

3. Rationalisation

Exercise 3.1

1. Question

Simplify each of the following :

$$(i) \sqrt[3]{4} \times \sqrt[3]{16} \quad (ii) \frac{\sqrt[3]{1250}}{\sqrt[3]{2}}$$

Answer

$$(i) \sqrt[3]{4} \times \sqrt[3]{16} \\ = \sqrt[3]{4 \times 16} = \sqrt[3]{64}$$

$$= \sqrt[3]{4^3} = (4^3)^{\frac{1}{3}} = 4$$

$$(ii) \frac{\sqrt[4]{1250}}{\sqrt[4]{2}} = \sqrt[4]{\frac{1250}{2}}$$

$$= \sqrt[4]{\frac{625 \times 2}{2}} = \sqrt[4]{625}$$

$$= \sqrt[4]{5^4} = (5^4)^{\frac{1}{4}} = 5$$

2. Question

Simplify the following expressions :

$$(i) (4 + \sqrt{7})(3 + \sqrt{2})$$

$$(ii) (3 + \sqrt{3})(5 - \sqrt{2})$$

$$(iii) (\sqrt{5} - 2)(\sqrt{3} - \sqrt{5})$$

Answer

$$(i) (4 + \sqrt{7})(3 + \sqrt{2})$$

$$= 4 \times 3 + 4 \times \sqrt{2} + \sqrt{7} \times 3 + \sqrt{7} \times \sqrt{2}$$

$$= 12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14}$$

$$(ii) (3 + \sqrt{3})(5 - \sqrt{2})$$

$$= 3 \times 5 + 3 \times (-\sqrt{2}) + \sqrt{3} \times 5 + \sqrt{3} \times (-\sqrt{2})$$

$$= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{3} \times 2$$

$$= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{6}$$

$$(iii) (\sqrt{5} - 2)(\sqrt{3} - \sqrt{5})$$

$$= \sqrt{5} \times \sqrt{3} + \sqrt{5} \times (-\sqrt{5}) + (-2) \times \sqrt{3} + (-2) \times (-\sqrt{5})$$

$$= \sqrt{5} \times 3 - \sqrt{5} \times 5 - 2\sqrt{3} + 2\sqrt{5}$$

$$= \sqrt{15} - \sqrt{5^2} - 2\sqrt{3} + 2\sqrt{5}$$

$$= \sqrt{15} - 5 - 2\sqrt{3} + 2\sqrt{5}$$

3. Question

Simplify the following expressions :

$$(i) (11 + \sqrt{11})(11 - \sqrt{11})$$

$$(ii) (5 + \sqrt{7})(5 - \sqrt{7})$$

$$(iii) (\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2})$$

$$(iv) (3 + \sqrt{3})(3 - \sqrt{3})$$

$$(v) (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$$

Answer

$$(i) (11 + \sqrt{11})(11 - \sqrt{11}) = (11)^2 - (\sqrt{11})^2$$

$$\text{Because } (a + b)(a - b) = a^2 - b^2$$

$$= 121 - 11 = 110$$

$$(ii) (5 + \sqrt{7})(5 - \sqrt{7}) = (5)^2 - (\sqrt{7})^2$$

$$= 25 - 7 = 18$$

$$(iii) (\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2}) = (\sqrt{8})^2 - (\sqrt{2})^2$$

$$= 8 - 2 = 6$$

$$(iv) (3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2$$

$$= 9 - 3 = 6$$

$$(v) (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2$$

$$= 5 - 2 = 3$$

4. Question

Simplify the following expressions:

(i) $(\sqrt{3} - \sqrt{7})^2$ (ii) $(\sqrt{5} - \sqrt{3})^2$ (iii) $(2\sqrt{5} - 3\sqrt{2})^2$

Answer

$$(i) (\sqrt{3} + \sqrt{7})^2 = (\sqrt{3})^2 + 2 \times \sqrt{3} \times \sqrt{7} + (\sqrt{7})^2$$

$$\text{Because: } (a + b)^2 = (a)^2 + 2 \times \sqrt{a} \times \sqrt{b} + (b)^2$$

$$= 3 + 2\sqrt{3 \times 7} + 7$$

$$= 10 + 2\sqrt{21}$$

$$(ii) (\sqrt{5} - \sqrt{3})^2 = (\sqrt{5})^2 - 2 \times \sqrt{5} \times \sqrt{3} + (\sqrt{3})^2$$

$$(a)^2 - 2 \times \sqrt{a} \times \sqrt{b} + (b)^2$$

$$= 5 - 2\sqrt{5 \times 3} + 3$$

$$= 8 - 2\sqrt{15}$$

$$(iii) (2\sqrt{5} + 3\sqrt{2})^2 = (2\sqrt{5})^2 + 2(2\sqrt{5}) \times (3\sqrt{2}) + (3\sqrt{2})^2$$

$$= 2^2 \times \sqrt{5}^2 + 2 \times 2 \times 3 \times \sqrt{5 \times 2} + 3^2 \times \sqrt{2}^2$$

$$= 4 \times 5 + 12\sqrt{5 \times 2} + 9 \times 2$$

$$= 20 + 12\sqrt{10} + 18$$

$$= 38 + 12\sqrt{10}$$

Exercise 3.2

1. Question

Rationalise the denominator of each of the following (i-vii) :

(i) $\frac{3}{\sqrt{5}}$ (ii) $\frac{3}{2\sqrt{5}}$ (iii) $\frac{1}{\sqrt{12}}$ (iv) $\frac{\sqrt{2}}{\sqrt{5}}$ (v) $\frac{\sqrt{3}-1}{\sqrt{2}}$ (vi) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$ (vii) $\frac{3\sqrt{2}}{\sqrt{5}}$

Answer

(i) As there is $\sqrt{5}$ in the denominator and we know that $\sqrt{5} \times \sqrt{5} = 5$

So, multiply numerator and denominator by $\sqrt{5}$,

$$\frac{3}{\sqrt{5}} = \frac{3 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{3\sqrt{5}}{5} = \frac{3}{5}\sqrt{5}$$

$$(ii) \frac{3}{2\sqrt{5}} \times \frac{2\sqrt{5}}{2\sqrt{5}} = \frac{3 \times 2\sqrt{5}}{(2\sqrt{5})^2} = \frac{6\sqrt{5}}{20} = \frac{3}{10}\sqrt{5}$$

$$\frac{1}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{12}}{12}$$

$$= \frac{\sqrt{3}\sqrt{4}}{12}$$

(iii)

$$= \frac{2\sqrt{3}}{12}$$

$$= \frac{\sqrt{3}}{6}$$

(iv) $\frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{2} \times \sqrt{5}}{(\sqrt{5})^2} = \frac{1}{5} \sqrt{10}$

(v) $\frac{\sqrt{3}+1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$

(vi) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}+\sqrt{15}}{3}$

(vii) $\frac{3\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{10}}{5}$

2. Question

Find the value to three places of decimals of each of the following. It is given that

$$\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236 \text{ and } \sqrt{10} = 3.162.$$

(i) $\frac{2}{\sqrt{3}}$ (ii) $\frac{3}{\sqrt{10}}$ (iii) $\frac{\sqrt{5}+1}{\sqrt{2}}$ (iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

(v) $\frac{2+\sqrt{3}}{3}$ (vi) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Answer

(i) Given that $\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$

So we have,

$\frac{2}{\sqrt{3}}$ Rationalising factor of denominator is $\sqrt{3}$

$$\frac{2}{\sqrt{3}} = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\frac{2 \times 1.732}{3} = \frac{3.464}{3}$$

$$= 1.15466667 = 1.54$$

(ii) we have $\frac{3}{\sqrt{10}}$ rationalisation factor of denominator is $\sqrt{10}$

$$\frac{3}{\sqrt{10}} = \frac{3 \times \sqrt{10}}{\sqrt{10} \times \sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\frac{3 \times 3.162}{10} = 0.9486$$

(iii) we have $\frac{\sqrt{5}+1}{\sqrt{2}}$ rationalisation factor of denominator is $\sqrt{2}$

$$\begin{aligned} &= \frac{\sqrt{5}+1}{\sqrt{2}} = \frac{\sqrt{5}+1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{(\sqrt{5}+1)\sqrt{2}}{(\sqrt{2})^2} = \frac{\sqrt{5} \times \sqrt{2} + 1 \times \sqrt{2}}{2} \end{aligned}$$

$$= \frac{\sqrt{5 \times 2} + \sqrt{2}}{2} = \frac{\sqrt{10} + \sqrt{2}}{2}$$

$$= \frac{3.162 + 1.414}{2} = \frac{4.576}{2} = 2.288$$

(iv) we have $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$ rationalisation factor of denominator is $\sqrt{2}$

$$= \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} = \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{(\sqrt{10}+\sqrt{15})\sqrt{2}}{(\sqrt{2})^2}$$

$$= \frac{\sqrt{10 \times 2} + \sqrt{15 \times 2}}{2} = \frac{\sqrt{10 \times 2} + \sqrt{15 \times 2}}{2}$$

$$= \frac{\sqrt{20} + \sqrt{30}}{2} = \frac{\sqrt{2 \times 10} + \sqrt{3 \times 10}}{2}$$

$$= \frac{\sqrt{2} \times \sqrt{10} + \sqrt{3} \times \sqrt{10}}{2} = \frac{1.414 \times 3.162 + 1.732 \times 3.162}{2}$$

$$= \frac{4.471068 + 5.476584}{2} = \frac{9.947652}{2}$$

$$= 4.973826 = 4.973$$

(v) We have $\frac{2+\sqrt{3}}{3}$

$$= \frac{2+\sqrt{3}}{3} = \frac{2+1.732}{3} = \frac{3.732}{3} = 1.244$$

(vi) We have $\frac{\sqrt{2}-1}{\sqrt{5}}$ rationalising factor of denominator is $\sqrt{5}$

$$= \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{(\sqrt{2} \times \sqrt{5}) - (1 \times \sqrt{5})}{(\sqrt{5})^2}$$

$$= \frac{\sqrt{2 \times 5} - 1\sqrt{5}}{5} = \frac{\sqrt{10} - \sqrt{5}}{5}$$

$$= \frac{3.162 - 2.236}{5} = \frac{0.926}{5}$$

$$= 0.185$$

3. Question

Express each one of the following with rational denominator:

(i) $\frac{1}{3+\sqrt{2}}$ (ii) $\frac{1}{\sqrt{6}-\sqrt{5}}$ (iii) $\frac{16}{\sqrt{41}-5}$

(iv) $\frac{30}{5\sqrt{3}-3\sqrt{5}}$ (v) $\frac{1}{2\sqrt{5}-\sqrt{3}}$ (vi) $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ (vii) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ (viii) $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$ (ix) $\frac{b^2}{\sqrt{a^2+b^2}+a}$

Answer

(i) we have $\frac{1}{3+\sqrt{2}}$ rationalizing factor of the denominator is $3-\sqrt{2}$

$$= \frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$= \frac{3-\sqrt{2}}{(3)^2 - (\sqrt{2})^2}$$

because $(a+b)(a-b) = (a)^2 - (b)^2$

$$= \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$$

(ii) we have $\frac{1}{\sqrt{6}-\sqrt{5}}$ rationalizing factor of the denominator is $\sqrt{6} + \sqrt{5}$

$$= \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$$

$$= \frac{\sqrt{6} + \sqrt{5}}{(\sqrt{6})^2 - (\sqrt{5})^2} = \frac{\sqrt{6} + \sqrt{5}}{6 - 5} = \frac{\sqrt{6} + \sqrt{5}}{1}$$

$$= \sqrt{6} + \sqrt{5}$$

Myclass24
Your Class. Your Pace.

(iii) we have $\frac{16}{\sqrt{41}-5}$ rationalizing factor of the denominator is $\sqrt{41} + 5$

$$\begin{aligned} &= \frac{16}{\sqrt{41}-5} \times \frac{\sqrt{41}+5}{\sqrt{41}+5} \\ &= \frac{16 \times (\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)} = \frac{16\sqrt{41}+5}{(\sqrt{41})^2 - (5)^2} \\ &= \frac{16\sqrt{41}+5}{41-25} = \frac{16\sqrt{41}+5}{16} = \sqrt{41} + 5 \end{aligned}$$

(iv) we have $\frac{30}{5\sqrt{3}-3\sqrt{5}}$ to rationalize factor of $5\sqrt{3} - 3\sqrt{5}$ is $5\sqrt{3} + 3\sqrt{5}$

$$\begin{aligned} &= \frac{30}{5\sqrt{3}-3\sqrt{5}} \times \frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}} = \frac{3(5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3})^2 - (3\sqrt{5})^2} \\ &= \frac{30(5\sqrt{3}+3\sqrt{5})}{5^2(\sqrt{3})^2 - 3^2(\sqrt{5})^2} = \frac{30(5\sqrt{3}+3\sqrt{5})}{25 \times 3 - 9 \times 5} \\ &= \frac{30(5\sqrt{3}+3\sqrt{5})}{75-45} = \frac{30(5\sqrt{3}+3\sqrt{5})}{30} \\ &= 5\sqrt{3} + 3\sqrt{5} \end{aligned}$$

(v) we have $\frac{1}{2\sqrt{5}-\sqrt{3}}$ to rationalize factor of $2\sqrt{5} - \sqrt{3}$ is $2\sqrt{5} + \sqrt{3}$

$$\begin{aligned} &= \frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}} = \frac{2\sqrt{5}+\sqrt{3}}{(2\sqrt{5})^2 - (\sqrt{3})^2} \\ &= \frac{2\sqrt{5}+\sqrt{3}}{2^2(\sqrt{5})^2 - (\sqrt{3})^2} = \frac{2\sqrt{5}+\sqrt{3}}{4 \times 5 - 3} \\ &= \frac{2\sqrt{5}+\sqrt{3}}{20-3} = \frac{2\sqrt{5}+\sqrt{3}}{17} \end{aligned}$$

(vi) we have $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ to rationalize factor of $2\sqrt{2} - \sqrt{3}$ is $2\sqrt{2} + \sqrt{3}$

$$\begin{aligned} &= \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} = \frac{(\sqrt{3}+1)(2\sqrt{2}+\sqrt{3})}{(2\sqrt{2})^2 - (\sqrt{3})^2} \\ &= \frac{\sqrt{3} \times 2\sqrt{2} + 2\sqrt{2} + \sqrt{3} \times \sqrt{3} + \sqrt{3}}{4 \times 2 - 3} \\ &= \frac{2\sqrt{2} \times \sqrt{3} + 2\sqrt{2} + 3 + \sqrt{3}}{8-3} \\ &= \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3}}{5} \end{aligned}$$

(vii) we have $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ to rationalize factor of $6 + 4\sqrt{2}$ is $6 - 4\sqrt{2}$

$$\begin{aligned}\frac{6 - 4\sqrt{2}}{6 + 4\sqrt{2}} &= \frac{6 - 4\sqrt{2}}{6 + 4\sqrt{2}} \times \frac{6 - 4\sqrt{2}}{6 - 4\sqrt{2}} \\ &= \frac{(6-4\sqrt{2})^2}{(6)^2 - (4\sqrt{2})^2}\end{aligned}$$

Because; $(a + b)(a - b) = a^2 - b^2$

$$(a - b)(a + b) = (a - b)^2$$

$$\begin{aligned}\text{so, } \frac{(6 - 4\sqrt{2})^2}{6^2 - (4\sqrt{2})^2} \\ &= \frac{6^2 - 2 \times 6 \times 4\sqrt{2} + (4\sqrt{2})^2}{36 - 4^2(\sqrt{2})^2} \\ &= \frac{36 - 48\sqrt{2} + 32}{36 - 32} = \frac{68 - 48\sqrt{2}}{4}\end{aligned}$$

$$= \frac{4(17 - 12\sqrt{2})}{4} = 17 - 12\sqrt{2}$$

(viii) we have $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$ to rationalize factor of $2\sqrt{5} - 3$ is $2\sqrt{5} + 3$

$$\begin{aligned}&= \frac{3\sqrt{2} + 1}{2\sqrt{5} - 3} \times \frac{2\sqrt{5} + 3}{2\sqrt{5} + 3} = \frac{(3\sqrt{2} + 1)(2\sqrt{5} + 3)}{(2\sqrt{5} - 3)(2\sqrt{5} + 3)} \\ &= \frac{3\sqrt{2} \times 2\sqrt{5} + 3\sqrt{2} \times 3 + 1 \times 2\sqrt{5} + 1 \times 3}{(2\sqrt{2})^2 - 3^2}\end{aligned}$$

$$= \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{20 - 9}$$

$$= \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{11}$$

(ix) we have $\frac{b^2}{\sqrt{a^2+b^2}+a}$ to rationalize factor of $\sqrt{a^2 + b^2} + a$ is $\sqrt{a^2 + b^2} - a$

$$= \frac{b^2}{\sqrt{a^2 + b^2} + a} \times \frac{\sqrt{a^2 + b^2} - a}{\sqrt{a^2 + b^2} - a} = \frac{b^2(\sqrt{a^2 + b^2} - a)}{(\sqrt{a^2 + b^2})^2 - (a)^2}$$

$$= \frac{b^2(\sqrt{a^2 + b^2})}{a^2 + b^2 - a^2} = \frac{b^2(\sqrt{a^2 + b^2} - a^2)}{b^2}$$

$$= (\sqrt{a^2 + b^2} - a^2)$$

4. Question

Rationalise the denominator and simplify :

$$(i) \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \quad (ii) \frac{5+2\sqrt{3}}{7-4\sqrt{3}} \quad (iii) \frac{1-\sqrt{2}}{3-2\sqrt{2}}$$

$$(iv) \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \quad (v) \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

$$(vi) \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$$

Answer

$$i) \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{3+2-2\sqrt{6}}{3-2} = 5 - 2\sqrt{6}.$$

$$ii) \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{35+14\sqrt{3}-20\sqrt{3}-24}{49-48} = 11 - 6\sqrt{3}.$$

$$iii) \frac{1+\sqrt{2}}{3-2\sqrt{2}} = \frac{1+\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+3\sqrt{2}+2\sqrt{2}+4}{9-8} = 7 + 5\sqrt{2}.$$

$$iv) \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \times \frac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}} = \frac{6\sqrt{30}-15+4\sqrt{36}-2\sqrt{30}}{45-24} = \frac{4\sqrt{30}+9}{21}.$$

$$v) \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} = \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}} = \frac{48+20\sqrt{6}-12\sqrt{6}-30}{48-18} = \frac{18+8\sqrt{6}}{30} = \frac{9+4\sqrt{6}}{15}.$$

$$vi) \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} \times \frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}} = \frac{4\sqrt{6}-2\sqrt{10}-18+3\sqrt{15}}{8-27} = \frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

5. Question

Simplify :

$$(i) \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}} \quad (ii) \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$(iii) \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

$$(iv) \frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$$

$$(v) \frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$$

Answer

$$i) \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}} = \frac{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}{18-12} + \frac{2\sqrt{3}(\sqrt{3}+\sqrt{2})}{3-2}$$

$$= \frac{30 - 12\sqrt{6}}{6} + (6 + 2\sqrt{6}) = (5 - 2\sqrt{6} + 6 + 2\sqrt{6}) = 11$$

$$\text{ii) } \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{[(\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})+(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})]}{5-3} = \frac{8+2\sqrt{15}+8-2\sqrt{15}}{2} = 8.$$

$$\text{iii) } \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

rationalising factors of denominators are $3 - \sqrt{5}$ and $3 + \sqrt{5}$

$$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{(7+3\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} - \frac{(7-3\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$$

$$\frac{7 \times 3 + 7 \times (-\sqrt{5}) + 3\sqrt{5} + 3 + 3\sqrt{5} \times (-\sqrt{5})}{3^2 - (\sqrt{5})^2} - \frac{7 \times 3 + 7 \times \sqrt{5} + (-3\sqrt{5}) \times 3 + (-3\sqrt{5}) \times \sqrt{5}}{3^2 - (\sqrt{5})^2}$$

$$= \frac{21 - 7\sqrt{5} + 9\sqrt{5} - 3 \times 5}{9 - 5} - \frac{21 + 7\sqrt{5} - 9\sqrt{5} - 3 \times 5}{9 - 5}$$

$$= \frac{21 - 15 + 2\sqrt{5}}{4} - \frac{21 - 15 - 2\sqrt{5}}{4}$$

$$= \frac{6 + 2\sqrt{5}}{4} - \frac{6 - 2\sqrt{5}}{4}$$

$$= \frac{6 + 2\sqrt{5} - (6 - 2\sqrt{5})}{4}$$

$$= \frac{6 + 2\sqrt{5} - 6 + 2\sqrt{5}}{4}$$

$$= \frac{4\sqrt{5}}{4} = \sqrt{5}$$

$$\text{(iv) } \frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$$

Rationalising factor for $2 + \sqrt{3}$ is $2 - \sqrt{3}$

For $\sqrt{5} - \sqrt{3}$ is $\sqrt{5} + \sqrt{3}$ and

For $2 - \sqrt{5}$ is $2 + \sqrt{5}$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{1}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}$$

$$= \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} + \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{2+\sqrt{5}}{2^2-(\sqrt{5})^2}$$

$$\begin{aligned}
&= \frac{2 - \sqrt{3}}{4 - 3} + \frac{2\sqrt{5} + 2\sqrt{3}}{5 - 3} + \frac{2 + \sqrt{5}}{4 - 5} \\
&= \frac{2 - \sqrt{3}}{1} + \frac{2\sqrt{5} + 2\sqrt{3}}{2} + \frac{2 + \sqrt{5}}{-1} \\
&= 2 - \sqrt{3} + 2 \frac{(\sqrt{5} + \sqrt{3})}{2} - (2 + \sqrt{5}) \\
&= 2 - \sqrt{3} + \sqrt{5} + \sqrt{3} - 2 - \sqrt{5} = 0
\end{aligned}$$

$$(v) \frac{2}{\sqrt{5} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{2}} - \frac{3}{\sqrt{5} + \sqrt{2}}$$

Rationalising factors for denominators are,

$$\text{For } \sqrt{5} + \sqrt{3} \text{ is } \sqrt{5} - \sqrt{3}$$

$$\text{For } \sqrt{3} + \sqrt{2} \text{ is } \sqrt{3} - \sqrt{2} \text{ and}$$

$$\text{For } \sqrt{5} + \sqrt{2} \text{ is } \sqrt{5} - \sqrt{2}$$

$$\begin{aligned}
&= \frac{2}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} - \frac{3}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} \\
&= \frac{2(\sqrt{5} - \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} + \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} - \frac{3(\sqrt{5} - \sqrt{2})}{(\sqrt{5})^2 - (\sqrt{2})^2} \\
&= \frac{2(\sqrt{5} - \sqrt{3})}{5 - 3} + \frac{\sqrt{3} - \sqrt{2}}{3 - 2} - \frac{3(\sqrt{5} - \sqrt{2})}{5 - 2} \\
&= \frac{2(\sqrt{5} - \sqrt{3})}{2} + \frac{\sqrt{3} - \sqrt{2}}{1} - \frac{3(\sqrt{5} - \sqrt{2})}{3} \\
&= \sqrt{5} - \sqrt{3} + \sqrt{3} - \sqrt{2} - \sqrt{5} + \sqrt{2} = 0
\end{aligned}$$

6. Question

In each of the following determine rational numbers a and b .

$$(i) \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = a - b\sqrt{3} \quad (ii) \frac{4 + \sqrt{2}}{2 + \sqrt{2}} = a - \sqrt{5}$$

$$(iii) \frac{3 + \sqrt{2}}{3 - \sqrt{2}} = a + b\sqrt{2} \quad (iv) \frac{5 + 3\sqrt{3}}{7 + 4\sqrt{3}} = a + b\sqrt{3}$$

$$(v) \frac{\sqrt{11} - \sqrt{7}}{\sqrt{11} + \sqrt{7}} = a - b\sqrt{77}$$

$$(vi) \frac{4 + 3\sqrt{5}}{4 - 3\sqrt{5}} = a + b\sqrt{5}$$

Answer

$$(i) \frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$$

Given,

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$$

Rationalising factor for denominator is $\sqrt{3}-1$

$$\begin{aligned} &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - (1)^2} \\ &= \frac{(\sqrt{3})^2 - 2\sqrt{3} \times 1 + (1)^2}{3-2} = \frac{3-2\sqrt{3}+1}{2} \\ &= \frac{4-2\sqrt{3}}{2} = \frac{2(2-\sqrt{3})}{2} = 2-\sqrt{3} \end{aligned}$$

we have, $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$

$$= 2 - \sqrt{3} = a - b\sqrt{3} = 2 - (1)\sqrt{3} = a - b\sqrt{3}$$

On equating rational and irrational parts,

We get $a = 2$ and $b = 1$

$$(ii) \frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b} \text{ rationalising factor for the denominator is } 2 - \sqrt{2}$$

$$\begin{aligned} &= \frac{4+\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{4 \times 2 + \sqrt{2} \times 2 + 4 \times (-\sqrt{2}) + \sqrt{2} \times (-\sqrt{2})}{2^2 - (\sqrt{2})^2} \\ &= \frac{8 + 2\sqrt{2} - 4\sqrt{2} - \sqrt{2}}{4-2} = \frac{6-2\sqrt{2}}{2} \\ &= \frac{2(3-\sqrt{2})}{2} = 3-\sqrt{2} \end{aligned}$$

We have $\frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b} = 3 - \sqrt{2} = a - \sqrt{b}$

On equating rational and irrational parts we get,

$a=3$ and $b=2$

$$(iii) \frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$$

Rationalising factor for the denominator is $3 + \sqrt{2}$

$$\begin{aligned}\frac{3 + \sqrt{2}}{3 - \sqrt{2}} \times \frac{3 + \sqrt{2}}{3 + \sqrt{2}} &= \frac{(3 + \sqrt{2})^2}{3^2 - (\sqrt{2})^2} \\ &= \frac{3^2 + 2 \times 3 \times \sqrt{2} + (\sqrt{2})^2}{9 - 2} = \frac{9 + 6\sqrt{2} + 2}{7} \\ &= \frac{11 + 6\sqrt{2}}{7} = \frac{11}{7} + \frac{6}{7}\sqrt{2}\end{aligned}$$

we have $\frac{3 + \sqrt{2}}{3 - \sqrt{2}} = a + b\sqrt{2}$

On equating rational and irrational parts we get,

$$a = \frac{11}{7}, \text{ and } b = \frac{6}{7}$$

(iv) $\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$ given,

Rationalising factor for denominator is $7 - 4\sqrt{3}$

$$\begin{aligned}&= \frac{5 + 3\sqrt{3}}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}} \\ &= \frac{5 \times 7 + 5 \times (-4\sqrt{3}) + 3\sqrt{3} \times 7 + 3\sqrt{3} \times (-4\sqrt{3})}{(7)^2 - (4\sqrt{3})^2} \\ &= \frac{35 - 20\sqrt{3} + 21\sqrt{3} - 12 \times 3}{49 - 48} \\ &= \frac{35 - 36 + \sqrt{3}}{1} = \frac{\sqrt{3} - 1}{1} = \sqrt{3} - 1\end{aligned}$$

We have $\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

$$\sqrt{3} - 1 = a + b\sqrt{3}$$

On equating rational and irrational parts we get,

$$a = -1 \text{ and } b = 1$$

(v) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$ given,

$$\begin{aligned}
&= \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} \times \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}} = \frac{(\sqrt{11}-\sqrt{7})^2}{(\sqrt{11})^2 - (\sqrt{7})^2} \\
&= \frac{(\sqrt{11})^2 - 2\sqrt{11}\times\sqrt{7} + (\sqrt{7})^2}{11-7} = \frac{11 - 2\sqrt{11\times 7} + 7}{4} \\
&= \frac{18 - 2\sqrt{77}}{4} = \frac{2(9 - \sqrt{77})}{4} \\
&= \frac{9 - \sqrt{77}}{2} = \frac{9}{2} - \frac{\sqrt{77}}{2}
\end{aligned}$$

We have $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$

$$\begin{aligned}
&= \frac{9}{2} - \frac{\sqrt{77}}{2} = a - b\sqrt{77} \\
&= \frac{9}{2} - \frac{1}{2}\sqrt{77} = a - b\sqrt{77}
\end{aligned}$$

On equating rational and irrational parts we get

$$a = \frac{9}{2}, b = \frac{1}{2}$$

(vi) $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$ given,

$$\begin{aligned}
&= \frac{4+3\sqrt{5}}{4-3\sqrt{5}} = \frac{4+3\sqrt{5}}{4-3\sqrt{5}} \times \frac{4+3\sqrt{5}}{4+3\sqrt{5}} \\
&= \frac{(4+3\sqrt{5})^2}{4^2 - (3\sqrt{5})^2} = \frac{4^2 + 2\times 4\times 3\sqrt{5} + (3\sqrt{5})^2}{16 - 3^2(\sqrt{5})^2} \\
&= \frac{16 + 24\sqrt{5} + 45}{16 - 45} = \frac{61 + 24\sqrt{5}}{-29} = \frac{-(61 + 24\sqrt{5})}{29} \\
&= \frac{-61}{29} - \frac{24}{29}\sqrt{5}
\end{aligned}$$

We have $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$

On equating rational and irrational parts we have,

$$a = \frac{-61}{29} \text{ and } b = \frac{-24}{29}$$

7. Question

If $x = 2 + \sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$.

Answer

Given $x = 2 + \sqrt{3}$ and given to find the value of $x^3 + \frac{1}{x^3}$

We have $x = 2 + \sqrt{3}$

$$= \frac{1}{x} = \frac{1}{2 + \sqrt{3}}$$

rationalising factor for denominator is $2 - \sqrt{3}$

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{2 - \sqrt{3}}{2^2 - (\sqrt{3})^2}$$

$$= \frac{2 - \sqrt{3}}{4 - 3} = \frac{2 - \sqrt{3}}{1} = 2 - \sqrt{3}$$

$$\therefore \frac{1}{x} = 2 - \sqrt{3}$$

and also, $\left(x + \frac{1}{x}\right) = 2 + \sqrt{3} + 2 - \sqrt{3}$

$$= 2 + 2 = 4$$

$$\therefore \left(x + \frac{1}{x}\right) = 4 \text{ equation (i)}$$

We know that ,

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right) \left(x^2 - x \times \frac{1}{x} + \frac{1}{x^2}\right)$$

$$= \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} - 1\right)$$

$$= \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} + 2 - 2 - 1\right)$$

$$= \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} - 3\right)$$

$$= x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} - 3\right)$$

$$= \left(x + \frac{1}{x}\right) \left(\left(x + \frac{1}{x}\right)^2 - 3\right)$$

By putting $\left(x + \frac{1}{x}\right) = 4$ we get

$$= x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right) \left(\left(x + \frac{1}{x}\right)^2 - 3\right)$$

$$= (4)(4^2 - 3)$$

$$= 4(16 - 3)$$

$$= 4(13) = 52$$

∴ The value of $x^3 + \frac{1}{x^3}$ is 52.

8. Question

If $x = 3 + \sqrt{8}$, find the value of $x^2 + \frac{1}{x^2}$.

Answer

Given that $x = 3 + \sqrt{8}$

And given to find the value of $x^2 + \frac{1}{x^2}$

We have $x = 3 + \sqrt{8}$

The rationalising factor for denominator is $3 - \sqrt{8}$

$$= \frac{1}{x} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$

$$= \frac{3 - \sqrt{8}}{3^2 - (\sqrt{8})^2} = \frac{3 - \sqrt{8}}{9 - 8} = \frac{3 - \sqrt{8}}{1} = 3 - \sqrt{8}$$

$$\therefore \frac{1}{x} = 3 - \sqrt{8}$$

$$\text{Also, } \left(x + \frac{1}{x}\right) = 3 + \sqrt{8} + 3 - \sqrt{8} = 3 + 3 = 6$$

$$\therefore \left(x + \frac{1}{x}\right) = 6$$

We know that,

$$= x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

by putting $x + \frac{1}{x} = 6$ in the above we get,

$$x^2 + \frac{1}{x^2} = (6)^2 - 2$$

$$= 36 - 2 = 34$$

∴ The value of $x^2 + \frac{1}{x^2}$ is 34.

9. Question

Find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$, it being given that $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$

Answer

$\frac{6}{\sqrt{5}+\sqrt{3}}$ Rationalising factor for the denominator is $\sqrt{5} + \sqrt{3}$

$$\begin{aligned} &= \frac{6}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{6(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} \\ &= \frac{6(\sqrt{5}+\sqrt{3})}{5-3} = \frac{6(\sqrt{5}+\sqrt{3})}{2} = 3(\sqrt{5}+\sqrt{3}) \end{aligned}$$

We have $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$

$$\frac{6}{\sqrt{5}-\sqrt{3}} = 3(2.236 + 1.732)$$

$$= 3(3.968)$$

$$= 11.904$$

∴ value of $\frac{6}{\sqrt{5}-\sqrt{3}}$ is 11.904

10. Question

Find the values of each of the following correct to three places of decimals, it being given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{6} = 2.4495$ and $\sqrt{10} = 3.162$.

(i) $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ (ii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

Answer

(i) We have $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ rationalising factor for denominator is $3 - 2\sqrt{5}$

$$= \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$= \frac{3 \times 3 + 3 \times (-2\sqrt{5}) + (-\sqrt{5})(3) + (-\sqrt{5})(-2\sqrt{5})}{3^2 - (2\sqrt{5})^2}$$

$$= \frac{9 - 6\sqrt{5} - 3\sqrt{5} + 2 \times 5}{9 - 20} = \frac{9 + 10 - 9\sqrt{5}}{-11}$$

$$= \frac{19 - 9\sqrt{5}}{-11} = \frac{9\sqrt{5} - 19}{11}$$

We have $\sqrt{5} = 2.236$

$$= \frac{3 - \sqrt{5}}{3 + 2\sqrt{5}} = \frac{9(2.236) - 19}{11} = \frac{20.124 - 19}{11}$$

$$= \frac{1.124}{11} = 0.102181818$$

$$= 0.102$$

$$= \text{the value of } \frac{3 - \sqrt{5}}{3 + 2\sqrt{5}} = 0.102$$

(ii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$ by putting the value of $\sqrt{2}$ in the equation we get,

$$= \frac{1 + \sqrt{2}}{3 - 2\sqrt{2}} = \frac{1 + 1.414}{3 - 2 \times 1.414} = \frac{2.414}{3 - 2.8284}$$

$$= \frac{2.4142}{0.1716} = 14.0687$$

$$= 14.068$$

$$= 14.070$$

11. Question

If $x = \frac{\sqrt{3}-1}{2}$, find the value of $4x^3 + 2x^2 - 8x + 7$.

Answer

Given $x = \frac{\sqrt{3}+1}{2}$ and given to find the value of $4x^3 + 2x^2 - 8x + 7$

$$x = \frac{\sqrt{3} + 1}{2}$$

$$= 2x = \sqrt{3} + 1$$

$$= (2x - 1) = \sqrt{3}$$

Squaring on both the sides we get,

$$= (2x - 1)^2 = (\sqrt{3})^2$$

$$= (2x)^2 - 2 \times 2x \times 1 + (1)^2 = 3$$

$$= 4x^2 - 4x + 1 = 3$$

$$= 4x^2 - 4x + 1 - 3 = 0$$

$$= 4x^2 - 4x - 2 = 0$$

$$= 2(2x^2 - 2x - 1) = 0$$

$$= 2x^2 - 2x - 1 = 0$$

Now take $4x^3 + 2x^2 - 8x + 7$

$$= 2x(2x^2 - 2x - 1) + 4x^2 + 2x + 2x^2 - 8x + 7$$

$$= 2x(2x^2 - 2x - 1) + 6x^2 - 6x + 7$$

$$= 2x(0) + 3(2x^2 - 2x - 1) + 7 + 3$$

$$= 0 + 3(0) + 10 = 10$$

The value of $4x^3 + 2x^2 - 8x + 7$ is 10.

CCE - Formative Assessment

1. Question

Write the value of $(2 + \sqrt{3})(2 - \sqrt{3})$.

Answer

$$(2 + \sqrt{3})(2 - \sqrt{3})$$

$$= (2)^2 - (\sqrt{3})^2 [(a+b)(a-b) = a^2 - b^2]$$

$$= 4 - 3 = 1.$$

2. Question

Write the reciprocal of $5 + \sqrt{2}$.

Answer

Reciprocal of $5 + \sqrt{2} = 1 / (5 + \sqrt{2})$

$$= \frac{1}{5 + \sqrt{2}} = \frac{1}{5 + \sqrt{2}} \times \frac{5 - \sqrt{2}}{5 - \sqrt{2}} = \frac{5 - \sqrt{2}}{25 - 2} = \frac{5 - \sqrt{2}}{23}$$

3. Question

Write the rationalisation factor of $7 - 3\sqrt{5}$.

Answer

Rationalizing factor of $7 - 3\sqrt{5}$

$$= \frac{1}{7 - 3\sqrt{5}} = 7 + 3\sqrt{5}.$$

4. Question

If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = x+y\sqrt{3}$, find the values of x and y .

Answer

$$\begin{aligned}\text{Given, } \frac{\sqrt{3}-1}{\sqrt{3}+1} &= x + y\sqrt{3} \\ &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{3-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}.\end{aligned}$$

So, $x = 2$, $y = -1$

5. Question

If $x = \sqrt{2} - 1$, then write the value of $\frac{1}{x}$.

Answer

Given . $x = \sqrt{2}-1$

$$\frac{1}{x} = \frac{1}{\sqrt{2}-1} = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1.$$

6. Question

Simplify $\sqrt{3-2\sqrt{2}}$.

Answer

Consider $\sqrt{(3+2\sqrt{2})}$,

$$\begin{aligned}\sqrt{(3+2\sqrt{2})} &= \sqrt{(2+1+2\sqrt{2})} \\ &= \sqrt{((\sqrt{2})^2 + (1)^2 + 2 \times 1 \times \sqrt{2})}\end{aligned}$$

As we know, $(a+b)^2 = a^2 + b^2 + 2ab$

$$\begin{aligned}&= \sqrt{(\sqrt{2}+1)^2} \\ &= \sqrt{2}+1\end{aligned}$$

7. Question

Simplify $\sqrt{3-2\sqrt{2}}$.

Answer

$$\sqrt{3-2\sqrt{2}} = \sqrt{(\sqrt{2})^2 + (1)^2 - 2 \times \sqrt{2} \times 1} = \sqrt{(\sqrt{2} - 1)^2} = \sqrt{2} - 1.$$

8. Question

If $a = \sqrt{2} + 1$, then find the value of $a - \frac{1}{a}$.

Answer

Given, $a = \sqrt{2} + 1$

$$= \frac{1}{a} = \frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = (\sqrt{2} - 1)$$

$$= a - \left(\frac{1}{a}\right) = \sqrt{2} + 1 - (\sqrt{2} - 1) = 2.$$

9. Question

If $x = 2 + \sqrt{3}$, find the value of $x + \frac{1}{x}$.

Answer

Given, $x = 2 + \sqrt{3}$

$$= \frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$$

$$= x + \frac{1}{x} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4.$$

10. Question

Write the rationalisation factor of $\sqrt{5} - 2$.

Answer

Rationalizing factor of $\sqrt{5} - 2$

$$= \frac{1}{\sqrt{5}-2} = \sqrt{5} + 2$$

11. Question

If $x = 3 + 2\sqrt{2}$, then find the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$.

Answer

Given $x = 3 + 2\sqrt{2}$

$$= \sqrt{x} = \sqrt{3 + 2\sqrt{2}} = \sqrt{(\sqrt{2} + 1)^2}$$

$$= \sqrt{x} = \sqrt{2} + 1$$

$$= \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{1} = \sqrt{2} - 1$$

$$\begin{aligned}\text{So, } \sqrt{x} - 1 / \sqrt{x} &= \sqrt{2+1} - (\sqrt{2} - 1) \\ &= 1 + 1 = 2.\end{aligned}$$

1. Question

$\sqrt{10} \times \sqrt{15}$ is equal to

- A. $5\sqrt{6}$
- B. $6\sqrt{5}$
- C. $\sqrt{30}$
- D. $\sqrt{25}$

Answer

$$\begin{aligned}\sqrt{10} \times \sqrt{15} &= (\sqrt{5 \times 2}) \times (\sqrt{5 \times 3}) \\ &= 5(\sqrt{6})\end{aligned}$$

2. Question

$\sqrt[5]{6} \times \sqrt[5]{6}$ is equal to

- A. $\sqrt[5]{36}$
- B. $\sqrt[5]{6 \times 6}$
- C. $\sqrt[5]{6}$
- D. $\sqrt[5]{12}$

Answer

$$\begin{aligned}\sqrt[5]{6} \times \sqrt[5]{6} &= (6)^{1/5} \times (6)^{1/5} = (36)^{1/5} \\ &= \sqrt[5]{36}\end{aligned}$$

3. Question

The rationalisation factor of $\sqrt{3}$ is

- A. $-\sqrt{3}$
- B. $\frac{1}{\sqrt{3}}$
- C. $2\sqrt{3}$
- D. $-2\sqrt{3}$

Answer

Rationalisation factor of $\sqrt{3} = 1/\sqrt{3}$

4. Question

The rationalisation factor of $2+\sqrt{3}$ is

- A. $2-\sqrt{3}$
- B. $2+\sqrt{3}$
- C. $\sqrt{2}-3$
- D. $\sqrt{3}-2$

Answer

Rationalisation factor of $2+\sqrt{3} = 1/2+\sqrt{3} = 2-\sqrt{3}$

5. Question

If $x = \sqrt{5} + 2$, then $x - \frac{1}{x}$ equals

- A. $2\sqrt{5}$
- B. 4
- C. 2
- D. $\sqrt{5}$

Answer

Given $x = \sqrt{5} + 2$

$$= \frac{1}{x} = \frac{1}{\sqrt{5}+2} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$$

$$\text{So, } x - \frac{1}{x} = \sqrt{5} + 2 - (\sqrt{5} - 2) = 4$$

6. Question

If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$, then

- A. $a = 2, b = 1$
- B. $a = 2, b = -1$
- C. $a = -2, b = 1$
- D. $a = b = 1$

Answer

Given $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}$$

So, $a = 2, b = 1$.

7. Question

The simplest rationalising of $\sqrt[3]{500}$ is

- A. $\sqrt[3]{2}$
- B. $\sqrt[3]{5}$
- C. $\sqrt{5}$
- D. none of these

Answer

$$\sqrt[3]{500} = \sqrt[3]{(125 \times 4)} = 5 \times \sqrt[3]{4}$$

8. Question

The simplest rationalising factor of $\sqrt{3} + \sqrt{5}$ is

- A. $\sqrt{3} - 5$
- B. $3 - \sqrt{5}$
- C. $\sqrt{3} - \sqrt{5}$
- D. $\sqrt{3} - \sqrt{5}$

Answer

Simplest rationalizing factor of $\sqrt{3} + \sqrt{5}$

$$1/(\sqrt{3} + \sqrt{5}) = \sqrt{3} - \sqrt{5}$$

9. Question

The simplest rationalising factor of $2\sqrt{5} - \sqrt{3}$ is

- A. $2\sqrt{5} + 3$
- B. $2\sqrt{5} +$
- C. $\sqrt{5} + \sqrt{3}$
- D. $\sqrt{5} - \sqrt{3}$

Answer

Simplest rationalizing factor of $2\sqrt{5} - \sqrt{3}$

$$= 1/(2\sqrt{5} - \sqrt{3})$$

$$= 2\sqrt{5} + \sqrt{3}$$

10. Question

If $x = \frac{2}{3 - \sqrt{5}}$, then $(x-3)^2 =$

- A. 1
- B. 3
- C. 6
- D. 7

Answer

Given $X = 2/(3+\sqrt{7})$

$$= \left(\frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} \right) = \frac{2(3-\sqrt{7})}{9-7} = 3 - \sqrt{7}$$

$$= (x - 3)^2 = (3 - \sqrt{7} - 3)^2 = \sqrt{7}^2 = 7$$

11. Question

If $x = 7+4\sqrt{3}$ and $xy=1$, then $= \frac{1}{x^2} + \frac{1}{y^2}$

- A. 64
- B. 134
- C. 194
- D. 1/49

Answer

Given $x = 7+4\sqrt{3}$, $xy = 1$

$$Y = 1/x = 1/7+ 4\sqrt{3} = 7-4\sqrt{3}$$

$$Y^2 = 1/x^2 = 49 + 48 - 56\sqrt{3} = 97 - 56\sqrt{3}$$

Similarly, $x = 1/y$

$$= x^2 = 1/y^2 = (7 + 4\sqrt{3})^2 = 49 + 48 + 56\sqrt{3} = 97 + 56\sqrt{3}$$

$$\text{So, } 1/x^2 + 1/y^2 = 97 + 56\sqrt{3} + 97 - 56\sqrt{3} = 194$$

12. Question

If $x + \sqrt{15} = 4$, then $x + \frac{1}{x} =$

- A. 2
- B. 4
- C. 8
- D. 1

Answer

Given $x + \sqrt{15} = 4$



$$X = 4 - \sqrt{15}$$

$$1/x = 1/(4 - \sqrt{15}) = (4 + \sqrt{15}) / (16 - 15) = 4 + \sqrt{15}$$

$$\text{So, } x + 1/x = 4 - \sqrt{15} + 4 + \sqrt{15} = 8$$

13. Question

$$\text{If } x = \sqrt[3]{2 + \sqrt{3}}, \text{ then } x^3 + \frac{1}{x^3} =$$

A. 2

B. 4

C. 8

D. 9

Answer

$$\text{Given } x = \sqrt[3]{2 + \sqrt{3}}$$

$$= x^3 = 2 + \sqrt{3}$$

$$\text{Similarly, } 1/x^3 = 2 - \sqrt{3}$$

$$X^3 + 1/x^3 = 2 + \sqrt{3} + 2 - \sqrt{3} = 4.$$

14. Question

$$\text{If } x = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \text{ and } y = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}, \text{ then } x + y + xy =$$

A. 9

B. 5

C. 17

D. 7

Answer

$$\text{Given } x = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}, y = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$

$$X = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15}$$

$$Y = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \frac{8 - 2\sqrt{15}}{2} = 4 - \sqrt{15}$$

$$Xy = 4^2 - \sqrt{15}^2 = 16 - 15 = 1$$

So,

$$X + y + xy = 4 + \sqrt{15} + 4 - \sqrt{15} + 1 = 9.$$

15. Question

If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then $x^2 + xy + y^2 =$

- A. 101
- B. 99
- C. 98
- D. 102

Answer

$$\text{Given } x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$X = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 5 - 2\sqrt{6}$$

$$X^2 = (5 - 2\sqrt{6})^2 = 25 + 24 - 20\sqrt{6} = 49 - 20\sqrt{6}$$

$$\text{Similarly } y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 5 + 2\sqrt{6}$$

$$Y^2 = (5 + 2\sqrt{6})^2 = 49 + 20\sqrt{6}$$

$$XY = (5 - 2\sqrt{6})(5 + 2\sqrt{6}) = 25 - 24 = 1$$

$$\text{So, } x^2 + xy + y^2 = 49 - 20\sqrt{6} + 1 + 49 + 20\sqrt{6} = 99.$$

16. Question

The value of $\sqrt{3-2\sqrt{2}}$ is

- A. $\sqrt{2} - 1$
- B. $\sqrt{2} + 1$
- C. $\sqrt{3} - \sqrt{2}$
- D. $\sqrt{3} + \sqrt{2}$

Answer

$$\sqrt{3-2\sqrt{2}}$$

(try to break the terms in form of $(a+b)^2$ or $(a - b)^2$)

$$\sqrt{(\sqrt{2})^2 + 1^2 - 2 \times \sqrt{2} \times 1} = \sqrt{(\sqrt{2}-1)^2} = \sqrt{2} - 1 .$$

17. Question

The value of $\sqrt{3-2\sqrt{2}}$ is

- A. $\sqrt{3} - \sqrt{2}$

- B. $\sqrt{3} + \sqrt{2}$
- C. $\sqrt{5} + \sqrt{5}$
- D. none of these

Answer

$$\sqrt{3 - 2\sqrt{2}}$$

(try to break the terms in form of $(a+b)^2$ or $(a - b)^2$)

$$\sqrt{(\sqrt{2})^2 + 1^2 - 2 \times \sqrt{2} \times 1} = \sqrt{(\sqrt{2}-1)^2} = \sqrt{2} - 1 .$$

18. Question

If $\sqrt{2} = 1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

- A. 0.1718
- B. 5.8282
- C. 0.4142
- D. 2.4142

Answer

Given $\sqrt{2} = 1.4142$

$$\sqrt{(\sqrt{2}-1)/\sqrt{2}+1} = \sqrt{(\sqrt{2}-1)^2} = \sqrt{2} - 1 = 1.4142 - 1 = 0.4142$$

19. Question

If $\sqrt{2} = 1.414$, then the value of $\sqrt{6} - \sqrt{3}$ upto three place of decimal is

- A. 0.235
- B. 0.707
- C. 1.414
- D. 0.471

Answer

Given , $\sqrt{2} = 1.414$

$$\sqrt{6} - \sqrt{3} = \sqrt{2} \times \sqrt{3} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1) = 1.732 (1.414 - 1) = 1.732 \times 0.414 = 0.707$$

20. Question

The positive square of $7 + \sqrt{48}$ is

- A. $7 + 2\sqrt{3}$



B. $7 + \sqrt{3}$

C. $2 + \sqrt{3}$

D. $3 + \sqrt{2}$

Answer

$$7 + \sqrt{48}$$

$$= 7 + \sqrt{(16 \times 3)} = 7 + 4\sqrt{3} \text{ (try to break it in form of } (a+b)^2 \text{)}$$

$$= (2)^2 + (\sqrt{3})^2 + 2 \times 2 \times \sqrt{3} = (2 + \sqrt{3})^2 = (2 + \sqrt{3})(2 + \sqrt{3}).$$

21. Question

$\frac{1}{\sqrt{9} - \sqrt{8}}$ is equal to

A. $3 + 2\sqrt{2}$

B. $\frac{1}{3 + 2\sqrt{2}}$

C. $3 - 2\sqrt{2}$

D. $\frac{3}{2} - \sqrt{2}$

Answer

$$1 / \sqrt{9 - 8}$$

$$= 1 / (\sqrt{9} - \sqrt{8}) \times (\sqrt{9} + \sqrt{8}) / (\sqrt{9} + \sqrt{8})$$

$$= \sqrt{9} + \sqrt{8} = 3 + 2\sqrt{2}$$

22. Question

The value of $\frac{\sqrt{48} + \sqrt{32}}{\sqrt{27} + \sqrt{18}}$ is

A. $\frac{4}{3}$

B. 4

C. 3

D. $\frac{3}{4}$

Answer

$$\sqrt{48} + \sqrt{32} / \sqrt{27} + \sqrt{18}$$

$$= 4\sqrt{3} + 4\sqrt{2} / 3\sqrt{3} + 3\sqrt{2} = (4\sqrt{3} + 4\sqrt{2}) / (3\sqrt{3} + 3\sqrt{2}) \times (3\sqrt{3} - 3\sqrt{2}) / (3\sqrt{3} - 3\sqrt{2})$$

$$= (36 + 12\sqrt{6} - 12\sqrt{6} - 24) / (27 - 18) = 12/9 = 4/3$$

23. Question

If $x = \sqrt{6} + \sqrt{5}$, then $x^2 + \frac{1}{x^2} - 2 =$

- A. $2\sqrt{5}$
- B. $2\sqrt{6}$
- C. 24
- D. 20

Answer

Given $x = \sqrt{6} + \sqrt{5}$

$$x^2 = 11 + 2\sqrt{11}$$

$$1/x^2 = 11 - 2\sqrt{11}$$

$$\text{So, } x^2 + 1/x^2 - 2 = 11 + 2\sqrt{11} + 11 - 2\sqrt{11} - 2 = 22 - 2 = 20.$$

24. Question

If $\sqrt{13 - a\sqrt{10}} = \sqrt{8} + \sqrt{5}$, then $a =$

- A. -5
- B. -6
- C. -4
- D. -2

Answer

$$\sqrt{13 - a\sqrt{10}} = \sqrt{8} + \sqrt{5}$$

Squaring both side,..

$$= 13 - a\sqrt{10} = 8 + 5 + 2 \times \sqrt{8} \times \sqrt{5}$$

$$= 13 - a\sqrt{10} = 13 + 2\sqrt{40}$$

$$= -a\sqrt{10} = 4\sqrt{10}$$

$$= a = -4$$

25. Question

If $\frac{5 - \sqrt{3}}{2 + \sqrt{3}} = x + y\sqrt{3}$, then

- A. $x = 13, y = -7$
- B. $x = -13, y = 7$
- C. $x = -13, y = -7$

D. $x = 13, y = 7$

Answer

$$5 - \sqrt{3} / 2 + \sqrt{3} = x + y\sqrt{3}$$

$$= (5 - \sqrt{3}) / (2 + \sqrt{3}) \times (2 - \sqrt{3}) / (2 - \sqrt{3})$$

$$= (10 - 5\sqrt{3} - 2\sqrt{3} + 3) / (4 - 3)$$

$$= 10 - 7\sqrt{3} + 3$$

$$= 13 - 7\sqrt{3} = x + y\sqrt{3}$$

So, $x = 13, y = -7$



Myclass24
Your Class. Your Pace.