NCERT Solutions for Class-XII Maths

Chapter-7.5

NCERT Maths Class 12

1. $\dfrac{x}{(x+1)(x+2)}$

1. Let $\dfrac{x}{(x+1)(x+2)} = \dfrac{A}{(x+1)} + \dfrac{B}{(x+2)}$

$\Rightarrow x = A(x+2) + B(x+1)$

Equating the coefficients of x and constant term, we obtain

A = B = 0

2A + B = 0

On solving, we obtain

A = –1 and B = 2

$\therefore \dfrac{x}{(x+1)(x+2)} = \dfrac{-1}{(x+1)} + \dfrac{2}{(x+2)}$

$\Rightarrow \int \dfrac{x}{(x+1)(x+2)} dx = \int \dfrac{-1}{(x+1)} + \dfrac{2}{(x+2)} dx$

$= \log|x+1|2 + \log|x+2| + C$

$= \log|x+2|^2 - \log|x+1| + C$

$= \log \dfrac{(x+2)^2}{(x+1)} + C$

2. $\dfrac{1}{x^2 - 9}$

2. Now, $\dfrac{1}{x^2-9} = \dfrac{1}{(x+3)(x-3)}$

Let $\dfrac{1}{(x+3)(x-3)} = \dfrac{A}{(x+3)} + \dfrac{B}{(x-3)}$

$\Rightarrow 1 = A(x - 3) + B(x + 3)$

On comparing the coefficients of x and constant term, we get,

A + B = 0

–3A + 3B = 1

On solving above two equations, we get,

$A = -\frac{1}{6} \text{and} B = \frac{1}{6}$

Thus,

$$\frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$\Rightarrow \int \frac{1}{(x+3)(x-3)} dx = \int \left\{ \frac{-1}{6(x+3)} + \frac{1}{6(x-3)} \right\} dx$$

$$= -\frac{1}{6} \log|x+3| + \frac{1}{6}|x-3| + C$$

$$= \frac{1}{6} \log\left|\frac{(x-3)}{(x+3)}\right| + C$$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

3. Let $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$3x - 1 = A(x - 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x - 2)$ …(1)

Equating the coefficients of x^2, x and constant term, we obtain

$A + B + C = 0$

$-5A - 4B - 3C = 3$

$6A + 3B + 2C = -1$

Solving these equations, we obtain

$A = 1$, $B = -5$, and $C = 4$

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)} \right\} dx$$

$$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

4. Let $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$\Rightarrow x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$ ….(1)

Substituting x = 1, 2 and 3 respectively in equation (1) , we get,

$A = \frac{1}{2}, B = -2 \text{and} C = \frac{3}{2}$

Thus,

$$\frac{x}{(x-1)(x-2)(x-3)}=\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{2(x-3)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)}dx=\int\left\{\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{2(x-3)}\right\}dx$$

$$=\frac{1}{2}\log|x-1|-2\log|x-2|+\frac{3}{2}\log|x-3|+C$$

5. $\frac{2x}{x^2+3x+2}$

5. Let $\frac{2x}{x^2+3x+2}=\frac{A}{(x+1)}+\frac{B}{(x+2)}$

$2x=A(x+2)+B(x+1)$

Equating the coefficients of x^2, x and constant term, we obtain

$A+B=2$

$2A+B=0$

Solving these equations, we obtain

$A=-2$ and $B=4$

$$\therefore \frac{2x}{(x+1)(x+2)}=\frac{-2}{(x+1)}+\frac{4}{(x+2)}$$

$$\Rightarrow \int \frac{2x}{(x+1)(x+2)}dx=\int\left\{\frac{4}{(x+2)}-\frac{2}{(x+1)}\right\}dx$$

$$=4\log|x+2|-2\log|x+1|+C$$

6. $\frac{1-x^2}{x(1-2x)}$

6. $\frac{1-x^2}{x(1-2x)}$

On dividing $1-x^2$ by $x(1-2x)$, we get,

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right) \qquad \text{……………….(1)}$$

Now, let $\frac{2-x}{x(1-2x)}=\frac{A}{x}+\frac{B}{(1-2x)}$

$$(2-x)=A(1-2x)+Bx \qquad \text{……………….(2)}$$

Now, substituting $x=0$ and $\frac{1}{2}$ in equation (2), we get,

$A=2$ and $B=3$

Thus, $\frac{2-x}{x(1-2x)}=\frac{2}{x}+\frac{3}{(1-2x)}$

Now, putting this value in equation (2), we get,

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{(1-2x)}\right)$$

$$\Rightarrow\int\frac{1-x^2}{x(1-2x)}dx=\int\left\{\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{(1-2x)}\right)\right\}dx$$

$$=\frac{x}{2}+\log|x|+\frac{3}{2(-2)}\log|1-2x|+C$$

$$=\frac{x}{2}+\log|x|+\frac{3}{4}\log|1-2x|+C$$

7. $\frac{x}{(x^2+1)(x-1)}$

7. Let $\frac{x}{(x^2+1)(x-1)}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x-1)}$

$$x=(Ax+B)(x-1)+C(x^2+1)$$

$$x=Ax^2-Ax+Bx-B+Cx^2+C$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A+C=0$

$-A+B=1$

$-B+C=0$

On solving these equations, we obtain

$A=-\frac{1}{2}, B=\frac{1}{2}, \text{and} C=\frac{1}{2}$

From equation (1), we obtain

$$\therefore\frac{x}{(x^2+1)(x-1)}=\frac{\left(-\frac{1}{2}x+\frac{1}{2}\right)}{x^2+1}+\frac{\frac{1}{2}}{(x-1)}$$

$$\Rightarrow\int\frac{x}{(x^2+1)(x-1)}=-\frac{1}{2}\int\frac{x}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x^2+1}dx\frac{1}{2}\int\frac{1}{x-1}dx$$

$$=\frac{1}{4}\int\frac{2x}{x^2+1}dx+\frac{1}{2}\tan^{-1}x+\frac{1}{2}\log|x-1|+C$$

Consider $\int\frac{2x}{x^2+1}dx$, let $(x^2+1)=t\Rightarrow 2xdx=dt$

$$\Rightarrow\int\frac{2x}{x^2+1}dx=\int\frac{dt}{t}=\log|t|=\log|x^2+1|$$

$$\therefore\int\frac{x}{(x^2+1)(x-1)}=-\frac{1}{4}\log|x^2+1|+\frac{1}{2}\tan^{-1}x+\frac{1}{2}\log|x-1|+C$$

$$=\frac{1}{2}\log|x-1||-\frac{1}{4}\log|x^2+1|+\frac{1}{2}\tan^{-1}x+C$$

8. $\frac{x}{(x-1)^2(x+2)}$

8. Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{((x-1)^2)} + \frac{C}{(x+2)}$

$\Rightarrow\ x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$ ………………(1)

Substituting x = -1 in equation (1) , we get,

$B = \frac{1}{3}$

Equating the coefficients of x^2 and constant term, we get,

A + C = 0

$\Rightarrow -2A + 2B + C = 0$

$A = \frac{3}{9}\text{andc} = \frac{-2}{9}$

Thus,

$\frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3((x-1)^2)} + \frac{2}{9(x+2)}$

$\Rightarrow \int \frac{x}{(x-1)^2(x+2)}dx = \int\left\{\frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{2}{9(x+2)}\right\}dx$

$= \frac{2}{9}\int\frac{1}{(x-1)}dx + \frac{1}{3}\int\frac{1}{(x-1)^2}dx - \frac{2}{9}\int\frac{1}{(x+2)}dx$

$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) - \frac{2}{9}\log|x+2| + C$

$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| + \frac{1}{3}\left(\frac{-1}{x-1}\right) + C$

9. $\frac{3x+5}{x^3-x^2-x+1}$

9. $\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)^2(x+1)}$

Let $\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$

$3x + 5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$3x + 5 = A(x^2-1) + B(x+1) + C(x^2+1-2x)$ ….(1)

Equating the coefficients of x^2, x and constant term, we obtain

A + C = 0

B – 2C = 3

–A + B + C = 5

On solving, we obtain

B = 4

$A=-\frac{1}{2}$ and $C=\frac{1}{2}$

$$\therefore \frac{3x+5}{(x-1)^2(x+1)}=\frac{-1}{2(x-1)}+\frac{4}{(x-1)^2}+\frac{1}{2(x+1)}$$

$$\Rightarrow \int \frac{3x+5}{(x-2)^2(x+1)}dx=-\frac{1}{2}\int\frac{1}{x-1}dx+4\int\frac{1}{(x-1)^2}dx+\frac{1}{2}\int\frac{1}{(x+1)}dx$$

$$=-\frac{1}{2}\log|x-1|+4\left(\frac{-1}{x-1}\right)+\frac{1}{2}\log|x+1|+C$$

$$=\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{(x-1)}+C$$

10. $\frac{2x-3}{(x^2-1)(2x+3)}$

10. $\frac{2x-3}{\left(x^2-1\right)\left(2x+3\right)}=\frac{2x-3}{(x+1)(x-1)\left(2x+3\right)}$

Let $\frac{2x-3}{(x+1)(x-1)\left(2x+3\right)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}+\frac{C}{(2x+3)}$

$$\Rightarrow 2x-3=A(x-1)(2x+3)+B(x+1)(2x+3)+C(x+1)(x-1)$$

$$\Rightarrow 2x-3=A\left(2x^2+x-3\right)+B\left(2x^2+5x+3\right)+C\left(x^2-1\right)$$

$$\Rightarrow 2x-3=(2A+2B+C)x^2+(A+5B)x+(-3A+3B-C)$$

Equating the coefficients of x^2 and x, we get,

$B=-\frac{1}{10}, A=\frac{5}{2}$ and $C=-\frac{24}{5}$

Thus,

$$\frac{2x-3}{(x+1)(x-1)\left(2x+3\right)}=\frac{5}{2(x+1)}-\frac{1}{10\left(x-1\right)}-\frac{24}{5(2x+3)}$$

$$\Rightarrow \int\frac{2x-3}{(x+1)(x-1)\left(2x+3\right)}dx=\int\left\{\frac{5}{2(x+1)}-\frac{1}{10\left(x-1\right)}-\frac{24}{5(2x+3)}\right\}dx$$

$$=\frac{5}{2}\int\frac{1}{(x+1)}dx-\frac{1}{10}\int\frac{1}{(x-1)}dx-\frac{24}{5}\int\frac{1}{(2x+3)}dx$$

$$=\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{24}{5\times 2}\log|2x+3|+C$$

$$=\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{24}{10}\log|2x+3|+C$$

11. $\dfrac{5x}{(x+1)(x^2-4)}$

11. $\dfrac{5x}{(x+1)(x^2-4)} = \dfrac{5x}{(x+1)(x+2)(x-2)}$

Let $\dfrac{5x}{(x+1)(x+2)(x-2)} = \dfrac{A}{(x+1)} + \dfrac{B}{(x+2)} + \dfrac{C}{(x-2)}$

$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2)$ …(1)

Equating the coefficients of x^2, x and constant, we obtain

$A + B = C = 0$

$-B + 3C = 5$ and

$-4A - 2B + 2C = 0$

On solving, we obtain

$A = \dfrac{5}{3}, B == -\dfrac{5}{3}, \text{and } C = \dfrac{5}{6}$

$\therefore \dfrac{5x}{(x+1)(x+2)(x-2)} = \dfrac{5}{3(x+1)} - \dfrac{5}{2(x+2)} + \dfrac{5}{6(x-2)}$

$\Rightarrow \int \dfrac{5x}{(x+1)(x^2-4)}dx = \dfrac{5}{3}\int \dfrac{1}{(x+1)}dx - \dfrac{5}{2}\int \dfrac{1}{(x+2)}dx + \dfrac{5}{6}\int \dfrac{1}{(x-2)}dx$

$= \dfrac{5}{3}\log|x+1| - \dfrac{5}{2}\log|x+2| + \dfrac{5}{6}\log|x-2| + C$

12. $\dfrac{x^3+x+1}{x^2-1}$

12. It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(x^3 + x + 1)$ by $x^2 - 1$, we obtain

$\dfrac{x^3+x+1}{x^2-1} = x + \dfrac{2x+1}{x^2-1}$

Let $\dfrac{2x+1}{x^2-1} = \dfrac{A}{(x+1)} + \dfrac{B}{(x-1)}$

$2x + 1 = A(x-1) + B(x+1)$ …(1)

Equating the coefficients of x and constant, we obtain

$A + B = 2$

$-A + B = 1$

On solving, we obtain

$A = \dfrac{1}{2} \text{ and } B = \dfrac{3}{2}$

$$\therefore \frac{x^3+x+1}{x^2-1}=x+\frac{1}{2(x+1)}+\frac{3}{2(x-1)}$$

$$\Rightarrow \int \frac{x^3+x+1}{x^2-1}dx \int x dx+\frac{1}{2}\int \frac{1}{(x+1)}dx+\frac{3}{2}\int \frac{1}{(x-1)}dx$$

$$=\frac{x^2}{2}+\frac{1}{2}\log|x+1|+\frac{3}{2}\log|x-1|+C$$

13. $\frac{2}{(1-x)(1+x^2)}$

13. Let $\frac{2}{(1-x)\left(1+x^2\right)}=\frac{A}{(1-x)}+\frac{B}{(1+x^2)}$

$$\Rightarrow 2 = A\left(1 - x^2\right) + (Bx + C)(1 - x)$$

$$\Rightarrow 2 = A + Ax^2 + Bx - Bx^2 + C - Cx$$

On comparing the coefficients of x2, x and constant term, we get,

A – B = 0

B – C = 0

A + C = 0

On solving these equations, we get,

A = 1, B =1 and C = 1

Thus,

$$\frac{2}{(1-x)\left(1+x^2\right)}=\frac{1}{(1-x)}+\frac{x+1}{(1+x^2)}$$

$$\Rightarrow \int \frac{2}{(1-x)\left(1+x^2\right)}dx=\int \frac{1}{(1-x)}dx+\int \frac{x}{(1+x^2)}dx+\int \frac{1}{(1+x^2)}dx$$

$$=-\int \frac{1}{(x-1)}dx+\frac{1}{2}\int \frac{2x}{(1+x^2)}dx+\int \frac{1}{(1+x^2)}dx=-\log|x-1|+\frac{1}{2}\log\left|1+x^2\right|+\tan^{-1}x+C$$

14. $\frac{3x-1}{(x+2)^2}$

14. Let $\frac{3x-1}{(x+2)^2}=\frac{A}{(x+2)}+\frac{B}{(x+2)^2}$

$$\Rightarrow 3x-1=A(x+2)+B$$

$$\therefore \frac{3x-1}{(x+2)^2}=\frac{3}{(x+2)}-\frac{7}{(x+2)^2}$$

$$\Rightarrow \int \frac{3x-1}{(x+2)^2}dx=3\int \frac{1}{(x+2)}dx-7\int \frac{x}{(x+2)^2}dx$$

$$= 3\log|x+2| - 7\left(\frac{-1}{(x+2)}\right) + C$$

$$= 3\log|x+2| + \frac{7}{(x+2)} + C$$

15. $\frac{1}{x^4 - 1}$

15. $\frac{1}{x^4 - 1} = \frac{1}{\left(x^2 - 1\right)\left(x^2 + 1\right)} = \frac{1}{(x+1)(x-1)\left(x^2+1\right)}$

Let $\frac{1}{(x+1)(x-1)\left(x^2+1\right)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{Cx+D}{\left(x^2+1\right)}$

$$1 = A(x-1)\left(x^2+1\right) + B(x+1)\left(x^2+1\right) + (Cx + D)\left(x^2 - 1\right)$$

$$1 = A\left(x^3 + x - x^2 - 1\right) + B\left(x^3 + x + x^2 + 1\right) + Cx^3 + Dx^2 - Cx - D$$

$$1 = (A + B + C)x^3 + (-A + B + D)x^2 + (A + B - C)x + (-A + B - D)$$

Equating the coefficients of x3, x2, x and constant term, we get,

(A + B + C) = 0

(–A + B + C) = 0

(A + B - C) = 0

(–A + B - D) = 0

On solving these equations, we get,

$A = -\frac{1}{4}, B = \frac{1}{4}, C = 0$ and $D = -\frac{1}{2}$

Therefore,

$$\frac{1}{x^4 - 1} = \frac{-1}{4(x+1)} + \frac{1}{4(x-1)} - \frac{1}{2(x^2+1)}$$

$$\Rightarrow \int \frac{1}{x^4 - 1}dx = \int \left\{\frac{-1}{4(x+1)} + \frac{1}{4(x-1)} - \frac{1}{2(x^2+1)}\right\}dx$$

$$= -\frac{1}{4}\log|x-1| + \frac{1}{4}\log|x-1| - \frac{1}{2}\tan^{-1}x + C$$

$$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + C$$

16. $\frac{1}{x(x^n + 1)}$

[Hint: multiply numerator and denominator by x^{n-1} and put $x^n = t$]

16. $\dfrac{1}{x(x^n+1)}$

Multiplying numerator and denominator by x^{n-1}, we obtain

$$\frac{1}{x(x^n+1)}=\frac{x^{n-1}}{x^{n-1}x(x^n+1)}=\frac{x^{n-1}}{x^n(x^n+1)}$$

Let $x^n = t \Rightarrow x^{n-1}\,dx = dt$

$$\int\frac{1}{x(x^n+1)}dx=\int\frac{x^{n-3}}{x^n(x^n+1)}dx=\frac{1}{n}\int\frac{1}{t(t+1)}dt$$

$$\text{Let}\frac{1}{t(t+1)}=\frac{A}{t}+\frac{B}{(t+1)}$$

1 = A (a + t) + Bt

Equating the coefficients of t and constant, we obtain

A = 1 and B = –1

$$\therefore\frac{1}{t(t+1)}=\frac{1}{t}-\frac{1}{(1+t)}$$

$$\Rightarrow\int\frac{1}{x(x^n+1)}dx=\frac{1}{n}\int\left[\frac{1}{t}-\frac{1}{(t+1)}\right]dx$$

$$=\frac{1}{n}\left[\log|t|-\log|t+1|\right]+C$$

$$=-\frac{1}{n}\left[\log|x^n|-\log|x^n+1|\right]+C$$

$$=\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+C$$

17. $\dfrac{\cos x}{(1-\sin x)(2-\sin x)}$ [Hint: Put sin x = t]

17. $\dfrac{\cos x}{(1-\sin x)(2-\sin x)}$

Let sinx = t

$\Rightarrow$ cosx dx = dt

Therefore,

$$\int\frac{\cos x}{(1-\sin x)(2-\sin x)}dx=\int\frac{dt}{(1-t)(2-t)}$$

$$\text{Let}\frac{1}{(1-t)(2-t)}=\frac{A}{(1-t)}+\frac{B}{(2-t)}$$

$1 = A(2 - t) + B(1 - t)$………….(1)

Substituting t = 2 and then t = 1 in equation (1), we get,

Therefore,

$$\frac{1}{(1-t)(2-t)}=\frac{1}{(1-t)}+\frac{1}{(2-t)}$$

$$\Rightarrow\int\frac{\cos x}{(1-\sin x)(2-\sin x)}dx=\int\left\{\frac{1}{(1-t)}+\frac{1}{(2-t)}\right\}dt$$

$$=-\log|1-t|+\log|2-t|+C$$

$$=\log\left|\frac{2-t}{1-t}\right|+C$$

$$=\log\left|\frac{2-\sin x}{1-\sin x}\right|+C$$

18. $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$

18. $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}=1-\frac{(4x^2+10)}{(x^2+3)(x^2+4)}$

Let $\frac{4x^2+10}{(x^2+3)(x^2+4)}=\frac{Ax+B}{(x^2+3)}+\frac{Cx+D}{(x^2+4)}$

$4x^2 + 10 = (Ax + B)(x^2 + 4) + (Cx + D)(x^2 + 3)$

$4x^2 + 10 = Ax^3 + 4Ax + Bx^2 + 4B + 4C^3 + 3Cx + Dx^2 + 3D$

$4x^2 + 10 = (A + C)x^3 + (B + D)x^2 + (4A + 3C)x + (4B + 3D)$

Equating the coefficients of x3, x2, x, and constant term, we obtain

$A + C = 0$

$B + D = 4$

$4A + 3C = 0$

$4B + 3D = 10$

On solving these equations, we obtain

$A = 0$, $B = -2$, $C = 0$, and $D = 6$

$$\therefore\frac{4x^2+10}{(x^2+3)(x^2+4)}=\frac{-2}{(x^2+3)}+\frac{6}{(x^2+4)}$$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}=1-\left(\frac{-2}{(x^2+3)}+\frac{6}{(x^2+4)}\right)$$

$$\Rightarrow\int\frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2+4)}dx=\int\left\{1+\frac{2}{(x^2+3)}-\frac{6}{(x^2+4)}\right\}$$

$$=\int\left\{1+\frac{2}{x^2+(\sqrt{3})^2}-\frac{6}{x^2+2^2}\right\}$$

$$=x+2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right)-6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right)+C$$

$$= x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - 3\tan^{-1} \frac{x}{2} + C$$

19. $\frac{2x}{(x^2+1)(x^2+3)}$

19. Now, $\frac{2x}{(x^2+1)(x^2+3)}$

Now, let $x^2 = t$

$\Rightarrow 2xdx = dt$

Thus, $\int \frac{2x}{(x^2+1)(x^2+3)} dx = \int \frac{dt}{(t+1)(t+3)}$ ....(1)

Let $\frac{1}{(t+1)(t+3)} = \frac{A}{t+1} + \frac{B}{t+3}$

$1 = A(t + 3) + B(t + 1)$ ....(2)

Substituting t = –3 and –1 in (2), we get

$A = \frac{1}{2}$ and $B = -\frac{1}{2}$

Therefore, $\frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$

$\Rightarrow \int \frac{2x}{(x^2+1)(x^2+3)} dx = \int \left\{ \frac{1}{2(t+1)} - \frac{1}{2(t+3)} \right\} dt$

$= \frac{1}{2} \log|(t+1)| - \frac{1}{2} \log|t+3| + C$

$= \frac{1}{2} \log\left|\frac{t+1}{t+3}\right| + C$

$= \frac{1}{2} \log\left|\frac{x^2+1}{x^2+3}\right| + C$

20. $\frac{1}{x(x^4-1)}$

20. $\frac{1}{x(x^4-1)}$

Multiplying numerator and denominator by x^3, we obtain

$\frac{1}{x(x^4-1)} = \frac{x^3}{x^4(x^4-1)}$

$\therefore \int \frac{1}{x(x^4-1)} dx = \int \frac{x^3}{x^4(x^4-1)} dx$

Let $x^4 = t \Rightarrow 4x^3\,dx = dt$

$1 = A\,(t - 1) + Bt$ ....(1)

Equating the coefficients of t and constant, we obtain

$A = -1$ and $B = 1$

$$\Rightarrow \frac{1}{t(t+1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{x(x^4-1)}dx = \frac{1}{4}\int\left\{\frac{-1}{t} + \frac{1}{t-1}\right\}dt$$

$$= \frac{1}{4}\left[-\log|t| + \log|t-1|\right] + C$$

$$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C$$

$$= \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$$

21. $\frac{1}{(e^x-1)}$ [Hint: Put $e^x = t$]

21. Now, $\frac{1}{\left(e^x-1\right)}$

Let $e^x = t$

$\Rightarrow e^x dx = dt$

$$\Rightarrow \int \frac{1}{\left(e^x-1\right)}dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)}dt$$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$

$1 = A(t - 1) + Bt$ ……………….(1)

Substituting t =1 and t = 0 in equation (1), we get,

$A = -1$ and $B = 1$

Therefore, $\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$

$$\Rightarrow \int \frac{1}{t(t-1)}dt = \log\left|\frac{t-1}{t}\right| + C$$

$$= \log\left|\frac{e^x-1}{e^x}\right| + C$$

22. $\int \frac{x\,dx}{(x-1)(x-2)}$ equals

(a) $\log\left|\frac{(x-1)^2}{x-2}\right|+C$ (b) $\log\left|\frac{(x-2)^2}{x-1}\right|+C$

(c) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$ (d) $\log|(x-1)(x-2)|+C$

22. Let $\frac{x}{(x-1)(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}$

$x=A(x-2)+B(x-1)$ **...(1)**

Equating the coefficients of x and constant, we obtain

A = – 1 and B = 2

$\therefore\frac{x}{(x-1)(x-2)}=-\frac{1}{(x-1)}+\frac{2}{(x-2)}$ $\Rightarrow\int\frac{x}{(x-1)(x-2)}dx=\int\left\{\frac{-1}{(x-1)}+\frac{2}{(x-2)}\right\}dx$

$=-\log|x-1|+2\log|x-2|+C$ $=\log\left|\frac{(x-2)^2}{x-1}\right|+C$

Hence, the correct Answer is B.

23. $\int\frac{dx}{x(x^2+1)}$ equals

(a) $\log|x|-\frac{1}{2}\log(x^2+1)+C$ (b) $\log|x|+\frac{1}{2}\log(x^2+1)+C$

(c) $-\log|x|+\frac{1}{2}\log(x^2+1)+C$ (d) $\frac{1}{2}\log|x|+\log(x^2+1)+C$

23. Let $\frac{1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$

$1 = A(X^2+1) + (Bx + C)$

Equating the coefficients of x^2, x and constant term, we get,

A + B = 0

C = 0

A = 1

On solving these equations, we get,

A = , B = -1 and C = 0

Therefore, $\frac{1}{x(x^2+1)}=\frac{1}{x}+\frac{-x}{x^2+1}$

$\Rightarrow\int\frac{1}{x(x^2+1)}=\int\left\{\frac{1}{x}+\frac{-x}{x^2+1}\right\}dx$

$=\log|x|-\frac{1}{2}\log\left|x^2+1\right|+C$