD}{2} = 90^\circ$$

$$\Rightarrow \angle EDC + \angle ECD = 90^\circ$$

In triangle ECD sum of angles = 180°

$$\Rightarrow \angle EDC + \angle ECD + \angle CED = 180^\circ$$

$$\Rightarrow \angle CED = 90^\circ$$

Similarly taking triangle BCF it can be prove that $\angle BFC = 90^\circ$

Now since

$$\angle BFC = \angle CED = 90^\circ$$

Therefore the lines DE and BF are parallel

Hence proved

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Solution 7:

Given: $ABCD$ is a parallelogram

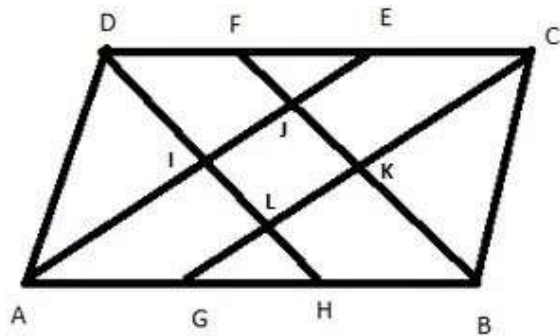
AE bisects $\angle BAD$

BF bisects $\angle ABC$

CG bisects $\angle BCD$

DH bisects $\angle ADC$

TO PROVE: $LKJI$ is a rectangle



Proof:

$\angle BAD + \angle ABC = 180^\circ$ [adjacent angles of a parallelogram are supplementary]

$\angle BAJ = \frac{1}{2} \angle BAD$ [AE bisects $\angle BAD$]

$\angle ABJ = \frac{1}{2} \angle ABC$ [DH bisects $\angle ABC$]

$\angle BAJ + \angle ABJ = 90^\circ$ [halves of supplementary angles are complementary]

$\triangle ABJ$ is a right triangle because its acute interior angles are complementary.

Similarly

$\angle DLC = 90^\circ$

$\angle AID = 90^\circ$

Then $\angle JIL = 90^\circ$ because $\angle AID$ and $\angle JIL$ are vertical angles

since 3 angles of quadrilateral $LKJI$ are right angles, so is the 4th one and so $LKJI$ is a rectangle, since its interior angles are all right angles

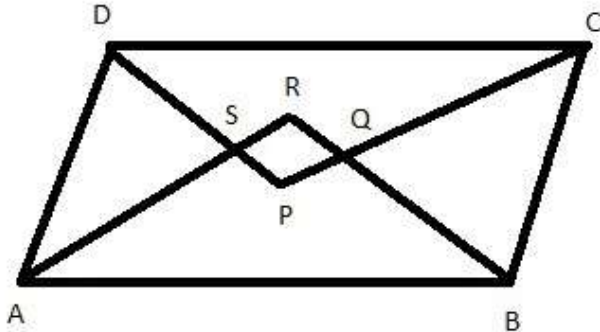
Hence proved

Solution 8:

Given: A parallelogram $ABCD$ in which AR, BR, CP, DP

are the bisectors of $\angle A, \angle B, \angle C, \angle D$ respectively forming quadrilateral $PQRS$.

To prove: $PQRS$ is a rectangle



Proof:

$$\angle DCB + \angle ABC = 180^\circ \text{ [co-interior angles of parallelogram are supplementary]}$$

$$\Rightarrow \frac{1}{2} \angle DCB + \frac{1}{2} \angle ABC = 90^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ$$

$$\Delta CQB, \angle 1 + \angle 2 + \angle CQB = 180^\circ$$

From the above equation we get

$$\angle CQB = 180^\circ - 90^\circ = 90^\circ$$

$$\angle RQP = 90^\circ \text{ [}\angle CQB = \angle RQP, \text{vertically opposite angles]}$$

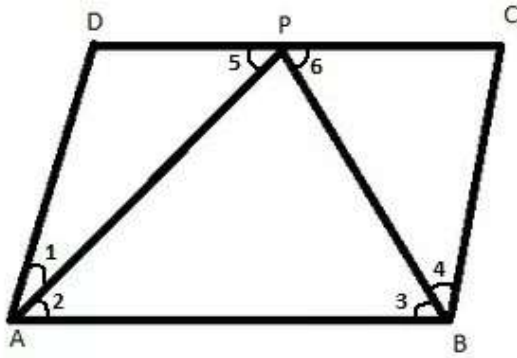
$$\angle QRP = \angle RSP = \angle SPQ = 90^\circ$$

Hence $PQRS$ is a rectangle

Also in

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Solution 9:



(i) Let $AD = x$

$$AB = 2AD = 2x$$

Also AP is the bisector $\angle A$

$$\angle 1 = \angle 2$$

Now,

$$\angle 2 = \angle 5 \text{ [alternate angles]}$$

Therefore $\angle 1 = \angle 5$

Now

$$AP = DP = x \text{ [sides opposite to equal angles are also equal]}$$



Therefore

$$AB=CD \text{ [opposite sides of parallelogram are equal]}$$

$$CD = 2x$$

$$\Rightarrow DP+PC=2x$$

$$\Rightarrow x+PC=2x$$

$$\Rightarrow PC = x$$

Also, $BC=x$

$\triangle BPC$

$$\text{In } \Rightarrow \angle 6 = \angle 4 \text{ [angles opposite to equal sides are equal]}$$

$$\Rightarrow \angle 6 = \angle 3$$

Therefore $\angle 3 = \angle 4$

Hence BP bisect $\angle B$

(ii)

Opposite angles are supplementary

Therefore

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

$$\Rightarrow 2\angle 2 + 2\angle 3 = 180^\circ \begin{cases} \angle 1 = \angle 2 \\ \angle 3 = \angle 4 \end{cases}$$

$$\Rightarrow \angle 2 + \angle 3 = 90^\circ$$

$\triangle APB$

$$\angle 2 + \angle 3 + \angle APB = 180^\circ$$

$$\Rightarrow \angle APB = 180^\circ - 90^\circ \text{ [by angle sum property]}$$

$$\Rightarrow \angle APB = 90^\circ$$

Hence proved

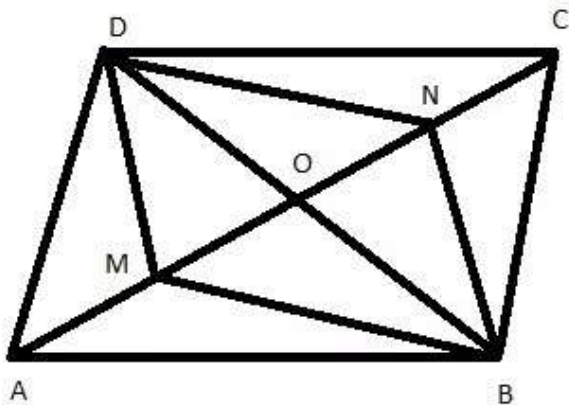
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Solution 10:

Points M and N are taken on the diagonal AC of a parallelogram $ABCD$ such that $AM=CN$.

Prove that $BMDN$ is a parallelogram



CONSTRUCTION: Join B to D to meet AC in O .

PROOF: We know that the diagonals of parallelogram bisect each other.

Now, AC and BD bisect each other at O .

$$OC=OA$$

$$AM=CN$$

$$\Rightarrow OA-AM=OC-CN$$

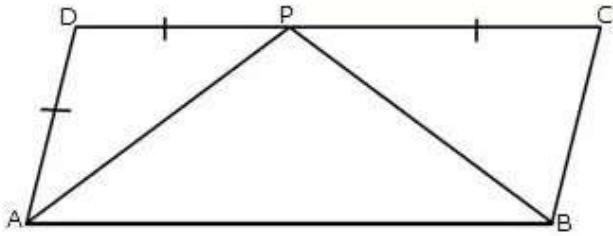
$$\Rightarrow OM=ON$$

Thus in a quadrilateral $BMDN$, diagonal BD and MN are such that $OM=ON$ and $OD=OB$.

Therefore the diagonals AC and PQ bisect each other.

Hence $BMDN$ is a parallelogram

Solution 11:



Consider $\triangle ADP$ and $\triangle BCP$

$AD=BC$ [since ABCD is a parallelogram]

$DC=AB$ [since ABCD is a parallelogram]

$\angle A = \angle C$ [opposite angles]

$\triangle ADP \cong \triangle BCP$ [SAS]

Therefore $AP=BP$

AP bisects $\angle A$

BP bisects $\angle B$

In $\triangle APB$

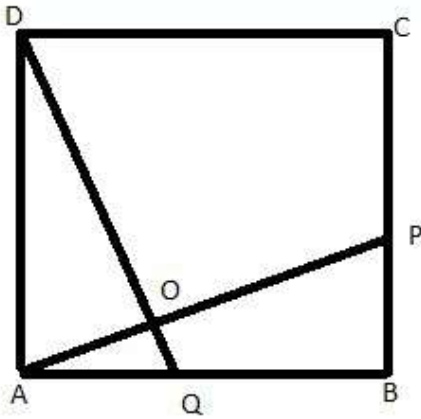
$AP=PB$

$\angle APB = \angle DAP + \angle BCP$

Hence proved



Solution 12:



ABCD is a square and $AP=PQ$

Consider $\triangle DAQ$ and $\triangle ABP$

$$\angle DAQ = \angle ABP = 90^\circ$$

$$DQ = AP$$

$$AD = AB$$

$$\triangle DAQ \cong \triangle ABP$$

$$\Rightarrow \angle PAB = \angle QDA$$

Now,

$$\angle PAB + \angle APB = 90^\circ$$

$$\text{also } \angle QDA + \angle APB = 90^\circ \quad [\angle PAB = \angle QDA]$$

Consider $\triangle AOQ$ By ASP

$$\angle QDA + \angle APB + \angle AOD = 180^\circ$$

$$\Rightarrow 90^\circ + \angle AOD = 180^\circ$$

$$\Rightarrow \angle AOD = 90^\circ$$

Hence AP and DQ are perpendicular.

Solution 13:

Given: $ABCD$ is quadrilateral,

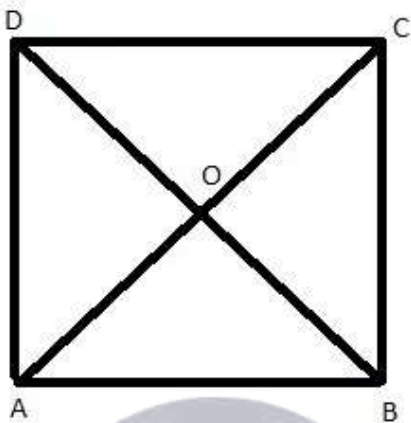
$$AB=AD$$

$$CB=CD$$

To prove: (i) AC bisects angle BAD .

(ii) AC is perpendicular bisector of BD .

Proof :



In $\triangle ABC$ and $\triangle ADC$

$$AB=AD \text{ [given]}$$

$$CB=CD \text{ [given]}$$

$$AC=AC \text{ [common side]}$$

$$\triangle ABC \cong \triangle ADC \text{ [SSS]}$$

Therefore AC bisects $\angle BAD$

$$OD=OB$$

$$OA=OA \text{ [diagonals bisect each other at O]}$$

Thus AC is perpendicular bisector of BD

Hence proved

Solution 14:

Given $ABCD$ is a trapezium, $AB \parallel DC$ and $AD=BC$

To prove (i) $\angle DAB = \angle CBA$

(ii) $\angle ADC = \angle BCD$

(iii) $AC = BD$

(iv) $OA = OB$ and $OC = OD$

Proof: (i) Since $AD \parallel CE$ and transversal AE cuts them at A and E respectively.

Therefore, $\angle A + \angle B = 180^\circ$

Since $AB \parallel CD$ and $AD \parallel BC$

Therefore $ABCD$ is a parallelogram

$\angle A = \angle C$

$\angle B = \angle D$ [since $ABCD$ is a parallelogram]

Therefore $\angle DAB = \angle CBA$

$\angle ADC = \angle BCD$

In $\triangle ABC$ and $\triangle BAD$, we have

$BC=AD$ [given]

$AB=BA$ [common]

$\angle A = \angle B$ [proved]

$\triangle ABC \cong \triangle BAD$ [SAS]

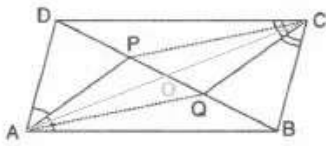
Since $\triangle ABC \cong \triangle BAD$

Therefore $AC=BD$ [corresponding parts of congruent triangles are equal]

Again $OA=OB$

$OC=OD$ [since diagonals bisect each other at O]

Hence proved

Solution 15:

Join AC to meet BD in O

We know that the diagonals of a parallelogram bisect each other. Therefore AC and BD bisect each other at O .

Therefore

$$OB = OD$$

But

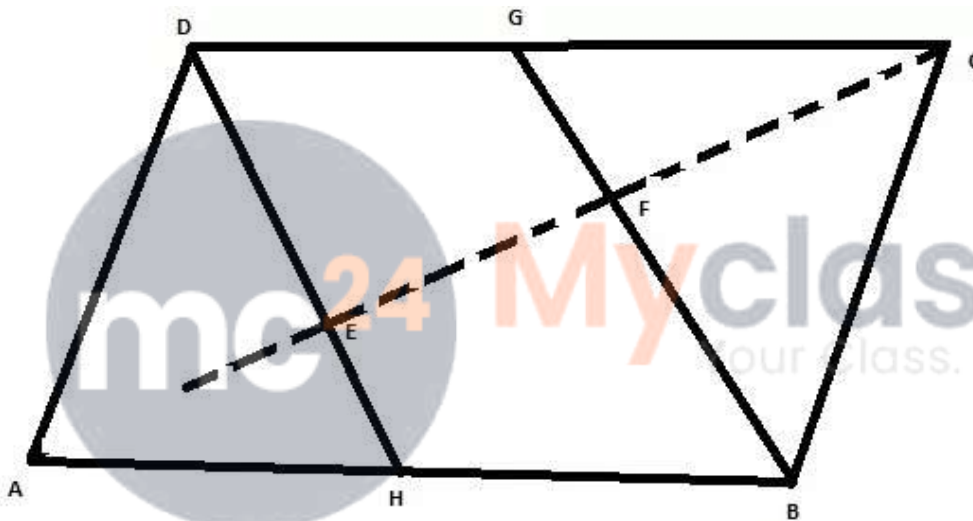
$$BQ = DP$$

$$OB - PQ = OD - DP$$

$$\Rightarrow OQ = OP$$

Thus in a quadrilateral $APCQ$ diagonals AC and PQ such that $OQ = OP$ and $OA = OC$. Since diagonals AC and PQ bisect each other.

Hence $APCQ$ is a parallelogram.

Solution 16:

$ABCD$ is a parallelogram, the bisectors of $\angle ADC$ and $\angle BCD$ meet at a point E and the bisectors of $\angle BCD$ AND $\angle ABC$ meet at F .

We have to prove that the $\angle CED = 90^\circ$ and $\angle CFG = 90^\circ$

Proof: In the parallelogram $ABCD$

$$\angle ADC + \angle BCD = 180^\circ \text{ [sum of adjacent angles of a parallelogram]}$$

$$\Rightarrow \frac{\angle ADC}{2} + \frac{\angle BCD}{2} = 90^\circ$$

$$\Rightarrow \angle BDC + \angle ECD + \angle CED = 180^\circ$$

$$\Rightarrow \angle CED = 90^\circ$$

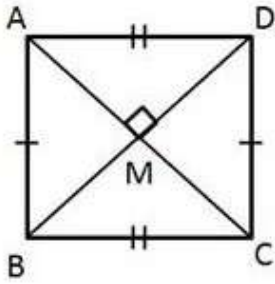
Similarly taking triangle BCF it can be proved that $\angle BFC = 90^\circ$

$$\text{Also } \angle BFC + \angle CFG = 180^\circ \text{ [adjacent angles on a line]}$$

$$\Rightarrow \angle CFG = 90^\circ$$

Now since $\angle CFG = \angle CED = 90^\circ$ [it means that the lines DE and BG are parallel]

Hence proved

Solution 17:

To prove : ABCD is a square,
 that is, to prove that sides of the quadrilateral are equal
 and each angle of the quadrilateral is 90° .
 ABCD is a rectangle,
 $\Rightarrow \angle A = \angle B = \angle C = \angle D = 90^\circ$ and diagonals bisect each other
 that is, $MD = BM \dots (i)$
 Consider $\triangle AMD$ and $\triangle AMB$,
 $MD = BM$ (from (i))
 $\angle AMD = \angle AMB = 90^\circ$ (given)
 $AM = AM$ (common side)
 $\triangle AMD \cong \triangle AMB$ (SAS congruence criterion)
 $\Rightarrow AD = AB$ (cpctc)
 Since ABCD is a rectangle, $AD = BC$ and $AB = CD$
 Thus, $AB = BC = CD = AD$ and $\angle A = \angle B = \angle C = \angle D = 90^\circ$
 \Rightarrow ABCD is a square.

Solution 18:

ABCD is a parallelogram
 \Rightarrow opposite angles of a parallelogram are congruent
 $\Rightarrow \angle DAB = \angle BCD$ and $\angle ABC = \angle ADC = 120^\circ$
 In ABCD,
 $\angle DAB + \angle BCD + \angle ABC + \angle ADC = 360^\circ$
(sum of the measures of angles of a quadrilateral)
 $\Rightarrow \angle BCD + \angle BCD + 120^\circ + 120^\circ = 360^\circ$
 $\Rightarrow 2\angle BCD = 360^\circ - 240^\circ$
 $\Rightarrow 2\angle BCD = 120^\circ$
 $\Rightarrow \angle BCD = 60^\circ$
 PQRS is a parallelogram
 $\Rightarrow \angle PQR = \angle PSR = 70^\circ$
 In $\triangle CMS$,
 $\angle CMS + \angle CSM + \angle MCS = 180^\circ$ (angle sum property)
 $\Rightarrow x + 70^\circ + 60^\circ = 180^\circ$
 $\Rightarrow x = 50^\circ$

Solution 19:

ABCD is a rhombus $\Rightarrow AD = CD$ and $\angle ADC = \angle ABC = 56^\circ$

DCFE is a square $\Rightarrow ED = CD$ and $\angle FED = \angle EDC = \angle DCF = \angle CFE = 90^\circ$

$\Rightarrow AD = CD = ED$

In $\triangle ADE$,

$AD = ED \Rightarrow \angle DAE = \angle AED \dots (i)$

$\angle DAE + \angle AED + \angle ADE = 180^\circ$

$\Rightarrow 2\angle DAE + 146^\circ = 180^\circ \dots (\text{Since } \angle ADE = \angle EDC + \angle ADC = 90^\circ + 56^\circ = 146^\circ)$

$\Rightarrow 2\angle DAE = 34^\circ$

$\Rightarrow \angle DAE = 17^\circ$

$\Rightarrow \angle DEA = 17^\circ \dots (ii)$

In ABCD,

$\angle ABC + \angle BCD + \angle ADC + \angle DAB = 360^\circ$

$\Rightarrow 56^\circ + 56^\circ + 2\angle DAB = 360^\circ \quad (\because \text{opposite angles of a rhombus are equal})$

$\Rightarrow 2\angle DAB = 248^\circ$

$\Rightarrow \angle DAB = 124^\circ$

We know that diagonals of a rhombus, bisect its angles.

$\Rightarrow \angle DAC = \frac{124^\circ}{2} = 62^\circ$

$\Rightarrow \angle EAC = \angle DAC - \angle DAE = 62^\circ - 17^\circ = 45^\circ$

Now, $\angle FEA = \angle FED - \angle DEA$

$= 90^\circ - 17^\circ \dots (\text{from (ii) and each angle of a square is } 90^\circ)$

$= 73^\circ$

We know that diagonals of a square bisect its angles.

$\Rightarrow \angle CED = \frac{90^\circ}{2} = 45^\circ$

So, $\angle AEC = \angle CED - \angle DEA$

$= 45^\circ - 17^\circ$

$= 28^\circ$

Hence, $\angle DAE = 17^\circ$, $\angle FEA = 73^\circ$, $\angle EAC = 45^\circ$ and $\angle AEC = 28^\circ$.