Exercise 4(B)

1. Represent the following inequalities on real number line:

(i) $2x - 1 < 5$ **(ii) $3x + 1 \geq -5$**

(iii) $2(2x - 3) \leq 6$ **(iv) $-4 < x < 4$**

(v) $-2 \leq x < 5$ **(vi) $8 \geq x > -3$**

(vii) $-5 < x \leq -1$

Solution:

(i) $2x - 1 < 5$

$2x < 6$

$x < 3$

Solution on the number line is as below

(ii) $3x + 1 \geq -5$

$3x \geq -5 - 1$

$3x \geq -6$

$x \geq -2$

Solution on the number line is as below

(iii) $2(2x - 3) \leq 6$

$4x - 6 \leq 6$

$4x \leq 12$

$x \leq 3$

Solution on the number line is as below

(iv) $-4 < x < 4$

Solution on the number line is as below

(v) $-2 \leq x < 5$

Solution on the number line is as below

(vi) $8 \geq x > -3$

Solution on the number line is as below

(vii) $-5 < x \leq -1$

Solution on the number line is as below

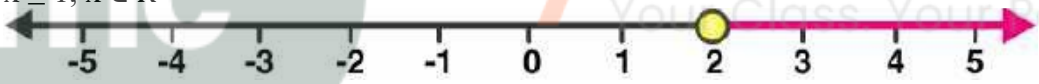
2. For each graph given alongside, write an inequation taking x as the variable:

Solution:

(i)

$x \leq 1, x \in R$

(ii)



$x \geq 2, x \in R$

(iii)

$-4 \leq x < 3, x \in R$

(iv)

$-1 < x \leq 5, x \in R$

3. For the following inequation, graph the solution set on the real number line:

(i) $-4 \leq 3x - 1 < 8$

(ii) $x - 1 < 3 - x \leq 5$

Solution:

(i) $-4 \leq 3x - 1 < 8$

$-4 \leq 3x - 1$ and $3x - 1 < 8$

$-3 \leq 3x$ and $3x < 9$

$-1 \leq x$ and $x < 3$

The solution on the number is as below

(ii) $x - 1 < 3 - x \leq 5$

$x - 1 < 3 - x$ and $3 - x \leq 5$

$2x < 4$ and $-x \leq 2$

$x < 2$ and $x \geq -2$

The solution on the number is as below

4. Represent the solution of each of the following inequations on the real number line:

(i) $4x - 1 > x + 11$

(ii) $7 - x \leq 2 - 6x$

(iii) $x + 3 \leq 2x + 9$

(iv) $2 - 3x > 7 - 5x$

(v) $1 + x \geq 5x - 11$

(vi) $(2x + 5)/3 > 3x - 3$

Solution:

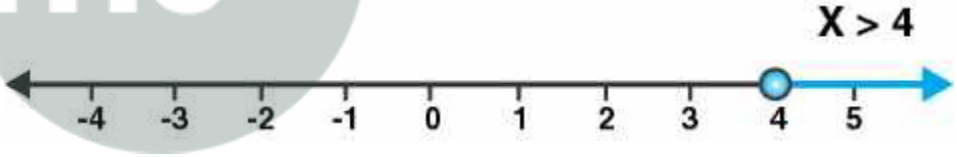
(i) $4x - 1 > x + 11$

$4x - x > 1 + 11$

$3x > 12$

$x > 4$

The solution on number line is as below



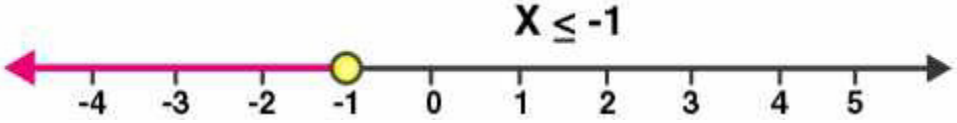
(ii) $7 - x \leq 2 - 6x$

$6x - x \leq 2 - 7$

$5x \leq -5$

$x \leq -1$

The solution on number line is as below



(iii) $x + 3 \leq 2x + 9$

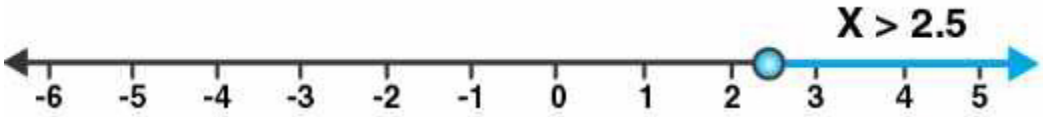
$x - 2x \leq -3 + 9$

$-x \leq 6$

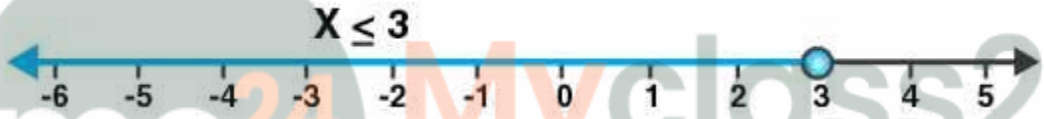
$x \geq -6$

The solution on number line is as below

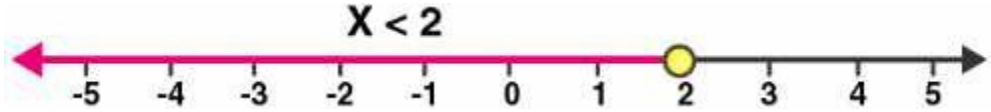
(iv) $2 - 3x > 7 - 5x$
$5x - 3x > 7 - 2$
$2x > 5$
$x > 5/2$
$x > 2.5$
The solution on number line is as below



(v) $1 + x \geq 5x - 11$
$12 \geq 4x$
$x \leq 3$
The solution on number line is as below

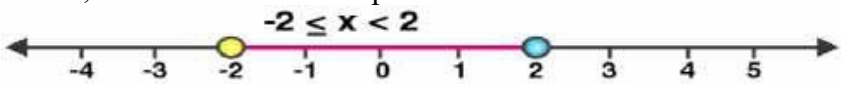


(vi) $(2x + 5)/3 > 3x - 3$
$2x + 5 > 3(3x - 3)$
$2x + 5 > 9x - 9$
$9 + 5 > 9x - 2x$
$7x < 14$
$x < 2$
The solution on number line is as below



5. $x \in$ {real numbers} and $-1 < 3 - 2x \leq 7$, evaluate x and represent it on a number line.
Solution:

$-1 < 3 - 2x \leq 7$
$-1 < 3 - 2x$ and $3 - 2x \leq 7$
$2x < 4$ and $-2x \leq 4$
$x < 2$ and $x \geq -2$
The solution set = $\{-2 \leq x < 2, x \in R\}$
Hence, the solution can be represented on a number line as:



6. List the elements of the solution set of the inequation
$-3 < x - 2 \leq 9 - 2x$; $x \in N$.
Solution:

$-3 < x - 2 \leq 9 - 2x$
$-3 < x - 2$ and $x - 2 \leq 9 - 2x$
$-1 < x$ and $3x \leq 11$
$-1 < x \leq 11/3$
As, $x \in N$
Hence, the solution set = {1, 2, 3}

7. Find the range of values of x which satisfies

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3};\ x \in R.$$

Graph these values of x on the number line.
Solution:

$$-2\frac{2}{3} \leq x + \frac{1}{3} \text{ and } x + \frac{1}{3} < 3\frac{1}{3}$$

$$\Rightarrow -\frac{8}{3} \leq x + \frac{1}{3} \text{ and } x + \frac{1}{3} < \frac{10}{3}$$

$$\Rightarrow -\frac{8}{3} - \frac{1}{3} \leq x \text{ and } x < \frac{10}{3} - \frac{1}{3}$$

$$\Rightarrow -\frac{9}{3} \leq x \text{ and } x < \frac{9}{3}$$

$\Rightarrow -3 \leq x$ and $x < 3$
Therefore, $3 \leq x < 3$
And the required graph of the solution set is as below:

8. Find the values of x, which satisfy the inequation:

$$-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6},\ x \in N.$$

Graph the solution on the number line.
Solution:

Given inequation,

$$-2 \le \frac{1}{2} - \frac{2x}{3} \le 1\frac{5}{6},\ x \in N$$

$$\Rightarrow -2 - \frac{1}{2} \le \frac{1}{2} - \frac{2x}{3} - \frac{1}{2} \le \frac{11}{6} - \frac{1}{2}$$

$$\Rightarrow -\frac{5}{2} \le -\frac{2x}{3} \le \frac{8}{6}$$

$$\Rightarrow -\frac{5}{2} \le -\frac{2x}{3} \text{ and } -\frac{2x}{3} \le \frac{8}{6}$$

$$\Rightarrow -15 \le -4x \text{ and } -2x \le 4$$

$$\Rightarrow 15 \ge 4x \text{ and } 2x \ge -4$$

$$\Rightarrow \frac{15}{4} \ge x \text{ and } x \ge -2$$

$$\Rightarrow 3.75 \ge x \text{ and } x \ge -2$$

Hence, the solution set is $\{x \in N: -2 \le x \le 3.75\}$
And as $x \in N$, the values of x are 1, 2, 3
The required graph of the solution on the number line is as below:

9. Given x ∈ {real numbers}, find the range of values of x for which -5 ≤ 2x - 3 < x + 2 and represent it on a real number line.
Solution:

Given inequation,
$-5 \le 2x - 3 < x + 2$
$-5 \le 2x - 3$ and $2x - 3 < x + 2$
$-2 \le 2x$ and $x < 5$
$-1 \le x$ and $x < 5$
Thus, the required range is $-1 \le x < 5$.
And the required graph is as below:

10. If 5x - 3 ≤ 5 + 3x ≤ 4x + 2, express it as a ≤ x ≤ b and then state the values of a and b.
Solution:

Given inequation,
$5x - 3 \le 5 + 3x \le 4x + 2$
$5x - 3 \le 5 + 3x$ and $5 + 3x \le 4x + 2$
$2x \le 8$ and $-x \le -3$
$x \le 4$ and $x \ge 3$
Hence, $3 \le x \le 4$.

Therefore, we have a = 3 and b = 4.

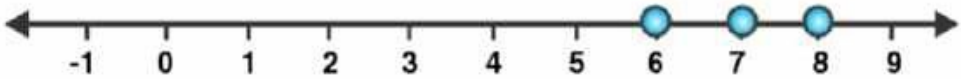
11. Solve the following inequation and graph the solution set on the number line:
$2x - 3 < x + 2 \leq 3x + 5, x \in R$.
Solution:

Given inequation,
$2x - 3 < x + 2 \leq 3x + 5$
$2x - 3 < x + 2$ and $x + 2 \leq 3x + 5$
$x < 5$ and $-3 \leq 2x$
$x < 5$ and $-1.5 \leq x$
So, the solution set = $\{-1.5 \leq x < 5\}$
And the solution set be graphed on the number line as below:

12. Solve and graph the solution set of:
(i) $2x - 9 < 7$ and $3x + 9 \leq 25, x \in R$
(ii) $2x - 9 \leq 7$ and $3x + 9 > 25, x \in I$
(iii) $x + 5 \geq 4(x - 1)$ and $3 - 2x < -7, x \in R$
Solution:

(i) $2x - 9 < 7$ and $3x + 9 \leq 25$
$2x < 16$ and $3x \leq 16$
$x < 8$ and $x \leq 16/3$
Thus, the solution set = $\{x \leq 16/3, x \in R\}$
And the required graph on number line is as below:

(ii) $2x - 9 \leq 7$ and $3x + 9 > 25$
$2x \leq 16$ and $3x > 16$
$x \leq 8$ and $x > 16/3$
Thus, the solution set = $\{16/3 < x \leq 8, x \in I\} = \{6, 7, 8\}$
And the required graph on number line is as below:

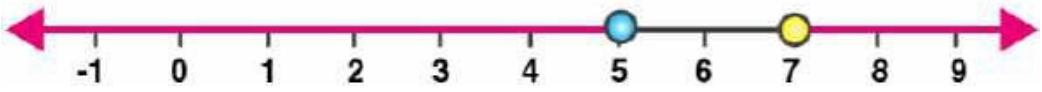


(iii) $x + 5 \geq 4(x - 1)$ and $3 - 2x < -7$
$9 \geq 3x$ and $-2x < -10$
$3 \geq x$ and $x > 5$
Hence, the solution set = Empty set (as there is no cross section)

13. Solve and graph the solution set of:
(i) $3x - 2 > 19$ or $3 - 2x \geq -7, x \in R$

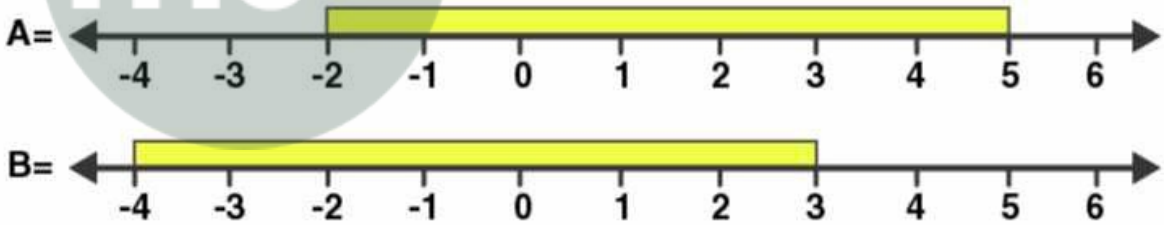
(ii) $5 > p - 1 > 2$ or $7 \leq 2p - \leq 17, p \in R$
Solution:

(i) $3x - 2 > 19$ or $3 - 2x \geq -7$
$3x > 21$ or $-2x \geq -10$
$x > 7$ or $x \leq 5$
The graph of solution set of $x > 7$ or $x \leq 5$ is equal to the graph of points which belong to $x > 7$ or $x \leq 5$ or both.
Thus, the graph of the solution set is as below:



(ii) $5 > p - 1 > 2$ or $7 \leq 2p - 1 \leq 17$
$6 > p > 3$ or $8 \leq 2p \leq 18$
$6 > p > 3$ or $4 \leq p \leq 9$
Now, we have to understand that:
Graph of solution set of $6 > p > 3$ or $4 \leq p \leq 9$
= Graph of points which belong to $6 > p > 3$ or $4 \leq p \leq 9$ or both
= Graph of points which belong to $3 < p \leq 9$
Thus, the graph of the solution set is as below:

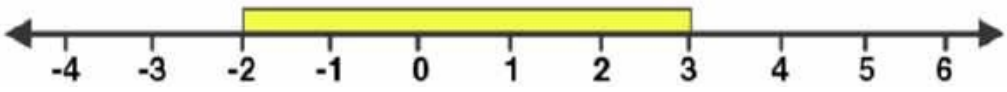
14. The diagram represents two inequations A and B on real number lines:



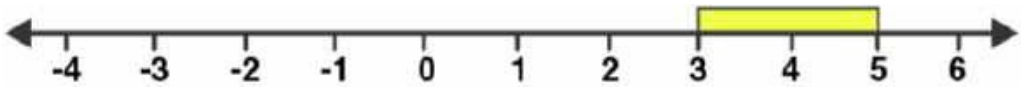
(i) Write down A and B in set builder notation.
(ii) Represent $A \cap B$ and $A \cap B'$ on two different number lines.
Solution:

(i) $A = \{x \in R: -2 \leq x < 5\}$
$B = \{x \in R: -4 \leq x < 3\}$

(ii) $A \cap B = \{x \in R: -2 \leq x < 3\}$
And this can be represented on number line as:

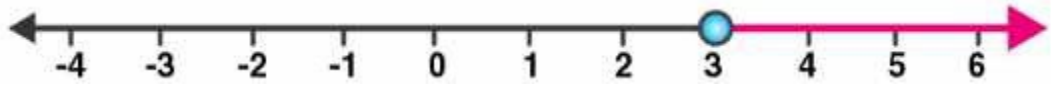


$B' = \{x \in R: 3 < x \leq -4\}$
$A \cap B' = \{x \in R: 3 \leq x < 5\}$
And this can be represented on number line as:



15. Use real number line to find the range of values of x for which:
(i) x > 3 and 0 < x < 6
(ii) x < 0 and -3 ≤ x < 1
(iii) -1 < x ≤ 6 and -2 ≤ x ≤ 3
Solution:

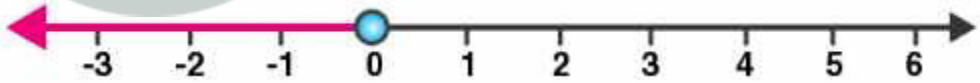
(i) x > 3 and 0 < x < 6
Both the given inequations are true in the range where their graphs on the real number lines overlap.
And, the graphs of these inequations can be drawn as below:
x > 3



0 < x < 6

Thus, from both the graphs, it is clear that their common range is
3 < x < 6

(ii) x < 0 and -3 ≤ x < 1
Both the given inequations are true in the range where their graphs on the real number lines overlap.
And, the graphs of these inequations can be drawn as below:
x < 0



-3 ≤ x < 1

Thus, from both the graphs, it is clear that their common range is
-3 ≤ x < 0

(iii) -1 < x ≤ 6 and -2 ≤ x ≤ 3
Both the given inequations are true in the range where their graphs on the real number lines overlap.
And, the graphs of these inequations can be drawn as below:
-1 < x ≤ 6

-2 ≤ x ≤ 3

Thus, from both the graphs, it is clear that their common range is
$-1 < x \leq 3$

16. Illustrate the set $\{x: -3 \leq x < 0 \text{ or } x > 2, x \in R\}$ on the real number line.
Solution:

We have to understand that:
Graph of solution set of $-3 \leq x < 0$ or $x > 2$ = Graph of points which belong to $-3 \leq x < 0$ or $x > 2$ or both
Therefore, the required graph is as below:

17. Given $A = \{x: -1 < x \leq 5, x \in R\}$ and $B = \{x: -4 \leq x < 3, x \in R\}$
Represent on different number lines:
(i) $A \cap B$
(ii) $A' \cap B$
(iii) $A - B$
Solution:

(i) $A \cap B = \{x: -1 < x < 3, x \in R\}$
And, it can be represented on a number line as:

(ii) Numbers which belong to B but do not belong to A = B - A
$A' \cap B = \{x: -4 \leq x \leq -1, x \in R\}$
And, it can be represented on a number line as:

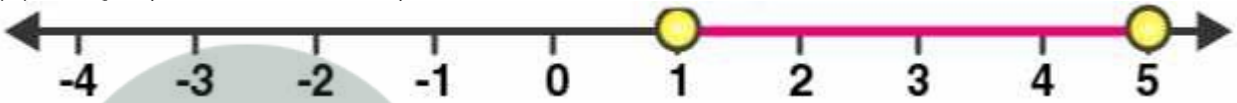
(iii) $A - B = \{x: 3 \leq x \leq 5, x \in R\}$
And, it can be represented on a number line as:

18. P is the solution set of $7x - 2 > 4x + 1$ and Q is the solution set of $9x - 45 \geq 5(x - 5)$; where $x \in R$.
Represent:
(i) $P \cap Q$
(ii) $P - Q$
(iii) $P \cap Q'$ on different number lines.
Solution:

P = {x: 7x – 2 > 4x + 1, x ∈ R}
7x – 2 > 4x + 1
7x – 4x > 1 + 2
3x > 3
x > 1
And,
Q = {x: 9x – 45 ≥ 5(x -5), x ∈ R}
9x – 45 ≥ 5x – 25
9x – 5x ≥ 45 – 25
4x ≥ 20
x ≥ 5

(i) P ∩ Q = {x: x ≥ 5, x ∈ R }

(ii) P - Q = {x: 1 < x < 5, x ∈ R}



(iii) P ∩ Q' = {x: 1 < x < 5, x ∈ R }