

EXERCISE 30.2

1. Differentiate each of the following from first principles:

(i) $2/x$

(ii) $1/\sqrt{x}$

(iii) $1/x^3$

(iv) $[x^2 + 1]/x$

(v) $[x^2 - 1]/x$

Solution:

(i) $2/x$

Given:

$$f(x) = 2/x$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x - 2x - 2h}{hx(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{-2h}{hx(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{x(x+h)}$$

When $h=0$, we get

$$= \frac{-2}{x^2}$$

$$= -2x^{-2}$$

\therefore Derivative of $f(x) = 2/x$ is $-2x^{-2}$

(ii) $1/\sqrt{x}$

Given:

$$f(x) = 1/\sqrt{x}$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

By using algebra of limits, we get

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h \rightarrow 0} \frac{x - x - h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

When $h=0$, we get

$$= \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x} + \sqrt{x})}$$

$$= \frac{-1}{x \times 2\sqrt{x}}$$

$$= \frac{-1}{2x^{\frac{3}{2}}}$$

$$= -\frac{1}{2}x^{-\frac{3}{2}}$$

\therefore Derivative of $f(x) = 1/\sqrt{x}$ is $-1/2 x^{-3/2}$

(iii) $1/x^3$

Given:

$$f(x) = 1/x^3$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^3 - (x+h)^3}{h(x+h)^3 x^3}$$

By using the formula $[a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$$= \lim_{h \rightarrow 0} \frac{x^3 - x^3 - 3x^2h - 3xh^2 - h^3}{h(x+h)^3 x^3}$$

$$= \lim_{h \rightarrow 0} \frac{-3x^2h - 3xh^2 - h^3}{h(x+h)^3 x^3}$$

$$= \lim_{h \rightarrow 0} \frac{h(-3x^2 - 3xh - h^2)}{h(x+h)^3 x^3}$$

$$= \lim_{h \rightarrow 0} \frac{(-3x^2 - 3xh - h^2)}{(x+h)^3 x^3}$$

When $h = 0$, we get

$$= \frac{-3x^2}{x^6}$$

$$= \frac{-3}{x^4}$$

$$= -3x^{-4}$$

\therefore Derivative of $f(x) = 1/x^3$ is $-3x^{-4}$

(iv) $[x^2 + 1]/x$

Given:

$$f(x) = [x^2 + 1]/x$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2+1}{x+h} - \frac{x^2+1}{x}}{h}$$

Upon expansion,

$$= \lim_{h \rightarrow 0} \frac{\frac{x^2+2xh+h^2+1}{x+h} - \frac{x^2+1}{x}}{h}$$

By using algebra of limits, we get

$$= \lim_{h \rightarrow 0} \frac{x^3 + 2x^2h + h^2x + x - x^3 - x^2h - x - h}{xh(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{x^2h + h^2x - h}{xh(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{h(x^2 + hx - 1)}{xh(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + hx - 1}{x(x+h)}$$

When $h = 0$, we get

$$= \frac{x^2 - 1}{x^2}$$

$$= 1 - 1/x^2$$

\therefore Derivative of $f(x) = 1 - 1/x^2$

(v) $[x^2 - 1] / x$

Given:

$$f(x) = [x^2 - 1] / x$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2-1}{x+h} - \frac{x^2-1}{x}}{h}$$

Upon expansion,

$$= \lim_{h \rightarrow 0} \frac{\frac{x^2 + 2xh + h^2 - 1}{x+h} - \frac{x^2 - 1}{x}}{h}$$

By using algebra of limits, we get

$$= \lim_{h \rightarrow 0} \frac{x^3 + 2x^2h + h^2x - x - x^3 - x^2h + x + h}{xh(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{x^2h + h^2x + h}{xh(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{h(x^2 + hx + 1)}{xh(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + hx + 1}{x(x+h)}$$

When $h = 0$, we get

$$= \frac{x^2 + 1}{x^2}$$

$$= 1 + 1/x^2$$

\therefore Derivative of $f(x) = 1 + 1/x^2$



2. Differentiate each of the following from first principles:

(i) e^{-x}

(ii) e^{3x}

(iii) e^{ax+b}

Solution:

(i) e^{-x}

Given:

$$f(x) = e^{-x}$$

By using the formula,

$$\frac{d}{dx} (f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$\frac{d}{dx} (e^{-x}) = \lim_{h \rightarrow 0} \frac{e^{-(x+h)} - e^{-x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{e^{-x} e^{-h} - e^{-x}}{h}$$

Taking e^{-x} common, we have

$$= \lim_{h \rightarrow 0} \frac{e^{-x} (e^{-h} - 1)}{h}$$

$$= \lim_{h \rightarrow 0} e^{-x} \times \lim_{h \rightarrow 0} \frac{e^{-h} - 1}{-h} \times (-1)$$

We know that, $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \log_e e = 1$

$$= -e^{-x} \lim_{h \rightarrow 0} \frac{e^{-h} - 1}{-h}$$

So,

$$= -e^{-x} (1)$$

$$= -e^{-x}$$

\therefore Derivative of $f(x) = -e^{-x}$

(ii) e^{3x}

Given:

$$f(x) = e^{3x}$$

By using the formula,

$$\frac{d}{dx} (f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$\frac{d}{dx} (e^{3x}) = \lim_{h \rightarrow 0} \frac{e^{3(x+h)} - e^{3x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{e^{3x} e^{3h} - e^{3x}}{h}$$

Taking e^{-x} common, we have

$$= \lim_{h \rightarrow 0} \frac{e^{3x} (e^{3h} - 1)}{3h}$$

By using algebra of limits,

$$= \lim_{h \rightarrow 0} e^{3x} \times \lim_{h \rightarrow 0} \frac{e^{3h} - 1}{h}$$

Since we cannot substitute the value of h directly, we take

$$= \lim_{h \rightarrow 0} e^{3x} \times \lim_{h \rightarrow 0} \frac{e^{3h} - 1}{3h} \times 3$$

We know that, $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \log_e e = 1$

$$= 3e^{3x} \lim_{h \rightarrow 0} \frac{e^{3h} - 1}{3h}$$

$$= 3e^{3x} (1)$$

$$= 3e^{3x}$$

\therefore Derivative of $f(x) = 3e^{3x}$

(iii) e^{ax+b}

Given:

$$f(x) = e^{ax+b}$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$\frac{d}{dx}(e^{ax+b}) = \lim_{h \rightarrow 0} \frac{e^{a(x+h)+b} - e^{ax+b}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{e^{ax+b} e^{ah} - e^{ax+b}}{h}$$

Taking e^{ax+b} common, we have

$$= \lim_{h \rightarrow 0} \frac{e^{ax+b} (e^{ah} - 1)}{h}$$

By using algebra of limits,

$$= \lim_{h \rightarrow 0} e^{ax+b} \times \lim_{h \rightarrow 0} \frac{e^{ah} - 1}{h}$$

Since we cannot substitute the value of h directly, we take

$$= \lim_{h \rightarrow 0} e^{ax+b} \times \lim_{h \rightarrow 0} \frac{e^{ah} - 1}{ah} \times a$$

We know that, $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \log_e e = 1$

$$\begin{aligned}
 &= ae^{ax+b} \lim_{h \rightarrow 0} \frac{e^{ah} - 1}{ah} \\
 &= ae^{ax+b} (1) \\
 &= ae^{ax+b}
 \end{aligned}$$

∴ Derivative of $f(x) = ae^{ax+b}$

3. Differentiate each of the following from first principles:

(i) $\sqrt{\sin 2x}$

(ii) $\sin x/x$

Solution:

(i) $\sqrt{\sin 2x}$

Given:

$$f(x) = \sqrt{\sin 2x}$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\sqrt{\sin(2x+2h)} - \sqrt{\sin 2x}}{h}$$

Multiply numerator and denominator by $\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}$, we have

$$= \lim_{h \rightarrow 0} \frac{\sqrt{\sin(2x+2h)} - \sqrt{\sin 2x}}{h} \times \frac{\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}}{\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}}$$

By using $a^2 - b^2 = (a+b)(a-b)$, we get

$$= \lim_{h \rightarrow 0} \frac{\sin(2x+2h) - \sin 2x}{h \left(\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x} \right)}$$

By using the formula,

$$\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$$

$$= \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{2x+2h+2x}{2}\right) \sin\left(\frac{2x+2h-2x}{2}\right)}{h \left(\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}\right)}$$

$$= \lim_{h \rightarrow 0} \frac{2 \cos(2x+h) \sin h}{h \left(\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}\right)}$$

By applying limits to each term, we get

$$= \lim_{h \rightarrow 0} 2 \cos(2x+h) \lim_{h \rightarrow 0} \frac{\sin h}{h} \lim_{h \rightarrow 0} \frac{1}{\left(\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}\right)}$$

$$= 2 \cos 2x (1) \frac{1}{\sqrt{\sin 2x} + \sqrt{\sin 2x}}$$

$$= \frac{2 \cos 2x}{2\sqrt{\sin 2x}}$$

$$= \frac{\cos 2x}{\sqrt{\sin 2x}}$$

\therefore Derivative of $f(x) = \cos 2x / \sqrt{(\sin 2x)}$

(ii) $\sin x/x$

Given:

$$f(x) = \sin x/x$$

By using the formula,

$$\frac{d}{dx} (f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x \sin(x+h) - (x+h) \sin x}{hx(x+h)}$$

By using algebra of limits,

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{x(\sin x \cos h + \cos x \sin h) - x \sin x - h \sin x}{hx(x+h)} \\
 &= \lim_{h \rightarrow 0} \frac{x \sin x \cos h + x \cos x \sin h - x \sin x - h \sin x}{hx(x+h)} \\
 &= \lim_{h \rightarrow 0} \frac{x \sin x \cos h - x \sin x + x \cos x \sin h - h \sin x}{hx(x+h)}
 \end{aligned}$$

By applying limits to each term, we get

$$\begin{aligned}
 &= x \sin x \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} + \frac{x \cos x}{x} \lim_{h \rightarrow 0} \frac{\sin h}{h} \lim_{h \rightarrow 0} \frac{1}{x+h} - \frac{\sin x}{x} \lim_{h \rightarrow 0} \frac{1}{x+h} \\
 &= x \sin x \lim_{h \rightarrow 0} \frac{-2 \sin^2 \frac{h}{2}}{h} + \frac{x \cos x}{x} \lim_{h \rightarrow 0} \frac{\sin h}{h} \lim_{h \rightarrow 0} \frac{1}{x+h} - \frac{\sin x}{x} \lim_{h \rightarrow 0} \frac{1}{x+h} \\
 &= x \sin x \lim_{h \rightarrow 0} \frac{-2 \sin^2 \frac{h}{2}}{\frac{h^2}{4}} \times \frac{h}{4} + \frac{x \cos x}{x} \lim_{h \rightarrow 0} \frac{\sin h}{h} \lim_{h \rightarrow 0} \frac{1}{x+h} - \frac{\sin x}{x} \lim_{h \rightarrow 0} \frac{1}{x+h} \\
 &= -x \sin x \times \lim_{h \rightarrow 0} \frac{h}{2} + \frac{x \cos x}{x} \lim_{h \rightarrow 0} \frac{\sin h}{h} \lim_{h \rightarrow 0} \frac{1}{x+h} - \frac{\sin x}{x} \lim_{h \rightarrow 0} \frac{1}{x+h}
 \end{aligned}$$

When $h=0$, we get

$$\begin{aligned}
 &= -x \sin x \left(\frac{1}{2} \right) (0) + \frac{\cos x}{x} - \frac{\sin x}{x^2} \\
 &= \frac{\cos x}{x} - \frac{\sin x}{x^2}
 \end{aligned}$$

By taking LCM, we get

$$= \frac{x \cos x - \sin x}{x^2}$$

\therefore Derivative of $f(x) = [x \cos x - \sin x]/x^2$

4. Differentiate the following from first principles:

(i) $\tan^2 x$

(ii) $\tan(2x + 1)$

Solution:

(i) $\tan^2 x$

Given:

$f(x) = \tan^2 x$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\tan^2(x+h) - \tan^2 x}{h}$$

By using $(a+b)(a-b) = a^2 - b^2$, we have

$$= \lim_{h \rightarrow 0} \frac{[\tan(x+h) + \tan x][\tan(x+h) - \tan x]}{h}$$

Replacing tan with sin/cos,

$$= \lim_{h \rightarrow 0} \frac{\left[\frac{\sin(x+h)}{\cos(x+h)} + \frac{\sin x}{\cos x} \right] \left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x} \right]}{h}$$

By taking LCM,

$$= \lim_{h \rightarrow 0} \frac{[\sin(x+h) \cos x + \cos(x+h) \sin x][\sin(x+h) \cos x - \cos(x+h) \sin x]}{h \cos^2 x \cos^2(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{[\sin(2x+h)][\sin h]}{h \cos^2 x \cos^2(x+h)}$$

By applying limits to each term, we get

$$= \frac{1}{\cos^2 x} \lim_{h \rightarrow 0} \sin(2x+h) \lim_{h \rightarrow 0} \frac{\sin h}{h} \lim_{h \rightarrow 0} \frac{1}{\cos^2(x+h)}$$

When $h=0$, we get

$$\begin{aligned} &= \frac{1}{\cos^2 x} \sin(2x) (1) \frac{1}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} 2 \sin x \cos x \frac{1}{\cos^2 x} \\ &= 2 \times \frac{\sin x}{\cos x} \times \frac{1}{\cos^2 x} \\ &= 2 \tan x \sec^2 x \end{aligned}$$

\therefore Derivative of $f(x) = 2 \tan x \sec^2 x$

(ii) $\tan(2x+1)$

Given:

$$f(x) = \tan(2x + 1)$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\tan(2x + 2h + 1) - \tan(2x + 1)}{h}$$

Replacing tan with sin/cos,

$$= \lim_{h \rightarrow 0} \frac{\frac{\sin(2x+2h+1)}{\cos(2x+2h+1)} - \frac{\sin(2x+1)}{\cos(2x+1)}}{h}$$

By taking LCM,

$$= \lim_{h \rightarrow 0} \frac{\sin(2x + 2h + 1) \cos(2x + 1) - \cos(2x + 2h + 1) \sin(2x + 1)}{h \cos(2x + 2h + 1) \cos(2x + 1)}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(2x + 2h + 1 - 2x - 1)}{h \cos(2x + 2h + 1) \cos(2x + 1)}$$

By applying limits to each term, we get

$$= \frac{1}{\cos(2x + 1)} \lim_{h \rightarrow 0} \frac{\sin(2h)}{2h} \times 2 \lim_{h \rightarrow 0} \frac{1}{\cos(2x + 2h + 1)}$$

When $h = 0$, we get

$$= \frac{1}{\cos(2x + 1)} \times 2 \times \frac{1}{\cos(2x + 1)}$$

$$= \frac{2}{\cos^2(2x + 1)}$$

$$= 2 \sec^2(2x + 1)$$

$$= 2 \sec^2(2x + 1)$$

\therefore Derivative of $f(x) = 2 \sec^2(2x + 1)$

5. Differentiate the following from first principles:

(i) $\sin \sqrt{2x}$

(ii) $\cos \sqrt{x}$

Solution:

(i) $\sin \sqrt{2x}$

Given:

$$f(x) = \sin \sqrt{2x}$$

$$f(x+h) = \sin \sqrt{2(x+h)}$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\sin \sqrt{2x+2h} - \sin \sqrt{2x}}{h}$$

By using the formula,

$$\begin{aligned} \sin C - \sin D &= 2 \sin \left(\frac{C-D}{2} \right) \cos \left(\frac{C+D}{2} \right) \\ &= \lim_{h \rightarrow 0} \frac{2 \sin \left(\frac{\sqrt{2x+2h} - \sqrt{2x}}{2} \right) \cos \left(\frac{\sqrt{2x+2h} + \sqrt{2x}}{2} \right)}{h} \end{aligned}$$

By using algebra of limits,

$$= \lim_{h \rightarrow 0} \frac{2 \times 2 \sin \left(\frac{\sqrt{2x+2h} - \sqrt{2x}}{2} \right) \cos \left(\frac{\sqrt{2x+2h} + \sqrt{2x}}{2} \right)}{2h + 2x - 2x}$$

To use the sandwich theorem to evaluate the limit, we need $\frac{\sqrt{2x+2h} - \sqrt{2x}}{2}$ in denominator.

$$= \lim_{h \rightarrow 0} \frac{2 \times 2 \sin \left(\frac{\sqrt{2x+2h} - \sqrt{2x}}{2} \right) \cos \left(\frac{\sqrt{2x+2h} + \sqrt{2x}}{2} \right)}{\left(\sqrt{2x+2h} - \sqrt{2x} \right) \sqrt{2x+2h} + \sqrt{2x}}$$

$$= \lim_{h \rightarrow 0} \frac{2 \times 2 \sin \left(\frac{\sqrt{2x+2h} - \sqrt{2x}}{2} \right) \cos \left(\frac{\sqrt{2x+2h} + \sqrt{2x}}{2} \right)}{2 \times \left(\frac{\sqrt{2x+2h} - \sqrt{2x}}{2} \right) \left(\sqrt{2x+2h} + \sqrt{2x} \right)}$$

By applying limits to each term, we get

$$= \lim_{h \rightarrow 0} \frac{\sin \left(\frac{\sqrt{2x+2h} - \sqrt{2x}}{2} \right)}{\left(\frac{\sqrt{2x+2h} - \sqrt{2x}}{2} \right)} \lim_{h \rightarrow 0} \frac{2 \cos \left(\frac{\sqrt{2x+2h} + \sqrt{2x}}{2} \right)}{\sqrt{2x+2h} + \sqrt{2x}}$$

When $h = 0$, we get

$$= 1 \times \frac{2 \cos \sqrt{2x}}{2\sqrt{2x}} \left[\because \lim_{h \rightarrow 0} \frac{\sin \left(\frac{\sqrt{2x+2h} - \sqrt{2x}}{2} \right)}{\left(\frac{\sqrt{2x+2h} - \sqrt{2x}}{2} \right)} = 1 \right]$$

$$= \frac{\cos \sqrt{2x}}{\sqrt{2x}}$$

∴ Derivative of $f(x) = \cos \sqrt{2x} / \sqrt{2x}$

(ii) $\cos \sqrt{x}$

Given:

$$f(x) = \cos \sqrt{x}$$

$$f(x+h) = \cos \sqrt{x+h}$$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \rightarrow 0} \frac{\cos \sqrt{x+h} - \cos \sqrt{x}}{h}$$

By using the formula,

$$\begin{aligned} \cos C - \cos D &= -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right) \\ &= \lim_{h \rightarrow 0} \frac{-2 \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \sin \left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)}{h} \end{aligned}$$

By using algebra of limits, we get

$$= \lim_{h \rightarrow 0} \frac{-2 \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \sin \left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)}{x+h-x}$$

To use the sandwich theorem to evaluate the limit, we need $\frac{\sqrt{x+h} - \sqrt{x}}{2}$ in denominator.

$$= \lim_{h \rightarrow 0} \frac{-2 \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \sin \left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)}{2 \times (\sqrt{x+h} + \sqrt{x}) \frac{(\sqrt{x+h} - \sqrt{x})}{2}}$$

By applying limits to each term, we get

$$= \lim_{h \rightarrow 0} \frac{\sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right)}{\frac{\sqrt{x+h} + \sqrt{x}}{2}} \lim_{h \rightarrow 0} \frac{-\sin \left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)}{\sqrt{x+h} + \sqrt{x}}$$

When $h = 0$, we get

$$= 1 \times \frac{-\sin \sqrt{x}}{2\sqrt{x}} \left[\because \lim_{h \rightarrow 0} \frac{\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{\frac{\sqrt{x+h}-\sqrt{x}}{2}} = 1 \right]$$

$$= \frac{-\sin \sqrt{x}}{2\sqrt{x}}$$

\therefore Derivative of $f(x) = -\sin \sqrt{x} / 2\sqrt{x}$



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