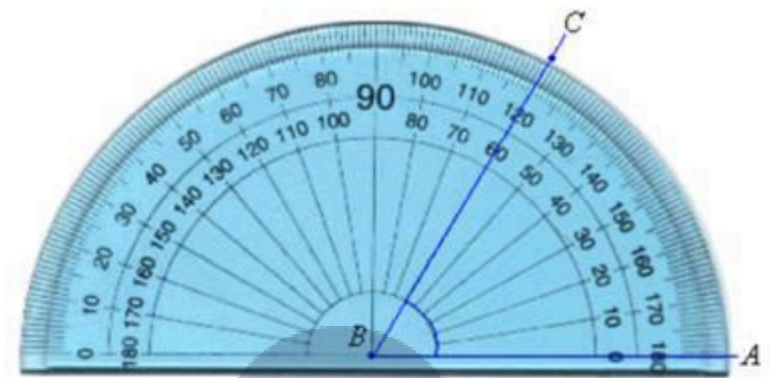
Measurement Of Angles

Exercise 14

Q. 1. A. Using a protractor, draw each of the following angles.

60°

Answer :



• Draw a straight line AB.

• Place a dot at B. This dot represents the vertex of the angle.

• Place the centre of the protractor at B and the baseline of the protractor along the arm BA.

• Find 60° on the scale and mark a small dot at the edge of the protractor.

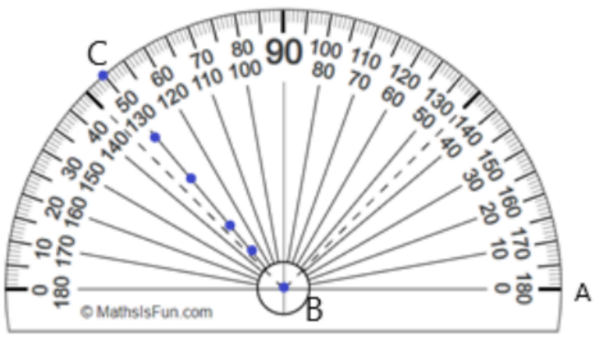
• Join the vertex B to the small dot with a ruler to form the second arm, BC, of the angle.

• Mark the angle with a small arc as shown below.

Q. 1. B. Using a protractor, draw each of the following angles.

130°

Answer :



• Draw a straight line AB.

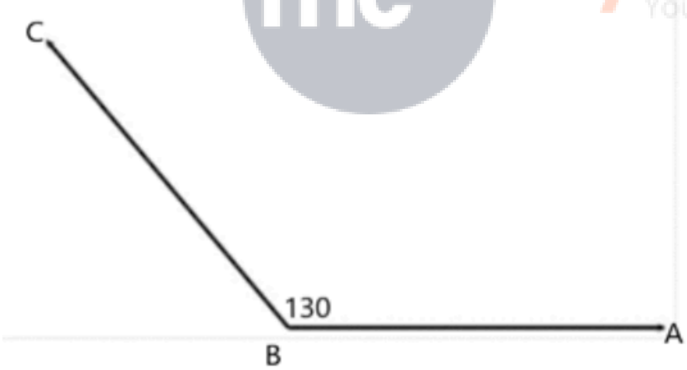
• Place a dot at B. This dot represents the vertex of the angle.

• Place the centre of the protractor at B and the baseline of the protractor along the arm BA.

• Find 130° on the scale and mark a small dot at the edge of the protractor.

• Join the vertex B to the small dot with a ruler to form the second arm, BC, of the angle.

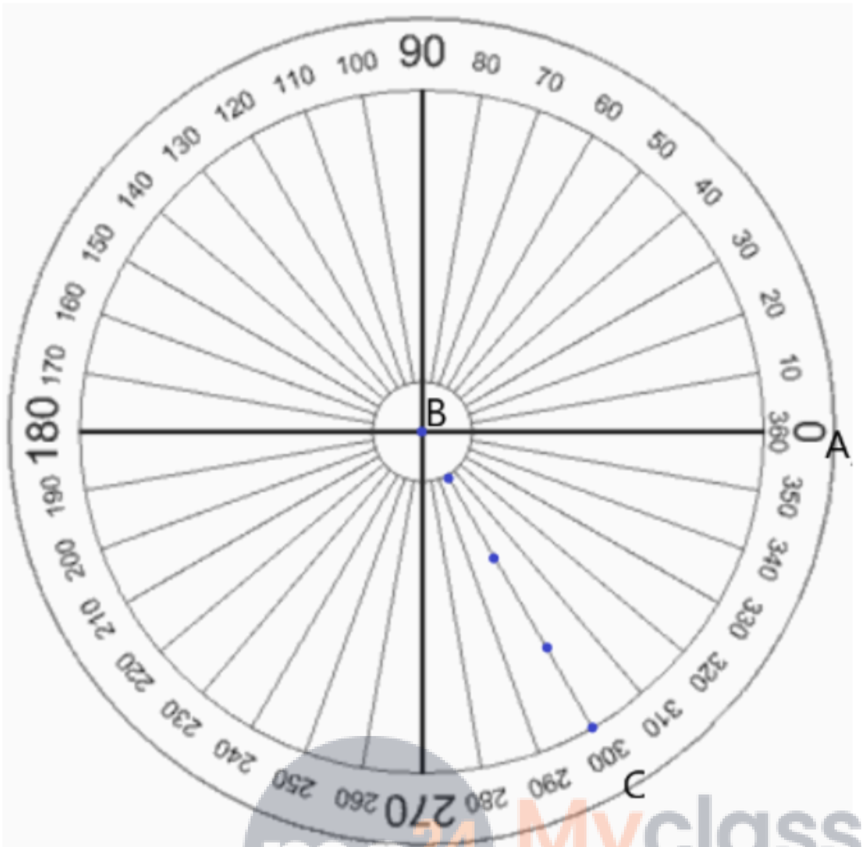
• Mark the angle with a small arc as shown below.



Q. 1. C. Using a protractor, draw each of the following angles.

300°

Answer :



• Draw a straight line AB.

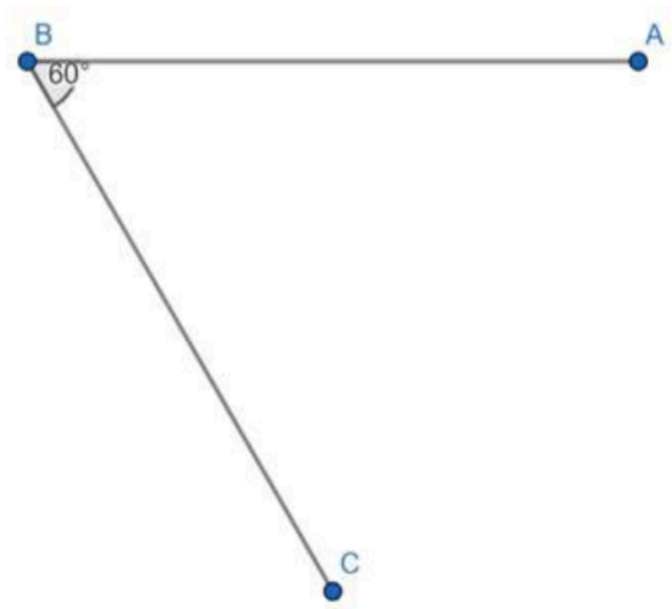
• Place a dot at B. This dot represents the vertex of the angle.

• Place the centre of the protractor at B and the baseline of the protractor along the arm BA.

• Find 300° on the scale and mark a small dot at the edge of the protractor.

• Join the vertex B to the small dot with a ruler to form the second arm, BC, of the angle.

• Mark the angle with a small arc as shown below.



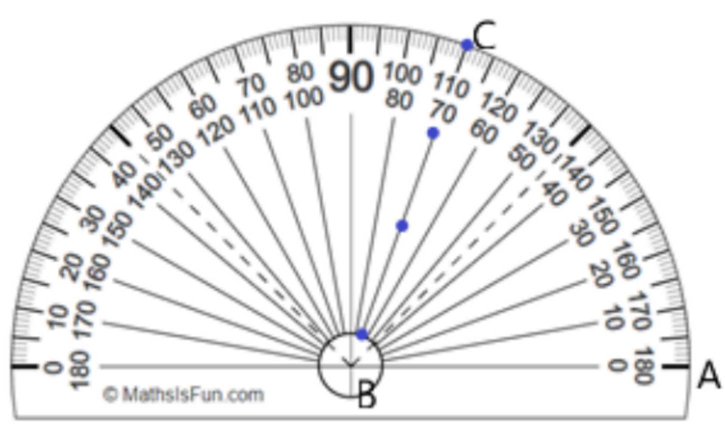
Q. 1. D. Using a protractor, draw each of the following angles.

430°

Answer : The given angle is greater than 360°

Adding or subtracting 360° from a particular angle does'nt changes its position.

Therefore, Angle can also be written at as = 430° – 360° = 70°



• Draw a straight line AB.

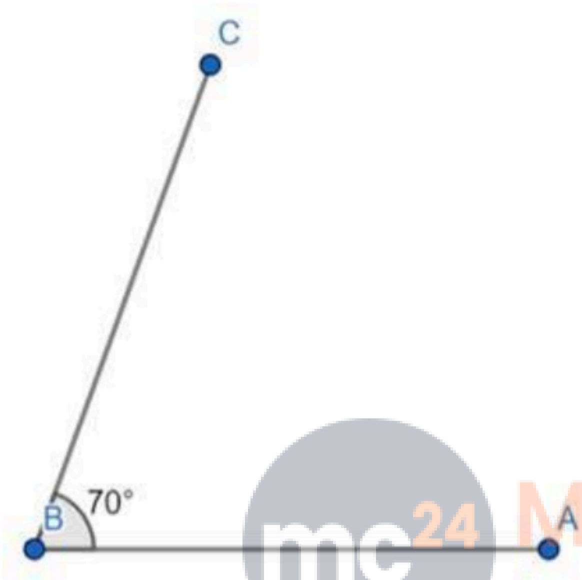
• Place a dot at B. This dot represents the vertex of the angle.

• Place the centre of the protractor at B and the baseline of the protractor along the arm BA.

• Find 70° on the scale and mark a small dot at the edge of the protractor.

• Join the vertex B to the small dot with a ruler to form the second arm, BC, of the angle.

• Mark the angle with a small arc as shown below.



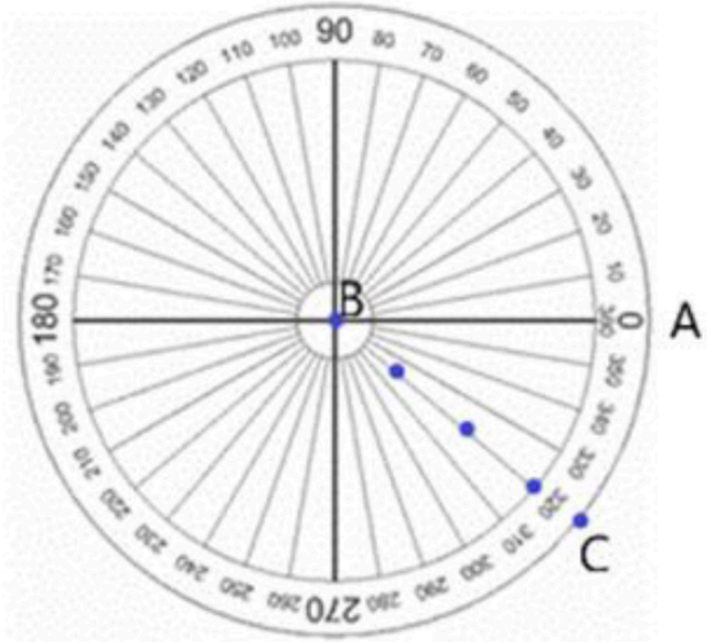
Q. 1. E. Using a protractor, draw each of the following angles.

-40°

Answer : The given angle is negative

Adding or subtracting 360° from a particular angle does'nt changes its position.

Therefore, Angle can also be written as=-40° + 360° = 320°



• Draw a straight line AB.

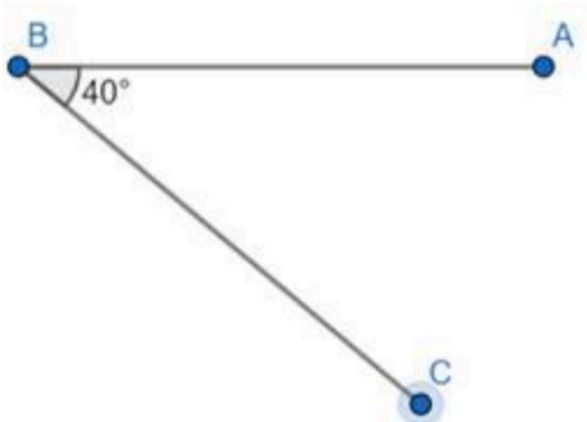
• Place a dot at B. This dot represents the vertex of the angle.

• Place the centre of the protractor at B and the baseline of the protractor along the arm BA.

• Find 320° on the scale and mark a small dot at the edge of the protractor.

• Join the vertex B to the small dot with a ruler to form the second arm, BC, of the angle.

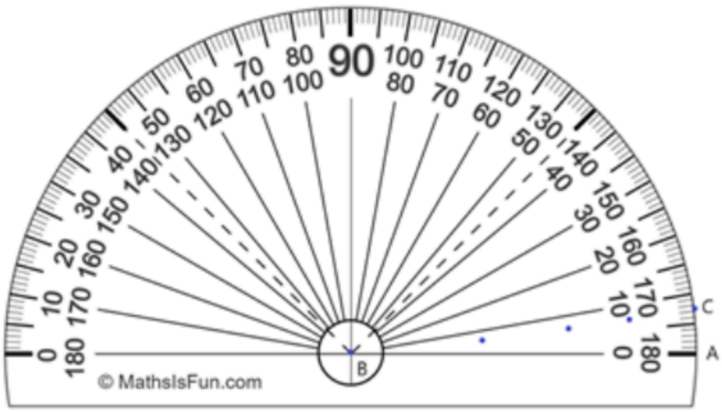
• Mark the angle with a small arc as shown below.



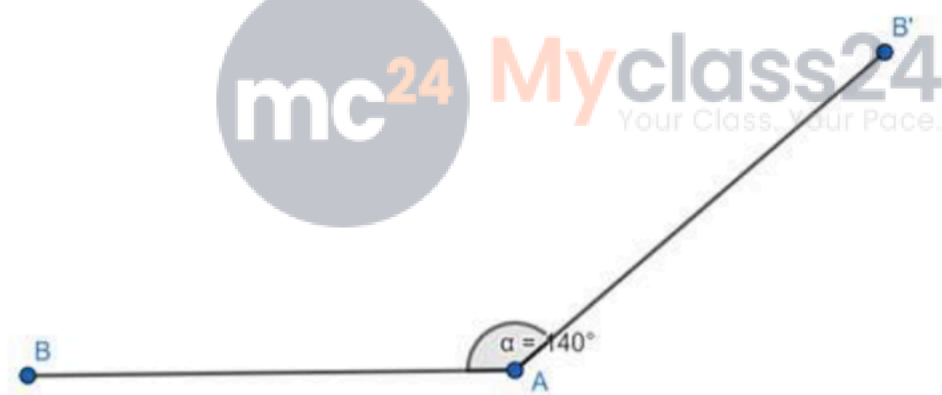
Q. 1. F. Using a protractor, draw each of the following angles.

-220°

Answer : Given angle can be completely written in degree as = -220°



-220° = 360° - 220° = 140°



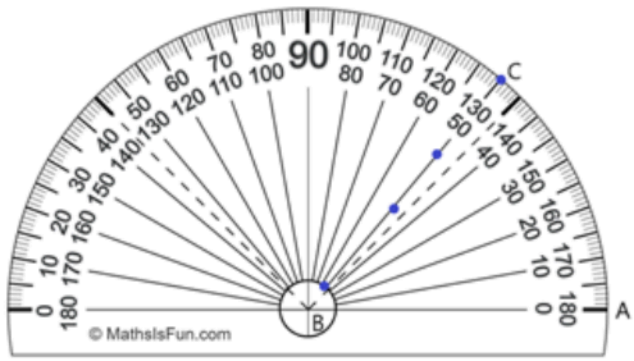
Q. 1. G. Using a protractor, draw each of the following angles.

-310°

Answer : The given angle is negative

Adding or subtracting 360° from a particular angle does’nt changes its position.

Therefore, Angle can also be written as=-310° + 360° = 50°



• Draw a straight line AB.

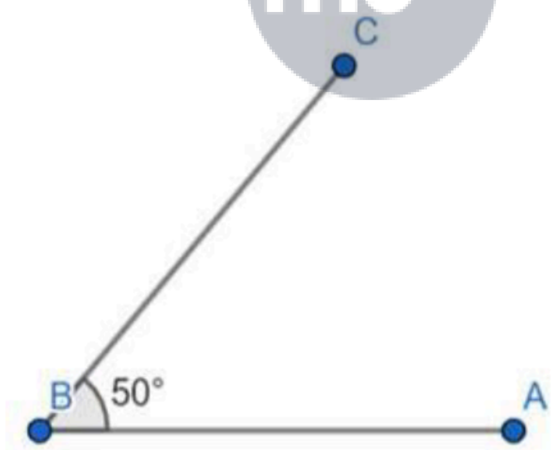
• Place a dot at B. This dot represents the vertex of the angle.

• Place the centre of the protractor at B and the baseline of the protractor along the arm BA.

• Find 50° on the scale and mark a small dot at the edge of the protractor.

• Join the vertex B to the small dot with a ruler to form the second arm, BC, of the angle.

• Mark the angle with a small arc as shown below.



Q. 1. H. Using a protractor, draw each of the following angles.

-400°

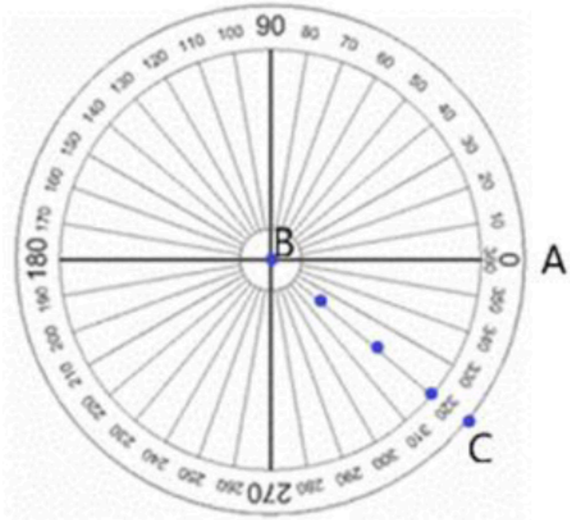
Answer : The given angle is negative

Adding or subtracting 360° from a particular angle does’nt changes its position.

Therefore, Angle can also be written as=-400° + 360° = -40°

The angle is still negative, so we will further add 360° to it.

Therefore, Angle can also be written as=-40° + 360°=320°



• Draw a straight line AB.

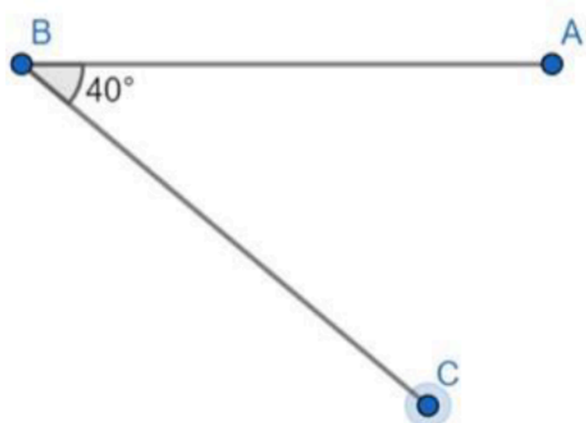
• Place a dot at B. This dot represents the vertex of the angle.

• Place the centre of the protractor at B and the baseline of the protractor along the arm BA.

• Find 320° on the scale and mark a small dot at the edge of the protractor.

• Join the vertex B to the small dot with a ruler to form the second arm, BC, of the angle.

• Mark the angle with a small arc as shown below.



Q. 1. Express each of the following angles in radians

36°

Answer : Formula : Angle in radians = Angle in degrees × $\frac{\pi}{180}$

Therefore, Angle in radians =. $36 \times \frac{\pi}{180} = \frac{\pi}{5}$

Q. 2. A. Express each of the following angles in radians

120°

Answer : Formula : Angle in radians = Angle in degrees × $\frac{\pi}{180}$

Therefore, Angle in radians = $120 \times \frac{\pi}{180} = \frac{2\pi}{3}$

Q. 2. C. Express each of the following angles in radians

225°

Answer : Formula : Angle in radians = Angle in degrees × $\frac{\pi}{180}$

Therefore, Angle in radians = $225 \times \frac{\pi}{180} = \frac{5\pi}{4}$

Q. 2. D. Express each of the following angles in radians

330°

Answer : Formula : Angle in radians = Angle in degrees × $\frac{\pi}{180}$

Therefore, Angle in radians = $330 \times \frac{\pi}{180} = \frac{11\pi}{6}$

Q. 2. E. Express each of the following angles in radians

400°

Answer : Formula : Angle in radians = Angle in degrees × $\frac{\pi}{180}$

Therefore, Angle in radians = $400 \times \frac{\pi}{180} = \frac{20\pi}{9}$

Q. 2. F. Express each of the following angles in radians

7°30.'

Answer : Formula : Angle in radians = Angle in degrees × $\frac{\pi}{180}$

The angle in radians = $\frac{\text{angle in minutes}}{60}$

Therefore, the total angle = $7 + \frac{30}{60} = 7.5$

Therefore, Angle in radians = $7.5 \times \frac{\pi}{180} = \frac{\pi}{24}$

Q. 2. G. Express each of the following angles in radians

-270°

Answer : Formula : Angle in radians = Angle in degrees × $\frac{\pi}{180}$

Therefore, Angle in radians = $-270 \times \frac{\pi}{180} = -\frac{3\pi}{2}$

Q. 2. H. Express each of the following angles in radians

-22°30'

Answer : Formula : Angle in radians = Angle in degrees × $\frac{\pi}{180}$

The angle in radians = $\frac{\text{angle in minutes}}{60}$

Therefore, the total angle = $-\left(22+\frac{30}{60}\right)=-22.5$

Therefore, Angle in radians = $-22.5\times\frac{\pi}{180}=-\frac{\pi}{8}$

Q. 3. Express each of the following angles in degrees.

(i) $\frac{5\pi}{12}$

(ii) $-\frac{18\pi}{5}$

(iii) $\frac{5}{5}$

(iv) -4

Answer : (i) Formula : Angle in degrees = Angle in degrees × $\frac{\pi}{180}$

Therefore, Angle in degrees = $\frac{5\pi}{12}\times\frac{180}{\pi}=75°$

(ii) Formula : Angle in degrees = $\text{Angle in radians}\times\frac{180}{\pi}$

Therefore, Angle in degrees = $-\frac{18\pi}{5}\times\frac{180}{\pi}=-648°$

(iii) Formula : Angle in degrees = $\text{Angle in radians}\times\frac{180}{\pi}$

The angle in minutes = Decimal of angle in radian x 60.'

The angle in seconds = Decimal of angle in minutes x 60.''

Therefore, Angle in degrees = $\frac{5}{6}\times\frac{180}{\pi}=\frac{150}{22/7}=47.7272°$

Angle in minutes = $0.7272\times60'=43.632'$

Angle in seconds = $0.632 \times 60'' = 37.92''$

Final angle = $47° \, 43' \, 38''$

(iv) Formula : Angle in degrees = $\text{Angle in radians} \times \frac{180}{\pi}$

The angle in minutes = Decimal of angle in radian x 60.'

The angle in seconds = Decimal of angle in minutes x 60.''

Therefore, Angle in degrees = $-4 \times \frac{180}{\pi} = -\frac{720}{22/7} = -229.0909°$

Angle in minutes = $0.0909 \times 60' = 5.4545'$

Angle in seconds = $0.4545 \times 60'' = 27.27''$

Final angle = $-229° \, 5' \, 27''$

Q. 4. The angles of a triangle are in AP, and the greatest angle is double the least. Find all the angles in degrees and radians.

Answer : Let a - d, a, a + d be the three angles of the triangle that form AP.
Given that the greatest angle is double the least. Now, a + d = 2(a - d) 2a - 2d = a + da = 3d …… (1)Now by angle sum property, (a - d) + a + (a + d) = 180°3a = 180°a = 60° …… (2) From (1) and (2), 3d = 60°d = 20°Now, the angles are, a - d = 60°- 20° = 40°a = 60°a + d = 60° + 20° = 80°.

Therefore the required angles are 40° 60° 80°

Q. 5. The difference between the two acute angles of a right triangle is $\left(\frac{\pi}{5}\right)^c$.

Answer : The angle in degree = $\frac{\pi}{5} \times \frac{180}{\pi} = 36°$

= 36°

Let, two acute angles are x and y

So,

ATQ, x - y= 36°......(1)

x+ y= 90°......(2)

Solving 1 & 2, we get;

⇒ 2x= 126°

⇒ x= 63°

Putting the value of x in 2, we get;

⇒ 63°+ y= 90°

⇒ y= 27°

So, Two acute angles are 63° & 27°

Q. 6. Find the radius of a circle in which a central angle of 45^0 intercepts an arc of length 33 cm. (Take $\pi = 22/7$)

Answer :

Angle in radians = Angle in degrees $\times \frac{\pi}{180}$

$\theta = \frac{l}{r}$ where θ is central angle, l=length of arc, r=radius

Therefore angle = $45 \times \frac{\pi}{180} = \frac{\pi}{4}$

Now,

$r = \frac{l}{\theta}$

$= \frac{33}{\pi/4} = \frac{132}{22/7} = \frac{924}{22} = 42$

Therefore radius is 42 cm

Q. 7. Find the length of an arc of a circle of radius 14 cm which subtends an angle of 36^0 at the centre

Answer : Angle in radians = Angle in degrees $\times \frac{\pi}{180}$

Angle in radians = Angle in degrees $\times \frac{\pi}{180}$

$\theta=\frac{l}{r}$ where θ is central angle, l=length of arc, r=radius

Therefore angle $= 36 \times \frac{\pi}{180} = \frac{\pi}{5}$

Now,

$l = r \times \theta$

$= 14 \times \frac{\pi}{5} = 14 \times \frac{22}{35} = \frac{44}{5} = 8.8$

Therefore the length of the arc is 8.8 cm

Q. 8. If the arcs of the same length in two circles subtend angles 75^0 and 120^0 at the centre, find the ratio of their radii

Answer : Angle in radians = Angle in degrees $\times \frac{\pi}{180}$

$\theta=\frac{l}{r}$ where θ is central angle, l=length of arc, r=radius

Therefore $\theta_1 = 75 \times \frac{\pi}{180} = \frac{5\pi}{12}$

$\theta_2 = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$

l = r × θ

Now, as the length is the same

Therefore, $r_1 \times \theta_1 = r_2 \times \theta_2$

$$r_1 \times \frac{5\pi}{12} = r_2 \times \frac{2\pi}{3}$$

$$\frac{r_1}{r_2} = \frac{12}{5\pi} \times \frac{2\pi}{3} = \frac{24}{15} = \frac{8}{5}$$

Therefore the ratio of their radii is 8 : 5

Q. 9. Find the degree measure of the angle subtended at the centre of a circle of diameter 60 cm by an arc of length 16.5 cm.

Answer : Angle in radians = Angle in degrees $\times \frac{\pi}{180}$

$\theta = \frac{l}{r}$ where θ is central angle, l=length of arc, r=radius

Now,

$\theta = \frac{l}{r}$ and r = 0.5 x diameter

$= \frac{16.5}{30}$ radians

$$\theta \text{ in degrees} = \frac{16.5}{30} \times \frac{180}{\pi} = \frac{16.5}{30} \times \frac{180}{22/7} = \frac{16.5}{30} \times \frac{180 \times 7}{22} = \frac{20790}{660} = 31.5°$$

θ in minutes = 0.5 x 60 = 30’

Therefore angle subtended at the center is 31° 30’

Q. 10. In a circle of diameter 30 cm, the length of a chord is 15 cm. Find the length of the minor arc of the chord.

Answer : Diameter = 30 cm

Length of chord = 15 cm

Radius = 15 cm [r = 0.5 x diameter]

Since the radius is equal to the length of the chord

Hence the formed triangle in the circle is an equilateral triangle.

$\theta = 60°$

We know that $l = r \times \theta$

$$l = 15 \times 60 \times \frac{\pi}{180} = 5 \times \pi = 5 \times 3.14 = 15.7$$

Therefore, the length of the minor arc is 15.7 cm

Q. 11. Find the angle in radians as well as in degrees through which a pendulum swings if its length is 45 cm and its tip describes an arc of length 11 cm

Answer : We know that $l = r \times \theta$

Here l = length of arc = 11 cm

R = radius = length of pendulum = 45 cm

We need to find θ

$11 = 45 \times \theta$

$$\theta = \frac{11}{45} \text{ radian}$$

$$\theta \text{ in degree} = \frac{11}{45} \times \frac{180}{\pi} = \frac{44}{22/7} = 14°$$

Q. 12. The large hand of a clock Is 42 cm long. How many centimetres does its extremity move in 20 minutes?

Answer : For 20 minutes = $\theta = 4 \times 30° = 120°$

We know that $l = r \times \theta$

l = $42 \times 120 \times \frac{\pi}{180} = 28 \times \frac{22}{7} = 88$

Therefore, the length is equal to 88 cm.

Q. 13. A wheel makes 180 revolutions in 1 minute. Through how many radians does it turn in 1 second?

Answer : Given that Number of revolutions per minute = 180

Then per second, it will be = 180/60 =3

We know that In one complete revolution, the wheel turns at an angle of 2 π rad.

Then for 3 complete revolutions, it will take 3 × 2 π = 6 π radians.

Q. 14. A train is moving on a circular curve of radius 1500 m at the rate of 66 km per hour. Through what angle has it turned in 10 seconds?

Answer : Radius = 1500 m.

Train speed at rate of 66km/hr = 18.33 m/s

Therefore, Distance covered in 1 second = 18.33 m

Distance covered in 10 second = 18.33 × 10 = 183.33m

We know that θ = Distance / radius

θ = 183.33 / 1500

= 0.122 radian

Therefore $\theta = 0.122 \times \frac{180}{\pi} = 7°$

Q. 15. A wire of length 121 cm is bent so as to lie along the arc of a circle of radius 180 cm. Find in degrees; the angle subtended at the centre by the arc.

Answer : θ will be in degrees.

Arc-length can be given by the formula : θ / 360° × 2πr

Hence it is given that 121 cm is the arc length.

⇒ 121 = θ / 360° × 2πr

= 121 = θ / 360° × 2 × 22 / 7 × 180

= 121 = θ / 360° × 360 × 22 / 7

= 121 = θ × 22 / 7

⇒ θ = 121 × 7 / 22

= 38.5°

Hence the angle subtended at the middle is 38.5°

Which can also be written as 38° 30.’

Q. 16. The angles of a quadrilateral are in AP, and the greatest angle is double the least. Express the least angle in radians.

Answer : Let the smallest term be x, and the largest term be 2x

Then AP formed= x, ?, ?, 2x

So,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[a + (a + (n-1)d)] = \frac{n}{2}[\text{First term} + (\text{Last term})]$$

360°= 4/2 [x+ 2x]....[We know that → a+(n-1) d= last term= 2x]

⇒ 180°= 3x

⇒ x= 60°

Now, 60° is least angle.

= 60°= π/180° × 60°

⇒ 60° = π/3 rad