NCERT Solutions for Class-XII Maths

Chapter-5.3

NCERT Math Class 12

Find $\frac{dy}{dx}$ in the following

1. 2x + 3y = sin x

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Differentiating both sides with respect to x, we get

2. 2x + 3y = sin y

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Differentiating both sides with respect to x, we get

$$\frac{d}{dx}(2x)\frac{d}{dx}(3y)=\frac{d}{dx}\sin y \qquad \Rightarrow 2+3\frac{dy}{dx}=\cos y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(\cos y-3)=2 \qquad \Rightarrow \frac{dy}{dx}=\frac{2}{\cos y-3}$$

3. ax + by^2 = cos y

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Differentiating both sides with respect to x, we get

$$\frac{d}{dx}(ax)\frac{d}{dx}(by^3)=\frac{d}{dx}\cos y \qquad \Rightarrow a+2by\frac{dy}{dx}=-\sin y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2by+\sin y)=-a \qquad \Rightarrow \frac{dy}{dx}=-\frac{a}{2by+\sin y}$$

4. xy + y^2 = tan x + y

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Differentiating both sides with respect to x, we get

$$\frac{d}{dx}(xy)+\frac{d}{dx}(y^2)=\frac{d}{dx}\tan x+\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx}+y+2y\frac{dy}{dx}=\sec^2 x+\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(x+2y-1)=\sec^2 x-y \qquad \Rightarrow \frac{dy}{dx}=\frac{\sec^2 x-y}{x+2y-1}$$

5. $x^2 + xy + y^2 = 100$

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Differentiating both sides with respect to x, we get

$$\frac{d}{dx}x^2 + \frac{d}{dx}(xy) + \frac{d}{dx}(100)$$

$$\Rightarrow 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(x+2y) = 2x + y \qquad \Rightarrow \frac{dy}{dx} = \frac{2x+y}{x+2y}$$

6. $x^3 + x^2y + xy^2 + y^3 = 81$

6. $x^3 + x^2y + xy^2 + y^3 = 81$

Differentiating both sides with respect to x, we get

$$\frac{d}{dx}x^3 + \frac{d}{dx}(x^2y) + \frac{d}{dx}(xy^2) + \frac{d}{dx}y^3 = \frac{d}{dx}81$$

$$\Rightarrow 3x^2 + x^2\frac{dy}{dx} + y.2x + x.2y\frac{dy}{dx} + y^2.1 + 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(x^2 + 2xy + 3y^2) = -(3x^2 + 2xy + y^2)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{3x^2 + 2xy + y^2}{x^2 + 2xy + 3y^2}$$

7. $\text{Sin}^2 y + \cos xy = k$

7. It is given that $\sin^2 y + \cos xy = \pi$

Differentiating both sides w.r.t. x, we get,

$$\frac{d}{dx}\left(\sin^2 y + \cos xy\right) = \frac{d}{dx}(\pi)$$

$$\Rightarrow \frac{d}{dx}\left(\sin^2 y\right) + \frac{d}{dx}(xy) = 0$$

$$\Rightarrow 2\sin y.\frac{d}{dx}(\sin y) + (-\cos xy.\frac{d}{dx}(xy) = 0$$

$$\Rightarrow 2\sin y\cos y\frac{dy}{dx} - \sin xy\left[y\frac{d}{dx}(x) + x\frac{dy}{dx}\right] = 0$$

$$\Rightarrow 2\sin y\cos y\frac{dy}{dx} - \sin xy\left[y.1 + x\frac{dy}{dx}\right] = 0$$

$$\Rightarrow 2\sin y\cos y\frac{dy}{dx} - y\sin xy - x\sin xy\frac{dy}{dx} = 0$$

$$\Rightarrow \left(2\sin y\cos y - x\sin xy\right)\frac{dy}{dx} = y\sin xy$$

$$\Rightarrow (\sin 2y - x\sin xy)\frac{dy}{dx} = y\sin xy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\sin xy}{(\sin 2y - x\sin xy)}$$

8. $\sin^2 x + \cos^2 y = 1$

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Differentiating both sides with respect to x, we get

$$\frac{d}{dx}\sin^2 x + \frac{d}{dx}\cos^2 y = \frac{d}{dx}1$$

$$\Rightarrow 2\sin x\cos x + 2\cos y(-\sin y)\frac{dy}{dx} = 0$$

$$\Rightarrow \sin 2x - \sin 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

$$y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

9. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

9. It is given that $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\Rightarrow \sin y = \frac{2x}{1+x^2}$$

Differentiating both sides w.r.t. x, we get,

$$\cos y\frac{dy}{dx} = \frac{(1+x^2).\frac{d}{dx}(2x) - 2x.\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$\Rightarrow \sqrt{1-\sin^2 y}\,\frac{dy}{dx} = \frac{(1+x^2)\times 2 - 2x.2x}{(1+x^2)^2}$$

$$\Rightarrow \sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}\,\frac{dy}{dx} = \left[\frac{(1-x^2)}{(1+x^2)^2}\right]$$

$$\Rightarrow \sqrt{\frac{(1+x^2)^2 - 4x^2}{(1+x^2)^2}}\,\frac{dy}{dx} = \frac{2(1-x^2)}{(1+x^2)^2}$$

$$\Rightarrow \sqrt{\frac{\left(1-x^2\right)^2}{\left(1+x^2\right)^2}}\frac{dy}{dx}=\frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \frac{1-x^2}{1+x^2}\frac{dy}{dx}=\frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2}{1+x^2}$$

10. $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), -\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$

10. $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

Let, x = tan θ

Therefore, $y=\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)=\tan^{-1}(\tan 3\theta)=3\theta=3\tan^{-1}x$

$\Rightarrow y=3\tan^{-1}x$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx}=\frac{3}{1+x^2}$$

11. $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0<x<1$

11. It is given that y = $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$\Rightarrow \cos y=\frac{1-x^2}{1+x^2}$$

$$\Rightarrow \frac{1-\tan^2\frac{y}{2}}{1+\tan^2\frac{y}{2}}=\frac{1-x^2}{1+x^2}$$

On comparing both sides, we get,

$$\tan\frac{y}{2}=x$$

Now, differentiating both sides, we get,

$$\sec^2\left(\frac{y}{2}\right).\frac{d}{dx}\left(\frac{y}{2}\right)=\frac{d}{dx}(x)$$

$$\Rightarrow \sec^2\frac{y}{2}\times\frac{1}{2}\frac{dy}{dx}=1$$

$$\Rightarrow \frac{dy}{dx}=\frac{2}{\sec^2\frac{y}{2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2}{1+\tan^2\frac{y}{2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2}{1+x^2}$$

12. $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0<x<1$

12. $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Let, $x=\tan\theta$

Therefore, $y=\sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$$=\sin^{-1}(\cos2\theta)=\sin^{-1}\left\{\sin\left(\frac{\pi}{2}-2\theta\right)\right\}=\frac{\pi}{2}-2\theta=\frac{\pi}{2}-2\tan^{-1}x$$

$$\Rightarrow y=\frac{\pi}{2}-2\tan^{-1}x$$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx}=0-\frac{2}{1+x^2}=-\frac{2}{1+x^2}$$

13. $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1<x<1$

13. It is given that $y=\cos^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\Rightarrow \cos y=\frac{2x}{1+x^2}$$

Differentiating both sides w.r.t. x, we get,

$$-\sin y\frac{dy}{dx}=\frac{(1+x^2).\frac{d}{dx}(2x)-2x.\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$\Rightarrow \sqrt{1-\cos^2 y}\,\frac{dy}{dx} = \frac{(1+x^2)\times 2 - 2x.2x}{(1+x^2)^2}$$

$$\Rightarrow \sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}\,\frac{dy}{dx} = \left[\frac{(1-x^2)}{(1+x^2)^2}\right]$$

$$\Rightarrow \sqrt{\frac{(1+x^2)^2-4x^2}{(1+x^2)^2}}\,\frac{dy}{dx} = \frac{-2(1-x^2)}{(1+x^2)^2}$$

$$\Rightarrow \sqrt{\frac{(1-x^2)^2}{(1+x^2)^2}}\,\frac{dy}{dx} = \frac{-2(1-x^2)}{(1+x^2)^2}$$

$$\Rightarrow \frac{1-x^2}{1+x^2}\frac{dy}{dx} = \frac{-2(1-x^2)}{(1+x^2)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{1+x^2}$$

14. $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

14. $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Let, $x = \sin\theta$

Therefore, $y = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$= \sin^{-1}(2\sin\theta\cos\theta) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$

$\Rightarrow y = 2\sin^{-1}x$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

15. $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$

15. $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

Let, $x = \cos\theta$

Therefore, $y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta) = 2\theta = 2\cos^{-1}x$

$\Rightarrow y = 2\cos^{-1} x$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = -\frac{2}{\sqrt{1 - x^2}}$$