NCERT Solutions for Class-XI Maths

Chapter-15 Exercise-15.2

NCERT Math Class 11

1. Find the mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12

1. $6,7,10,12,13,4,8,12$

Mean, $\bar{x} = \frac{\sum_{i=1}^{8} x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

The following table is obtained.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	–3	9
7	–2	4
10	–1	1
12	3	9
13	4	16
4	–5	25
8	–1	1
12	3	9

$$\text{Variance}(\sigma^2) = \frac{1}{n}\sum_{i=1}^{8}(x_i - \bar{x})^2 = \frac{1}{8}\times 74 = 9.25$$

2. Find the mean and variance for the first n natural numbers

2. We know that Mean $= \frac{\text{Sum of all observations}}{\text{Number of observations}}$

$$\therefore \text{Mean, } \bar{x} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

We know that Variance, $\sigma^2 = = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \frac{n+1}{2}\right)^2$$

We know that $(a-b)^2 = a^2 - 2ab + b^2$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i)^2 - \frac{1}{n}\sum_{i=1}^{n}2x_i(\frac{n+1}{2}) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2$$

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} - \frac{n+1}{n}\left[\frac{n(n+1)}{2}\right] + \frac{(n+1)^2}{4n}\times n$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$=(n+1)\left[\frac{4n+2-3n-3}{12}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

We know that $(a + b)(a - b) = a^2 - b^2$

$$\sigma^2 = \frac{n^2-1}{12}$$

Ans. Mean $= \frac{n+1}{2}$ and Variance $= \frac{n^2-1}{12}$

3. Find the mean and variance for the first 10 multiples of 3

3. The first 10 multiples of 3 are

$3, 6, 9, 12, 15, 18, 21, 24, 27, 30$

Here, number of observations, $n = 10$

Mean, $\bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{165}{10} = 16.5$

The following table is obtained.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
3	−13.5	182.25
6	−10.5	110.25
9	−7.5	56.25
12	−4.5	20.25
15	−1.5	2.25

18	1.5	2.25
21	4.5	20.25
24	7.5	56.25
27	10.5	110.25
30	13.5	182.25
		742.5

$$\text{Variance}\left(\sigma^2\right)=\frac{1}{n}\Sigma_{i=1}^{10}\left(x_i-\bar{x}\right)^2=\frac{1}{10}\times 742.5=74.25$$

4. Find the mean and variance for the data

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

4. Presenting the data in the tabular form, we get

x_i	f_i	$f_i x_i$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x}$
6	2	12	6 – 19 = -13	169	338
10	4	40	10 – 19 = -9	81	324
14	7	98	14 – 19 = -5	25	175
18	12	216	18 – 19 = -1	1	12
24	8	192	24 – 19 = 5	25	200
28	4	112	28 – 19 = 9	81	324
30	3	90	30 – 19 = 11	121	363
	$\Sigma f_i = N = 40$	$\Sigma f_i x_i = 760$			$\Sigma f_i(x_i - \bar{x})$ 6

We know that Mean, $\bar{x}=\dfrac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\therefore \bar{x}=\dfrac{760}{40}=19$

We know that Variance, $\sigma^2 = \dfrac{1}{N}\sum_{i=1}^{a} f_i\left(x_i-\bar{x}\right)^2$

$\therefore \sigma^2 = (1/40) \times 1736 = 43.4$

Ans. Mean = 19 and Variance = 43.4

5. Find the mean and variance for the data

xi	92	93	97	98	102	104	109
fi	3	2	3	2	6	3	3

5.

The data is obtained in tabular form as follows.

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
92	3	276	−8	64	192
93	2	186	−7	49	98
97	3	291	−3	9	27
98	2	196	−2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243

Here, $N = 22, \sum_{i=1}^{7} f_i x_i = 2200$

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{7} f_i x_i = \frac{1}{22} \times 2200 = 100$$

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{N}\sum_{i=1}^{7} f_i \left(x_i - \bar{x}\right)^2 = \frac{1}{22} \times 640 = 29.09$$

6. Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

6. Let the assumed mean, A = 64 and h = 1

We obtain the following table from the given data:

x_i	Frequency f_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
60	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12

66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
	$\Sigma f_i = N = 100$			$\Sigma f_i y_i = 0$	$\Sigma f_i y_i^2 = 28$

We know that Mean, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

$\therefore \ \bar{x} = 64 + \frac{0}{100} \times 1 = 64 + 0 = 64$

We know that Variance $\sigma^2 = \frac{h^2}{N^2}\left[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2\right]$

$\therefore \sigma^2 = \frac{1^2}{100^2}\left[100(286) - 0^2\right] = \frac{1}{10000}[28600 - 0] = \frac{28600}{10000} = 2.86$

We know that Standard Deviation = σ

$\therefore \sigma = \sqrt{2.86} = 1.691$

Ans. Mean = 64 and Standard Deviation = 1.691

7. Find the mean and variance for the following frequency distribution.

Classes	0 – 30	30 – 60	60 – 90	90 – 120	120 – 150	150 – 180	180 – 210
Frequencies	2	3	5	10	3	5	2

7.

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 105}{30}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0 – 30	2	15	–3	9	–6	18
30 – 60	3	45	–2	4	–6	12
60 – 90	5	75	–1	1	–5	5
90 – 120	10	105	0	0	0	0
120 – 150	3	165	2	4	10	20
150 – 180	5			1	3	3
180 – 210	2				6	18

$$\text{Mean, } \bar{x} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 105 + \frac{2}{30} \times 30 = 105 + 2 = 107$$

$$\text{Variance}\left(\sigma^2\right) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2 \right]$$

$$= \frac{(30)^2}{(30)^2}\left[30 \times 76 - (2)^2\right]$$

$$= 2280 - 4$$

$$= 2276$$

8.

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

8.

Presenting the data in the tabular form, we get

Classes	Frequency f_i	Midpoint x_i	$f_i x_i$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
0-10	5	5	25	-22	484	2420
10-20	8	15	120	-12	144	1152
20-30	15	25	375	-2	4	60
30-40	16	35	560	8	64	1024
40-50	6	45	270	18	324	1944
	$\Sigma f_i = N = 50$		$\Sigma f_i x_i = 1350$			$\Sigma f_i(x_i - \bar{x})^2 = 6600$

We know that Mean, $\bar{x} = \dfrac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\therefore \bar{x} = \dfrac{1350}{50} = 27$

We know that Variance, $\sigma^2 = \dfrac{1}{N}\sum_{i=1}^{a} f_i\left(x_i - \bar{x}\right)^2$

$\therefore \sigma^2 = (1/50) \times 6600 = 132$

Ans. Mean = 27 and Variance = 132

9. Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	105-115
No. of children	3	4	7	7	15	9	6	6	3

9.

Let the assumed mean, A = 92.5 and h = 5

We obtain the following table from the given data:

Height(Class)	Number of children(Frequency) f_i	Midpoint x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	f_iy_i	$f_iy_i^2$
70-75	3	72.5	-4	16	-12	48
75-80	4	77.5	-3	9	-12	36
80-85	7	82.5	-2	4	-14	28
85-90	7	87.5	-1	1	-7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	$\Sigma f_i = N = 60$				$\Sigma f_iy_i = 6$	$\Sigma f_iy_i^2 = 254$

We know that Mean, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

$\therefore \ \bar{x} = 92.5 + \frac{6}{60} \times 5 = 92.5 + \frac{1}{2} = 92.5 + 0.5 = 93$

We know that Variance $\sigma^2 = \frac{h^2}{N^2}\left[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2\right]$

$\therefore \sigma^2 = \frac{5^2}{60^2}\left[60(254) - 6^2\right] = \frac{1}{144}[15240 - 36] = \frac{15204}{144} = \frac{1267}{12} = 105.583$

We know that Standard Deviation = σ

$\therefore \sigma = \sqrt{105.583} = 10.275$

Ans. Mean = 93, Variance = 105.583 and Standard Deviation = 10.275

10. The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

[Hint: First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 - 48.5, 48.5 - 52.5 and then proceed.]

10. Let the assumed mean, A = 42.5 and h = 4

We obtain the following table from the given data:

Height (Class)	Number of children (Frequency) f_i	Midpoint x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	f_iy_i	$f_iy_i^2$

32.5-36.5	15	34.5	-2	4	-30	60
36.5-40.5	17	38.5	-1	1	-17	17
40.5-44.5	21	42.5	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	25	50.5	2	4	50	100
	$\Sigma f_i = N = 100$				$\Sigma f_i y_i = 25$	$\Sigma f_i y_i^2 = 199$

We know that Mean, $\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$

$\therefore\ \bar{x} = 42.5 + \frac{25}{100} \times 4 = 42.5 + 1 = 43.5$

We know that Variance $\sigma^2 = \frac{h^2}{N^2}\left[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2\right]$

$\therefore \sigma^2 = \frac{4^2}{100^2}\left[100(199) - 25^2\right] = \frac{1}{625}[19900 - 625] = \frac{19275}{625} = \frac{771}{25} = 30.84$

We know that Standard Deviation = σ

$\therefore \sigma = \sqrt{30.84} = 5.553$

Ans. Mean = 43.5, Variance = 30.84 and Standard Deviation = 5.553