where c is the integrating constant.

11. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{1+\sin x}\, dx = ?$

A. $-\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$

B. $\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$

C. $-2\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$

D. none of these

Answer

Given = $\int \sqrt{1+\sin x}$

So,

$$\int \sqrt{1+\sin x}\,dx$$

$$= \int \sqrt{1+\sin x}\,\frac{\sqrt{1-\sin x}}{\sqrt{1-\sin x}}dx$$

$$= \int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\sin x}}\,dx$$

$$= \int \frac{\cos x}{\sqrt{1-\sin x}}dx$$

Let 1 - sinx = u^2

So, -cosxdx = 2udu

$$-\int \frac{2u}{u}du = -2\int du = -2u + c = -2\sqrt{1-\sin x} + c$$

where c is the integrating constant.

12. Question

Mark (√) against the correct answer in each of the following:

$\int \sin^3 x\, dx = ?$

A. $-\frac{3}{4}\cos x + \frac{\cos 3x}{12} + C$

B. $\frac{3}{4}\cos x + \frac{\cos 3x}{12} + C$

C. $-\frac{3}{4}\cos x - \frac{\cos 3x}{12} + C$

D. none of these

Answer

Given = $\int \sin^3 x dx$

So,

$$\int \sin^3 x dx$$
$$= \int \sin^2 x \sin x dx$$
$$= \int (1-\cos^2 x) \sin x dx$$

Let cosx = u

So, -sinxdx = du

$$-\int (1-u^2) du$$
$$= -\int du + \int u^2 du$$
$$= -u + \frac{u^3}{3} + c$$
$$= -\cos x + \frac{\cos^3 x}{3} + c$$

where c is the integrating constant.

13. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\log x}{x} dx = ?$$

A. $\frac{1}{2}(\log x)^2 + C$

B. $-\frac{1}{2}(\log x)^2 + C$

C. $\frac{2}{x^2} + C$

D. $\frac{-2}{x^2} + C$

Answer

Given = $\int \frac{\log x}{x}$

Let, logx = u

So, $\frac{1}{x} dx = du$

So,

$$\int \frac{\log x}{x} dx$$
$$= \int u du$$
$$= \frac{u^2}{2} + c$$
$$= \frac{(\log x)^2}{2} + c$$

where c is the integrating constant.

14. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2(\log x)}{x} dx = ?$$

A. log (tan x) + C

B. - log (tan x) + C

C. tan (tan x) + C

D. - tan (log x) + C

Answer

Given = $\int \frac{\sec^2(\log x)}{x}$

Let, logx = z

$$\Rightarrow \frac{dx}{x} = dz$$

So,

$$\int \frac{\sec^2(\log x)}{x} dx$$
$$= \int \sec^2 z dz$$
$$= \tan z + c$$
$$= \tan(\log x) + c$$

where c is the integrating constant.

15. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x(\log x)} dx = ?$$

A. $\log|x| + C$

B. $\frac{-2}{x^2} + C$

C. $(\log x)^2 + C$

D. $\log|\log x| + C$

Answer

Given $= \int \frac{1}{x(\log x)}$

Let, $\log x = z$

$\Rightarrow \frac{dx}{x} = dz$

So,

$\int \frac{1}{x(\log x)} dx$

$= \int \frac{1}{z} dz$

$= \log z + c$

$= \log(\log x) + c$

where c is the integrating constant.

16. Question

Mark (√) against the correct answer in each of the following:

$\int e^{x^3} x^2 dx = ?$

A. $e^{x^3} + C$

B. $\frac{1}{3} e^{x^3} + C$

C. $\frac{1}{6} e^{x^3} + C$

D. none of these

Answer

Given $= \int e^{x^3} x^2$

Let, $x^3 = z$

$\Rightarrow 3x^2 dx = dz$

$\Rightarrow x^2 dx = \frac{dz}{3}$

So,

$$\int e^{x^3} x^2 dx$$

$$= \frac{1}{3}\int e^z dz$$

$$= \frac{1}{3}e^z + c$$

$$= \frac{1}{3}e^{x^3} + c$$

where c is the integrating constant.

17. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = ?$$

A. $e^{\sqrt{x}} + C$

B. $\frac{1}{2}e^{\sqrt{x}} + C$

C. $2e^{\sqrt{x}} + C$

D. none of these

Answer

Given = $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}$

Let, $x = z^2$

$\Rightarrow dx = 2zdz$

So,

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$= \int \frac{e^z}{z} 2zdz$$

$$= 2\int e^z dz$$

$$= 2e^z + c$$

$$= 2e^{\sqrt{x}} + c$$

where c is the integrating constant.

18. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^{\tan^{-1} x}}{(1+x^2)} dx = ?$$

A. $\frac{e^{\tan^{-1}x}}{x}+C$

B. $e^{\tan^{-1}x}+C$

C. $e^{x}\tan^{-1}x+C$

D. none of these

Answer

Given = $\int \frac{e^{\tan^{-1}x}}{\left(1+x^{2}\right)}$

Let, $\tan^{-1}x = z$

$\Rightarrow \frac{1}{1+x^{2}}dx = dz$

So,

$\int \frac{e^{\tan^{-1}x}}{\left(1+x^{2}\right)}dx$

$= \int e^{z}dz$

$= e^{z}+c$

$= e^{\tan^{-1}x}+c$

where c is the integrating constant.

19. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = ?$

A. $2\cos\sqrt{x}+C$

B. $-2\cos\sqrt{x}+C$

C. $-\frac{\cos\sqrt{x}}{2}+C$

D. $\frac{\cos\sqrt{x}}{2}+C$

Answer

Given = $\int \frac{\sin\sqrt{x}}{\sqrt{x}}$

Let, $x = z^{2}$

$\Rightarrow dx = 2zdz$

So,

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$$

$$= \int \frac{\sin z}{z}2zdz$$

$$= 2\int \sin zdz$$

$$= -2\cos z + c$$

$$= -2\cos\sqrt{x} + c$$

where c is the integrating constant.

20. Question

Mark (√) against the correct answer in each of the following:

$$\int\left(\sqrt{\sin x}\right)\cos x\,dx = ?$$

A. $\frac{2}{3}(\cos x)^{3/2} + C$

B. $\frac{3}{2}(\cos x)^{3/2} + C$

C. $\frac{2}{3}(\sin x)^{3/2} + C$

D. $\frac{3}{2}(\sin x)^{3/2} + C$

Answer

Given = $\int\left(\sqrt{\sin x}\right)\cos x$

Let, sinx = z^2

⇒ cosxdx = 2zdz

So,

$$\int\left(\sqrt{\sin x}\right)\cos xdx$$

$$= 2\int z^2 dz$$

$$= 2\frac{z^3}{3} + c$$

$$= \frac{2}{3}\sin^{3/2} x + c$$

where c is the integrating constant.

21. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}$$

A. $\frac{1}{2}\log\left|\tan^{-1} x\right| + C$

B. $2\sqrt{\tan^{-1} x} + C$

C. $\frac{1}{2\sqrt{\tan^{-1} x}} + C$

D. none of these

Answer

Given = $\int \frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}$

Let, $\tan^{-1}x = z^2$

$\Rightarrow \frac{1}{1+x^2}dx = 2zdz$

So,

$$\int \frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}dx$$

$$= \int \frac{2z}{z}dz$$

$$= 2\int dz$$

$$= 2z + c$$

$$= 2\sqrt{\tan^{-1} x} + c$$

where c is the integrating constant.

22. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cot x}{\log(\sin x)}dx = ?$$

A. log |cot x| + C

B. log |cot x cosec x| + C

C. log |log sin x| + C

D. none of these

Answer

Given = $\int \frac{\cot x}{\log(\sin x)}$

Let, sinx = z

$\Rightarrow$ cosxdx = dz

So,

$$\int \frac{\cot x}{\log(\sin x)} dx$$

$$= \int \frac{\cos x}{\sin x \log(\sin x)} dx$$

$$= \int \frac{dz}{z \log z}$$

Let, logz = u

$$\Rightarrow \frac{1}{z} dz = du$$

So,

$$\int \frac{dz}{z \log z}$$

$$= \int \frac{du}{u}$$

$$= \log u + c$$

$$= \log|\log z| + c$$

where c is the integrating constant.

23. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x \cos^2(1 + \log x)} dx = ?$$

A. tan (1 + log x) + C

B. cot (1 + log x) + C

C. sec (1 + log x) + C

D. none of these

Answer

Given = $\int \frac{1}{x \cos^2(1 + \log x)}$

Let, 1 + logx = z

$$\Rightarrow \frac{1}{x} dx = dz$$

So,

$$\int \frac{1}{x\cos^2(1+\log x)}dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(1+\log x) + c$$

where c is the integrating constant.

24. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2 \tan^{-1} x^3}{(1+x^6)} dx = ?$$

A. $\frac{1}{3}(\tan^{-1} x^3) + C$

B. log $|\tan^{-1} x^3| + C$

C. $\frac{1}{6}(\tan^{-1} x^3)^2 + C$

D. none of these

Answer

Given = $\int \frac{x^2 \tan^{-1} x^3}{(1+x^6)} dx$

Let, $\tan^{-1}x^3 = z$

$$\Rightarrow \frac{1}{1+x^6} \times 3x^2 dx = dz$$

$$\Rightarrow \frac{x^2}{1+x^6} dx = \frac{dz}{3}$$

So,

$$\frac{1}{3}\int z dz$$

$$= \frac{1}{3}\frac{z^2}{2} + c$$

$$= \frac{z^2}{6} + c$$

$$= \frac{(\tan^{-1} x^3)^2}{6} + c$$

where c is the integrating constant.

25. Question

Mark (√) against the correct answer in each of the following:

$$\int \sec^5 x \tan x \, dx = ?$$

A. 5 $\tan^5 x$ + C

B. $\frac{1}{5}\tan^5 x + C$

C. 5 log |cos x| + C

D. none of these

Answer

Given = $\int \sec^5 x \tan x$

So, $\int \sec^5 \tan x dx = \int \sec^4 x(\sec x \tan x) dx$

Let, secx = z

⇒ secxtanxdx = dz

$$\int \sec^4 x(\sec x \tan x) dx$$
$$= \int z^4 dz$$
$$= \frac{z^5}{5} + c$$
$$= \frac{\sec^5 x}{5} + c$$

where c is the integrating constant.

26. Question

Mark (√) against the correct answer in each of the following:

$$\int \text{cosec}^3(2x+1)\cot(2x+1) dx = ?$$

A. $\frac{1}{4}\text{cosec}^4(2x+1) + C$

B. $-\frac{1}{3}\text{cosec}^3(2x+1) + C$

C. $-\frac{1}{6}\text{cosec}^3(2x+1) + C$

D. $\frac{1}{2}\text{cosec}(2x+1)\cot(2x+1) + C$

Answer

Given = $\int \cos ec^3(2x+1)\cot(2x+1)$

So,

$$\int \operatorname{cosec}^3(2x+1)\cot(2x+1)dx$$

$$= \int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$

Let, cosec(2x + 1) = z

⇒ -2cosec(2x + 1)cot(2x + 1)dx = dz

$$\int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$

$$= \int z^2 \frac{dz}{-2} =$$

$$= -\frac{1}{2}\frac{z^3}{3} + c$$

$$= -\frac{\operatorname{cosec}^6(2x+1)}{6} + c$$

where c is the integrating constant.

27. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\tan(\sin^{-1}x)}{\sqrt{1-x^2}}dx = ?$$

A. log |sec ($\sin^{-1}$ x)| + C

B. log |cos ($\sin^{-1}$ x)| + C

C. tan ($\sin^{1}$ x) + C

D. none of these

Answer

Given = $\int \frac{\tan(\sin^{-1}x)}{\sqrt{1-x^2}}$

Let, $\sin^{-1}x = z$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dz$$

So,

$$\int \frac{\tan(\sin^{-1}x)}{\sqrt{1-x^2}}dx$$

$$= \int \tan z dz$$

$$= \log|\sec z| + c$$

$$= \log\left|\sec(\sin^{-1}x)\right| + c$$

where c is the integrating constant.

28. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\tan(\log x)}{x} dx = ?$$

A. x tan (log x) + C

B. log |tan x| + C

C. log |cos (log x)| + C

D. - log |cos (log x)| + C

Answer

Given = $\int \frac{\tan(\log x)}{x}$

Let, logx = z

$\Rightarrow \frac{1}{x} dx = dz$

So,

$$\int \frac{\tan(\log x)}{x} dx$$
$$= \int \tan z dz$$
$$= \log|\sec z| + c$$
$$= \log|\sec(\log x)| + c$$
$$= -\log|\cos(\log x)| + c$$

where c is the integrating constant.

29. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x \cot(e^x) dx = ?$$

A. cot (e^x) + C

B. log |sin e^x| + C

C. log |cosec e^x| + C

D. none of these

Answer

Given = $\int e^x \cot(e^x) dx$

Let, $e^x = z$

$\Rightarrow e^x dx = dz$

So,

$\int e^x \cot(e^x) dx$

$= \int \cot z dz$

$= \log|\sin z| + c$

$= \log\left|\sin(e^x)\right| + c$

where c is the integrating constant.

30. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{e^x}{\sqrt{1+e^x}} dx = ?$

A. $2\sqrt{1+e^x} + C$

B. $\frac{1}{2}\sqrt{1+e^x} + C$

C. $\frac{1}{\sqrt{1+e^x}} + C$

D. none of these

Answer

Given = $\int \frac{e^x}{\sqrt{1+e^x}}$

Let, $1 + e^x = z^2$

$\Rightarrow e^x dx = 2z dz$

So,

$\int \frac{e^x}{\sqrt{1+e^x}} dx$

$= \int \frac{2z dz}{z}$

$= 2\int dz$

$= 2z + c$

$= 2\sqrt{1+e^x} + c$

where c is the integrating constant.

31. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x}{\sqrt{1-x^2}} dx = ?$

A. $\sin^{-1} x + C$

B. $\sin^{-1}\sqrt{x}+C$

C. $\sqrt{1-x^2}+C$

D. $-\sqrt{1-x^2}+C$

Answer

Given = $\int \frac{x}{\sqrt{1-x^2}}dx$

Let, $1 - x^2 = z^2$

$\Rightarrow -2xdx = 2zdz$

So,

$$\int \frac{x}{\sqrt{1-x^2}}dx$$
$$=-\int \frac{zdz}{z}$$
$$=-\int dz$$
$$=-z+c$$
$$=-\sqrt{1-x^2}+c$$

where c is the integrating constant.

32. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx=?$$

A. $\tan(xe^x) + C$

B. $\cot(xe^x) + C$

C. $ex^x \tan x + C$

D. none of these

Answer

Given = $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$

Let, $xe^x = z$

$\Rightarrow e^x(1 + x)dx = dz$

So,

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(xe^x) + c$$

where c is the integrating constant.

33. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(e^x + e^{-x})} = ?$$

A. cot-1 (e^x) + C

B. tan-1 (e^x) + C

C. log |e^x + 1| + C

D. none of these

Answer

Given =

$$\int \frac{dx}{(e^x + e^{-x})}$$

$$= \int \frac{e^x}{(e^x+1)}dx$$

Let, $e^x + 1 = z$

$\Rightarrow e^x dx = dz$

So,

$$\int \frac{e^x dx}{(e^x+1)}$$

$$= \int \frac{dz}{z}$$

$$= \log|z| + c$$

$$= \tan|e^x+1| + c$$

where c is the integrating constant.

34. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{2^x}{1-4^x} dx = ?$$

A. $\sin^{-1}(2^x) + C$

B. $(\log e^2) \sin^{-1}(2^x) + C$

C. $(\log e^2) \cos^{-1}(2^x) + C$

D. $\log_2 e) \sin^{-1}(2^x) + C$

Answer

Given =

$$\int \frac{2^x dx}{1-4^x}$$

$$= \int \frac{2^x}{1-\left(2^x\right)^2} dx$$

Let, $2^x = z$

$\Rightarrow 2^x(\log 2)dx = dz$

So,

$$\int \frac{2^x dx}{1-\left(2^x\right)^2}$$

$$= \frac{1}{\log 2} \int \frac{dz}{1-z^2}$$

$$= \frac{1}{\log 2} \sin^{-1} z + c$$

$$= \frac{\sin^{-1} 2x}{\log 2} + c$$

where c is the integrating constant.

35. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(e^x - 1\right)} = ?$$

A. $\log |e^x - 1| + C$

B. $\log |1 - e^{-x}| + C$

C. $\log |e^x - 1| + C$

D. none of these

Answer

Given =

$$\int \frac{dx}{e^x - 1}$$

$$= -\int \frac{-1 + e^x - e^x}{e^x - 1} dx$$

$$= -\int \frac{e^x - 1}{e^x - 1} dx + \int \frac{e^x}{e^x - 1} dx$$

$$= -\int dx + \int \frac{e^x}{e^x - 1} dx$$

Let, $e^x - 1 = z$

$\Rightarrow e^x dx = dz$

So,

$$-\int dx + \int \frac{e^x}{e^x - 1} dx$$

$$= -x + \int \frac{dz}{z}$$

$$= -x + \log z + c$$

$$= -x + \log\left|e^x - 1\right| + c$$

where c is the integrating constant.

36. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\left(\sqrt{x} + x\right)} dx = ?$$

A. $\log\left|1 + \sqrt{x}\right| + C$

B. $2\log\left|1 + \sqrt{x}\right| + C$

C. $\frac{1}{\sqrt{x}} \tan^{-1} \sqrt{x} + C$

D. none of these

Answer

Given =

$$\int \frac{dx}{\left(\sqrt{x} + x\right)}$$

$$= \int \frac{1}{\sqrt{x}} \frac{1}{\left(1 + \sqrt{x}\right)} dx$$

Let, $1 + \sqrt{x} = z$

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dz$

So,

$$\int \frac{1}{\sqrt{x}} \frac{1}{\left(1+\sqrt{x}\right)} dx$$

$$= 2\int \frac{dz}{z}$$

$$= 2\log|z| + c$$

$$= 2\tan\left|1+\sqrt{x}\right| + c$$

where c is the integrating constant.

37. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1+\sin x)} = ?$$

A. tan x + sec x + C

B. tan x – sec x + C

C. $\frac{1}{2}\tan\frac{x}{2} + C$

D. none of these

Answer

Given

$$\int \frac{dx}{1+\sin x}$$

$$= \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \int \frac{dx}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= \int \frac{\sec^2\frac{x}{2}dx}{\left(\tan\frac{x}{2} + 1\right)^2}$$

Let, $\tan\frac{x}{2} + 1 = z$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dz$

So,

$$\int \frac{2dz}{z^2}$$

$$= -\frac{2}{z} + c$$

$$= -\frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

38. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1 + \sin x)} dx = ?$$

A. x + tan x – sec x + C

B. x – tan x – sec x + C

C. x – tan x + sec x + C

D. none of these

Answer

Given

$$\int \frac{\sin x}{1 + \sin x} dx$$

$$= \int dx - \int \frac{dx}{1 + \sin x}$$

$$= x - \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= x - \int \frac{dx}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= x - \int \frac{\sec^2\frac{x}{2}dx}{\left(\tan\frac{x}{2} + 1\right)^2}$$

Let, $\tan\frac{x}{2} + 1 = z$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dz$

So,

$$x - \int \frac{2dz}{z^2}$$

$$= x + \frac{2}{z} + c$$

$$= x + \frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

39. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1 - \sin x)} dx = ?$$

A. - x + sec x – tan x + C

B. x + cos x - sin x + C

C. - log |1 – sin x| + C

D. none of these

Answer

Given

$$\int \frac{\sin x}{1 - \sin x} dx$$

$$= -\int dx + \int \frac{dx}{1 - \sin x}$$

$$= -x + \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= -x + \int \frac{dx}{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2}$$

$$= -x + \int \frac{\sec^2\frac{x}{2}dx}{\left(\tan\frac{x}{2} - 1\right)^2}$$

Let, $\tan\frac{x}{2} - 1 = z$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dz$

So,

$-x+\int\frac{2dz}{z^2}$

$=-x-\frac{2}{z}+c$

$=-x-\frac{2}{\tan\frac{x}{2}+1}+c$

where c is the integrating constant.

40. Question

Mark (√) against the correct answer in each of the following:

$\int\frac{dx}{(1+\cos x)}=?$

A. $\frac{1}{2}\tan\frac{x}{2}+C$

B. $-\cot\frac{x}{2}+C$

C. $\tan\frac{x}{2}+C$

D. none of these

Answer

Given

$\int\frac{dx}{1+\cos x}$

$=\int\frac{dx}{1+2\cos^2\frac{x}{2}-1}$

$=\frac{1}{2}\int\frac{dx}{\cos^2\frac{x}{2}}$

$=\frac{1}{2}\int\sec^2\frac{x}{2}dx$

$=\frac{1}{2}2\tan\frac{x}{2}+c$

$=\tan\frac{x}{2}+c$

where c is the integrating constant.

41. Question

Mark (√) against the correct answer in each of the following:

$\int\frac{dx}{(1-\cos x)}=?$

A. $\frac{1}{(x-\sin x)}+C$

B. log |x - sin x| + C

C. $\log\left|\tan\frac{x}{2}\right|+C$

D. $-\cot\frac{x}{2}+C$

Answer

Given

$$\int\frac{dx}{1-\cos x}$$

$$=\int\frac{dx}{1-1+2\sin^2\frac{x}{2}}$$

$$=\frac{1}{2}\int\frac{dx}{\sin^2\frac{x}{2}}$$

$$=\frac{1}{2}\int\cos ec^2\frac{x}{2}dx$$

$$=-\frac{1}{2}2\cot\frac{x}{2}+c$$

$$=-\cot\frac{x}{2}+c$$

where c is the integrating constant.

42. Question

Mark (√) against the correct answer in each of the following:

$$\int\left\{\frac{1-\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}\right\}dx=?$$

A. $2\log\left|\sec\frac{x}{2}\right|+C$

B. $2\log\left|\cos\frac{x}{2}\right|+C$

C. $2\log\left|\sec\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|+C$

D. $2\log\left|\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|+C$

Answer

Given

$$\int \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}dx$$

$$= \int \frac{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}dx$$

$$= \int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}dx$$

Let, $\cos\frac{x}{2}+\sin\frac{x}{2} = z$

$\Rightarrow \left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx = dz$

So,

$$\int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}dx$$

$$= \int \frac{dz}{z}$$

$$= \log z + c$$

$$= \log\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right) + c$$

where c is the integrating constant.

43. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{e^x}\,dx = ?$$

A. $\sqrt{e^x} + C$

B. $2\sqrt{e^x} + C$

C. $\frac{1}{2}\sqrt{e^x} + C$

D. none of these

Answer

Given

$$\int \sqrt{e^x}\,dx$$

$$= \int \left(e^x\right)^{\frac{1}{2}} dx$$

$$= \int e^{\frac{1}{2}x} dx$$

$$= 2e^{\frac{1}{2}x} + c$$

$$= 2\sqrt{e^x} + c$$

where c is the integrating constant.

44. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{(1+\cos x)} dx = ?$$

A. $x + \tan\frac{x}{2} + C$

B. $-x + \tan\frac{x}{2} + C$

C. $x - \tan\frac{x}{2} + C$

D. none of these

Answer

Given

$$\int \frac{\cos x\,dx}{1+\cos x}$$

$$= \int \frac{1+\cos x - 1}{1+\cos x} dx$$

$$= \int dx - \int \frac{dx}{1+\cos x}$$

$$= x - \tan\frac{x}{2} + c$$

[From Question no. 40] where c is the integrating constant.

45. Question

Mark (√) against the correct answer in each of the following:

$$\int \sec^2 x \ \mathrm{cosec}^2 x\,dx = ?$$

A. tan x – cot x + C

B. tan x + cot x + C

C. - tan x + cot x + C

D. none of these

Answer

Given

$$\int \sec^2 x \operatorname{cosec}^2 x dx$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

46. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1-\cos 2x)}{(1+\cos 2x)} dx = ?$$

A. tan x + x + C

B. tan x – x + C

C. - tan x + x + C

D. none of these

Answer

Given

$$\int \frac{(1-\cos 2x)}{(1+\cos 2x)} dx$$

$$= \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$$

$$= \int \tan^2 \frac{x}{2} dx$$

$$= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= 2\tan \frac{x}{2} - x + c$$

where c is the integrating constant.

47. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{(1+\cos x)}{(1-\cos x)} dx = ?$

A. $-2\cot\frac{x}{2} - x + C$

B. $-2\cot\frac{x}{2} + x + C$

C. $2\cot\frac{x}{2} + x + C$

D. none of these

Answer

Given

$$\int \frac{(1+\cos 2x)}{(1-\cos 2x)} dx$$

$$= \int \frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}} dx$$

$$= \int \cot^2\frac{x}{2} dx$$

$$= \int \left(\operatorname{cosec}^2\frac{x}{2} - 1\right) dx$$

$$= -2\cot\frac{x}{2} - x + c$$

where c is the integrating constant.

48. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{1}{\sin^2 x \cos^2 x} dx = ?$

A. tan x + cot x + C

B. tan x – cot x + C

C. - tan x + cot x + C

D. none of these

Answer

Given

$$\int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

49. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = ?$$

A. cot x + tan x + C

B. - cot x + tan x + C

C. cot x – tan x + C

D. - cot x – tan x + C

Answer

Given

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx$$

$$= \int \operatorname{cosec}^2 x dx - \int \sec^2 x dx$$

$$= -\tan x - \cot x + c$$

where c is the integrating constant.

50. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = ?$$

A. sin x + x cos α + C

B. 2sin x + x cos α + C

C. 2 sin x + 2x cos α + C

D. none of these

Answer

Given

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos\alpha)} dx$$

$$= \int \frac{-2\sin\left(\frac{2x+2\alpha}{2}\right)\sin\left(\frac{2x-2\alpha}{2}\right)}{-2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right) \times 2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 2\int 2\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\int \cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right) + \cos\left(\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right)$$

$$= 2\int (\cos x + \cos\alpha) dx$$

$$= 2[\sin x + x\cos\alpha] + c$$

where c is the integrating constant.

51. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}\left\{\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right\} dx = ?$$

A. $2x^2 + C$

B. $\frac{x^2}{2} + C$

C. $\frac{2}{(1+x^2)} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \cos 2x = 2\cos^2 x$; $1 - \cos 2x = 2\sin^2 x$

Therefore ,

$$\Rightarrow \int \tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}} dx = \int \tan^{-1}\sqrt{\frac{2\sin^2 x}{2\cos^2 x}} dx = \int \tan^{-1}\tan x \, dx$$

$\Rightarrow \int x\,dx = \frac{x^2}{2} + c$

52. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}(\sec x + \tan x)\,dx = ?$$

A. $\frac{\pi x}{4} + \frac{x^2}{4} + C$

B. $\frac{\pi x}{4} - \frac{x^2}{4} + C$

C. $\frac{1}{(1+x^2)} + C$

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \sin x = (\cos\frac{x}{2} + \sin\frac{x}{2})^2$

$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$; $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

Therefore ,

$\Rightarrow \int \tan^{-1}(\sec x + \tan x)\,dx = \int \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) dx$

$\Rightarrow \int \tan^{-1} \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\,dx = \int \tan^{-1} \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{(\cos\frac{x}{2}+\sin\frac{x}{2})(\cos\frac{x}{2}-\sin\frac{x}{2})}\,dx$

$\Rightarrow \int \tan^{-1} \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^1}{(\cos\frac{x}{2}-\sin\frac{x}{2})}\,dx = \int \tan^{-1} \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\,dx$

(Multiply by $\sec\frac{x}{2}$ in numerator and denominator)

$\Rightarrow \int \tan^{-1} \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\,dx = \int \tan^{-1} \frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{\tan\frac{\pi}{4}-\tan\frac{\pi}{4}\tan\frac{x}{2}}\,dx = \int \tan^{-1}\tan(\frac{\pi}{4}+\frac{x}{2})\,dx$

$\Rightarrow \int (\frac{\pi}{4}+\frac{x}{2})\,dx = \frac{\pi x}{4} + \frac{x^2}{4} + c$

53. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1+\sin x)}{(1-\sin x)}\,dx = ?$$

A. 2 tan x + x - 2sec x + C

B. 2 tan x - x + 2 sec x + C

C. 2 tan x - x - 2sec x + C

D. none of these

Answer

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x\,dx = \tan x$