NCERT Solutions for Class 10 Maths Chapter 8 Introduction to Trigonometry EXERCISE 8.3

1. Evaluate:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(iii) Cos 48° – sin 42°

(iv) Cosec 31° – sec 59°

(i) Solution: (i) $\frac{Sin18^\circ}{Cos72^\circ}$

$= \frac{Sin(90^\circ - 72^\circ)}{Cos72^\circ}$

$= \frac{Cos72^\circ}{Cos72^\circ}$

= 1

(ii) $\frac{tan26^\circ}{Cot64^\circ}$

$= \frac{tan(90^\circ - 64^\circ)}{Cot64^\circ}$

$= \frac{Cot64^\circ}{Cot64^\circ}$

$= 1$

(iii) cos 48° - sin 42°

= cos (90° - 42°) – sin 42°

= sin 42° - sin 42°

= 0

(iv) $\text{cosec } 31^\circ - \sec 59^\circ$

$= \text{cosec}(90^\circ - 59^\circ) – \sec 59^\circ$

$= \sec 59^\circ - \sec 59^\circ$

$= 0$

2. Show that:

(i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

(ii) $\text{Cos } 38° \text{ Cos } 52° – \sin 38° \sin 52° = 0$

Solution: (i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

LHS: $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

$= \tan (90^\circ - 42^\circ) \times \tan 23^\circ \times \tan 42^\circ \times \tan (90^\circ - 23^\circ)$

$= \cot 42^\circ \times \tan 23^\circ \times \tan 42^\circ \times \cot 23^\circ$

$= \cot 42^\circ \times \tan 42^\circ \times \tan 23^\circ \times \cot 23^\circ$

$= 1 \times 1$

$= 1 = \text{RHS}$

Hence Proved

(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

LHS= $\cos(90^\circ - 52^\circ) \cos(90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ$

$= \sin 38^\circ \sin 52^\circ - \sin 38^\circ \sin 52^\circ$

$= 0 = \text{RHS}$

Hence proved

3. If $\tan 2A = \cot (A – 18°)$, where $2A$ is an acute angle, find the value of A

Solution: tan 2A = cot (90° – 2A)

Hence,

90° – 2A = A – 18°

108° – 2A = A

3A = 108°

A = 108/3 = 36°

4. If tan A = cot B, prove that A + B = 90°

Solution: tan A = cot (90°- A) = cot B

Therefore,

90° - A = B

A + B = 90°

Hence proved

5. If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A

Solution: sec 4A = cosec (90° - 4A) = cosec (A - 20°)

Therefore,

90° – 4A = A - 20°

110°– 4A = A

5A = 110°

A = 22°

6. If A, B and C are interior angles of a triangle ABC, then show that

$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$.

Solution: We know,

A + B + C = 90°

B + C = 90°- A

Therefore,

$Sin\left(\frac{B+C}{2}\right)$

$= Sin\left(\frac{90°-A}{2}\right)$

$= cos\,\frac{A}{2}$

Hence proved

7. Express sin 67° + Cos 75° in terms of trigonometric ratios of angles between 0° and 45°

Solution: sin 67° + cos 75°

=Sin (90° – 23°) + cos (90° – 15°)

= cos 23° + sin 15°   (Since, sin(90°- A) = cosA)

EXERCISE 8.4:

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A

Solution: sinA can be expressed in terms of cotA as:

$sinA = \frac{1}{cosecA}$

= $1/(\sqrt{cosec^2A})$

= $1/\sqrt{(1 + cot^2A)}$

Now,

Sec can be expressed in terms of cot A as:

secA = $\sqrt{sec^2A}$

= $\sqrt{(1 + tan^2A)}$

$= \sqrt{[1 + (1/\cot^2 A)]}$

$= \sqrt{[(1 + \cot^2 A)/\cot^2 A]}$

$= [\sqrt{(1 + \cot^2 A)}]/\cot A$

And,

TanA can be expressed in terms of cotA as:

$\tan A = \frac{1}{cotA}$

2. Write all the other trigonometric ratios of ∠ A in terms of sec A

1. Solution: SinA can be expressed in terms of sec A as:

$\sin A = \sqrt{\sin^2 A}$

$= \sqrt{(1 - \cos^2 A)}$

$= \sqrt{[1 - (1/\cos^2 A)]}$

$= \sqrt{[(\sec^2 A - 1)/\sec^2 A]}$

$= [\sqrt{(\sec^2 A - 1)}]/\sec A$

Now,

cosA can be expressed in terms of secA as:

$\cos A = \frac{1}{secA}$

TanA can be expressed in the form of secA as:

$\tan A = \sqrt{(\sec^2 A - 1)}$

cosec A can be expressed in the form of sec A as:

$\text{cosec } A = \frac{1}{sinA}$

$= 1/\sqrt{\sin^2 A}$

$= 1/\sqrt{(1 - \cos^2 A)}$

$= \sec A/\sqrt{(\sec^2 A - 1)}$

Cot A can be expressed in terms of sec A as:

cot A $=\frac{1}{tanA}$

=1/(√tan^2)

= 1/√(sec^{2}A -1)

3. Evaluate:

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) Sin 25° Cos 65° + Cos 25° Sin 65°

Solution: (i) [Sin^{2}63º + sin^2 (90º -63º)]/ [coa^{2}17º + cos^2 (90º -17)º]

Sin^{2}63º + cos^2 (63º)/([cos^{2}17º + sin^{2}17º]

= 1

(ii) sin 25°cos 65°+ cos 25°sin 65°

= sin 25° cos (90°-25°) + cos 25°sin 65°

= sin^2 25° + cos 25° cos (90°-25°)

= sin^2 25° + cos^2 25°

=1

4. Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

A. 1 B. 9

C. 8 D. 0

Solution: (B) is the correct option. Following is the explanation:

$9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1$

$= 9$

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)=$

A. 0 B. 1

C. 2 D. −1

Solution: (C) is the correct option

$$(1 + tan\theta + sec\theta)(1 + cot\theta - cosec\theta)$$

$= \left(1 + \frac{sin\theta}{cos\theta} + \frac{1}{cos\theta}\right)(1 + \frac{sin\theta}{cos\theta} - \frac{1}{sin\theta}$

$= \left(\frac{1+cos\theta+sin\theta}{cos\theta}\right)\left(\frac{sin\theta+cos\theta-1}{sin\theta}\right),$

$= (\sin^2\theta + 2\sin\theta cos\theta + \cos^2\theta + sin\theta + cos\theta)/(sin\theta cos\theta)$

$= \frac{2sin\theta cos\theta}{2sin\theta cos\theta}$

$= 2$

(iii) (Sec A + tan A) (1 – sin A) =

A. sec A B. sin A

C. cosec A D. Cos A

Solution: (D) is the correct option.

$(secA + tanA)(1 - sinA)$

$= \left(\frac{1}{cosA} + \frac{SinA}{cosA}\right)(1 - sinA)$

$= \frac{(1+sinA)(1-sinA)}{cos\theta}$

$= (1 - \sin^2 A)/\cos A$

$= \cos^2 A/\cos A$

$= \cos A$

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A} =$

A. $\sec^2 A$ B. -1

C. $\cot^2 A$ D. $\tan^2 A$

Solution: $\frac{1+\frac{sinA}{cosA}*\frac{sinA}{cosA}}{1+\frac{cosA}{sinA}*\frac{cosA}{sinA}}$

$= \frac{\frac{1}{cosA*cosA}}{\frac{1}{sinA*sinA}}$

$= \frac{sinA*sinA}{cosA*cosA}$

$= \tan^2 A$

Therefore, option (D) is correct

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Solution: LHS = (cosec θ - cot θ)

$= (\frac{1}{sin\theta} - \frac{cos\theta}{sin\theta})^2$

$= \frac{(1-\cos\theta)\,(1-\cos\theta)}{\sin\theta*\sin\theta}$

$= (1 - \cos\theta)^2/\sin^2\theta$

$= (1 - \cos\theta)^2/1 - \cos^2\theta$

$= (1 - c0s\,\theta)^2/\ (1 - \cos\theta)\ (1 + \cos\theta)$

$= \frac{1-cos\theta}{1+\cos\theta}$

= RHS

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\ \sec A$

Solution: LHS = $\frac{\cos A}{1+sinA} + \frac{1+sinA}{\cos A}$

$= \cos^2 A + (1 + \sin^2 A)/(1 + \sin A)\ (\cos A)$

$= \cos^2 A + 1 + \sin^2 A + 2\sin A/(1 + \sin A)\ (\cos A)$

$= \sin^2 A + \cos^2 A + 1 + 2\sin A/(1 + \sin A)\ (\cos A)$

$= \frac{1+1+2sinA}{(1+sinA)(\cos A)}$

$= \frac{2+2\ sinA}{(1+sinA)(\cos A)}$

$= \frac{2\ (1+sinA)}{(1+sinA)(\cos A)}$

$= \frac{2}{\cos A}$

= 2 sec A

= RHS

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

[Hint: Write the expression in terms of sin θ and Cos θ]

Solution: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{sin\theta}{cos\theta}}{1-\frac{cos\theta}{sin\theta}} + \frac{\frac{cos\theta}{sin\theta}}{1-\frac{sin\theta}{cos\theta}}$$

$$= \frac{\frac{sin\theta}{cos\theta}}{\frac{sin\theta}{sin\theta}-\frac{cos\theta}{sin\theta}} + \frac{\frac{cos\theta}{sin\theta}}{\frac{cos\theta}{cos\theta}-\frac{sin\theta}{cos\theta}}$$

= $\sin^2 \theta/[\cos \theta (\sin \theta - \cos \theta)] + \cos^2 \theta/ [\sin \theta (\cos \theta - \sin \theta)]$

= $\sin^2 \theta/[\cos \theta (\sin \theta - \cos \theta)] - \cos^2 \theta/[\sin \theta (\sin \theta - \cos \theta)]$

= $1/(\sin \theta - \cos \theta) [(\sin^2 \theta/\cos \theta) - (\cos^2 \theta/\sin \theta)]$

= $1/(\sin \theta - \cos \theta) * [(\sin^3 \theta - \cos^3 \theta)/\sin \theta \cos \theta]$

= $[(\sin \theta - \cos \theta) (\sin^2 \theta + \cos^2 \theta + \sin \theta\cos \theta)]/ [\sin \theta - \cos \theta) \sin \theta \cos \theta]$

= $(1 + \sin \theta \cos \theta)/\sin \theta \cos \theta$

= $1/\sin \theta \cos \theta + 1$

= $1 + \sec \theta \operatorname{cosec} \theta$

= RHS

(iv) $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$

[Hint: Simplify LHS and RHS separately]

Solution: LHS = $(1 + \sec A)/ \sec A$

= $(1 + 1/\cos A)/1/\cos A$

= $(\cos A + 1)/\cos A/1/\cos A$

= $\cos A + 1$

RHS = $\sin^2 A/(1 - \cos A)$

= $(1 - \cos^2 A)/(1 - \cos A)$

= $(1 - \cos A) (1 + \cos A)/ (1 - \cos A)$

= cos A + 1

LHS = RHS

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A,$ using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A.$

Solution: LHS = (cos A – sin A + 1)/ (cos A + sin A – 1)

Dividing Numerator and Denominator by sin A

= (c0s A – sin A + 1) / sin A/ (cos A sin A – 1) / sin A

= (cot A – 1 + cosec A)/ (cot A + 1 – cosec A)

= (cot A – cosec^{2}A + cot^{2}A + cosec A)/ (cot A + 1- cosec A) [Using cosec^{2}A – cot^{2}A = 1]

= [(cot A + cosec A) – (cosec A + cot A) (cosec A – cot A)]/ (1 – cosec A + cot A)

= (cot A + cosec A) (1 – cosec A + cot A)/ (1 – cosec A + cot A)

= cot A + cosec A

= RHS

(vi) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

Solution: Dividing numerator and enominator of LHS by cos A

$= \frac{\sqrt{\frac{1}{cosA} + \frac{sinA}{cosA}}}{\sqrt{\frac{1}{cosA} - \frac{sinA}{cosA}}}$

$= \frac{\sqrt{\sec A + \tan A}}{\sqrt{\sec A - \tan A}}$

$= \frac{\sqrt{\sec A + \tan A}}{\sqrt{\sec A - \tan A}} * \frac{\sqrt{\sec A + \tan A}}{\sqrt{\sec A + \tan A}}$

= [(sec A + tan A)^2/sec^{2}A – tan^{2}A]$^{1/2}$

$= \frac{secA+tanA}{1}$

= sec A + tan A

= RHS

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Solution: (sin θ – 2 $\sin^3$ θ)/ (2$\cos^3$ θ – cos θ)

= [sin θ (1 – 2$\sin^2$ θ)]/[cos θ (2$\cos^2$ θ – 1)]

= sin θ [1 – 2 (1 – $\cos^2$ θ)] / [cos θ (2 $\cos^2$ θ – 1)]

= [sin θ (2$\cos^2$ θ – 1)] / [cos θ (2$\cos^2$ θ – 1)]

= tan θ = RHS

(viii) $(\sin A + \operatorname{cosec} A)^2 (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

Solution: LHS = (sin A + cosec A)^2 + (cos A + sec A)^2

= ($\sin^2$A + cosec^2A + 2sin A cosec A) + ($\cos^2$A + $\sec^2$A + 2 cos A sec A)

= ($\sin^2$A + $\cos^2$A) + 2 sin A (1/sinA) + 2cos A (1/cosA) + 1 + $\tan^2$A + 1 + $\cot^2$A

= 1 + 2 + 2 + 2 + $\tan^2$A + $\cot^2$A

= 7 + $\tan^2$A + $\cot^2$A

= RHS

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

[Hint: Simplify LHS and RHS separately]

Solution: LHS = (cosec A – sin A) (sec A – cos A)

= (1/sin A – sin A) (1/cos A – cos A)

$= [(1 – \sin^2A)/ \sin A] [(1 – \cos^2A)/\cos A]$

$= (\cos^2A/\sin A) * (\sin^2A/\cos A)$

$= \cos A \sin A$

RHS $= 1/(\tan A + \cot A)$

$= 1 / (\sin A/\cos A + \cos A / \sin A)$

$= 1/[(\sin^2A + \cos^2A)/\sin A \cos A]$

$= \cos A \sin A$

LHS = RHS

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan\ A}{1-\cot\ A}\right)^2 = \tan^2 A$

Solution: LHS $= (1 + \tan^2A/ 1 + \cot^2A)$

$= (1 + \tan^2A / 1 + 1/\tan^2A)$

$= 1 + \tan^2A/ [(1 + \tan^2A)/\tan^2A]$

$= \tan^2A$