Exercise 8(B)

1. Express in terms of log 2 and log 3:
(i) log 36
(ii) log 144
(iii) log 4.5
(iv) log 26/51 - log 91/119
(v) log 75/16 - 2log 5/9 + log 32/243
Solution:

(i) log 36 = log (2 x 2 x 3 x 3)
= log (2^2 x 3^2)
= log 2^2 + log 3^2 [Using $\log_a mn = \log_a m + \log_a n$]
= 2log 2 + 2log 3 [Using $\log_a m^n = n\log_a m$]

(ii) log 144 = log (2 x 2 x 2 x 2 x 3 x 3)
= log (2^4 x 3^2)
= log 2^4 + log 3^2 [Using $\log_a mn = \log_a m + \log_a n$]
= 4log 2 + 2log 3 [Using $\log_a m^n = n\log_a m$]

(iii) log 4.5 = log 45/10
= log (5 x 3 x 3)/ (5 x 2)
= log 3^2/2
= log 3^2 - log 2 [Using $\log_a m/n = \log_a m - \log_a n$]
= 2log 3 - log 2 [Using $\log_a m^n = n\log_a m$]

(iv) log 26/51 - log 91/119 = log (26/51)/ (91/119) [Using $\log_a m - \log_a n = \log_a m/n$]
= log [(26/51) x (119/91)]
= log (2 x 13 x 7 x 117)/ (3 x 17 x 7 x 13)
= log 2/3
= log 2 – log 3 [Using $\log_a m/n = \log_a m - \log_a n$]

(v) log 75/16 - 2log 5/9 + log 32/243
= log 75/16 - log $(5/9)^2$ + log 32/243 [Using $n\log_a m = \log_a m^n$]
= log 75/16 - log 25/81 + log 32/243
= log [(75/16)/ (25/81)] + log 32/243 [Using $\log_a m - \log_a n = \log_a m/n$]
= log (75 x 81)/ (16 x 25) + log 32/243
= log (3 x 81)/16 + log 32/243
= log 243/16 + log 32/243
= log (243/16) x (32/243) [Using $\log_a m + \log_a n = \log_a mn$]
= log 32/16
= log 2

2. Express each of the following in a form free from logarithm:
(i) 2 log x - log y = 1
(ii) 2 log x + 3 log y = log a
(iii) a log x - b log y = 2 log 3

Solution:

(i) We have, 2 log x - log y = 1
Then,
log x^2 - log y = 1 [Using $n\log_a m = \log_a m^n$]
log x^2/y = 1 [Using $\log_a m - \log_a n = \log_a m/n$]
Now, on removing log we have
$x^2/y = 10^1$
⇒ $x^2 = 10y$

(ii) We have, 2 log x + 3 log y = log a
Then,
log x^2 + log y^3 = log a [Using $n\log_a m = \log_a m^n$]
log x^2y^3 = log a [Using $\log_a m + \log_a n = \log_a mn$]
Now, on removing log we have
$x^2y^3 = a$

(iii) a log x - b log y = 2 log 3
Then,
log x^a - log y^b = log 3^2 [Using $n\log_a m = \log_a m^n$]
log x^a/y^b = log 3^2 [Using $\log_a m - \log_a n = \log_a m/n$]
Now, on removing log we have
$x^a/y^b = 3^2$
⇒ $x^2 = 9y^b$

3. Evaluate each of the following without using tables:
(i) log 5 + log 8 - 2log 2
(ii) $\log_{10} 8 + \log_{10} 25 + 2\log_{10} 3 - \log_{10} 18$
(iii) log 4 + 1/3 log 125 – 1/5 log 32
Solution:

(i) We have, log 5 + log 8 - 2log 2
= log 5 + log 8 - log 2^2 [Using $n\log_a m = \log_a m^n$]
= log 5 + log 8 - log 4
= log (5 x 8) - log 4 [Using $\log_a m + \log_a n = \log_a mn$]
= log 40 - log 4
= log 40/4 [Using $\log_a m - \log_a n = \log_a m/n$]
= log 10
= 1

(ii) We have, $\log_{10} 8 + \log_{10} 25 + 2\log_{10} 3 - \log_{10} 18$
= $\log_{10} 8 + \log_{10} 25 + \log_{10} 3^2 - \log_{10} 18$ [Using $n\log_a m = \log_a m^n$]
= $\log_{10} 8 + \log_{10} 25 + \log_{10} 9 - \log_{10} 18$
= $\log_{10} (8 \times 25 \times 9) - \log_{10} 18$ [Using $\log_a l + \log_a m + \log_a n = \log_a lmn$]
= $\log_{10} 1800 – \log_{10} 18$
= $\log_{10} 1800/18$ [Using $\log_a m - \log_a n = \log_a m/n$]

$= \log_{10} 100$
$= \log_{10} 10^2$
$= 2\log_{10} 10$ [Using $\log_a m^n = n\log_a m$]
$= 2 \times 1$
$= 2$

(iii) We have, $\log 4 + 1/3\log 125 – 1/5\log 32$
$= \log 4 + \log (125)^{1/3} – \log (32)^{1/5}$ [Using $n\log_a m = \log_a m^n$]
$= \log 4 + \log (5^3)^{1/3} – \log (2^5)^{1/5}$
$= \log 4 + \log 5 – \log 2$
$= \log (4 \times 5) – \log 2$ [Using $\log_a m + \log_a n = \log_a mn$]
$= \log 20 – \log 2$
$= \log 20/2$ [Using $\log_a m - \log_a n = \log_a m/n$]
$= \log 10$
$= 1$

4. Prove that:
2log 15/18 – log 25/162 + log 4/9 = log 2
Solution:

Taking L.H.S.,
$= 2\log 15/18 – \log 25/162 + \log 4/9$
$= \log (15/18)^2 – \log 25/162 + \log 4/9$ [Using $n\log_a m = \log_a m^n$]
$= \log 225/324 – \log 25/162 + \log 4/9$
$= \log [(225/324)/(25/162)] + \log 4/9$ [Using $\log_a m - \log_a n = \log_a m/n$]
$= \log (225 \times 162)/(324 \times 25) + \log 4/9$
$= \log (9 \times 1)/(2 \times 1) + \log 4/9$
$= \log 9/2 + \log 4/9$ [Using $\log_a m + \log_a n = \log_a mn$]
$= \log (9/2 \times 4/9)$
$= \log 2$
= R.H.S.

5. Find x, if:
x - log 48 + 3 log 2 = 1/3 log 125 - log 3.
Solution:

We have,
$x - \log 48 + 3 \log 2 = 1/3 \log 125 - \log 3$
Solving for x, we have
$x = \log 48 – 3 \log 2 + 1/3 \log 125 - \log 3$
$= \log 48 – \log 2^3 + \log 125^{1/3} – \log 3$ [Using $n\log_a m = \log_a m^n$]
$= \log 48 – \log 8 + \log (5^3)^{1/3} – \log 3$
$= (\log 48 – \log 8) + (\log 5 – \log 3)$
$= \log 48/8 + \log 5/3$ [Using $\log_a m - \log_a n = \log_a m/n$]
$= \log (48/8 \times 5/3)$ [Using $\log_a m + \log_a n = \log_a mn$]
$= \log (2 \times 5)$

= log 10
= 1
Hence, x = 1

6. Express $\log_{10} 2 + 1$ in the form of $\log_{10} x$.
Solution:

Given, $\log_{10} 2 + 1$
$= \log_{10} 2 + \log_{10} 10$ [As, $\log_{10} 10 = 1$]
$= \log_{10} (2 \times 10)$ [Using $\log_a m + \log_a n = \log_a mn$]
$= \log_{10} 20$

7. Solve for x:
(i) $\log_{10} (x - 10) = 1$
(ii) $\log (x^2 - 21) = 2$
(iii) log (x - 2) + log (x + 2) = log 5
(iv) log (x + 5) + log (x - 5) = 4 log 2 + 2 log 3
Solution:

(i) We have, $\log_{10} (x - 10) = 1$
Then,
$x - 10 = 10^1$
x = 10 + 10
Hence, x = 20

(ii) We have, $\log (x^2 - 21) = 2$
Then,
$x^2 - 21 = 10^2$
$x^2 - 21 = 100$
$x^2 = 100 + 21$
$x^2 = 121$
Taking square root on both sides,
Hence, x = ±11

(iii) We have, log (x - 2) + log (x + 2) = log 5
Then,
log (x - 2)(x + 2) = log 5 [Using $\log_a m + \log_a n = \log_a mn$]
$\log (x^2 - 2^2) = \log 5$ [As $(x - a)(x + a) = x^2 - a^2$]
$\log (x^2 - 4) = \log 5$
Removing log on both sides, we get
$x^2 - 4 = 5$
$x^2 = 5 + 4$
$x^2 = 9$
Taking square root on both sides,
x = ±3

(iv) We have, log (x + 5) + log (x - 5) = 4 log 2 + 2 log 3
Then,
log (x + 5) + log (x - 5) = log 2^4 + log 3^2 [Using $n\log_a m = \log_a m^n$]
log (x + 5)(x - 5) = log 16 + log 9 [Using $\log_a m + \log_a n = \log_a mn$]
log (x^2 - 5^2) = log (16 x 9) [As $(x - a)(x + a) = x^2 – a^2$]
log (x^2 – 25) = log 144
Removing log on both sides, we have
x^2 – 25 = 144
x^2 = 144 + 25
x^2 = 169
Taking square root on both sides, we get
x = ±13

8. Solve for x:
(i) log 81/log 27 = x
(ii) log 128/log 32 = x
(iii) log 64/log 8 = log x
(iv) log 225/log 15 = log x
Solution:

(i) We have, log 81/log 27 = x
x = log 81/log 27
= log (3 x 3 x 3 x 3)/ log (3 x 3 x 3)
= log 3^4/log 3^3
= (4log 3)/(3log 3) [Using $\log_a m^n = n\log_a m$]
= 4/3
Hence, x = 4/3

(ii) We have, log 128/log 32 = x
x = log 128/log 32
= log (2 x 2 x 2 x 2 x 2 x 2 x 2)/ log (2 x 2 x 2 x 2 x 2)
= log 2^7/log 2^5
= (7log 2)/(5log 2) [Using $\log_a m^n = n\log_a m$]
= 7/5
Hence, x = 7/5

(iii) log 64/log 8 = log x
log x = log 64/log 8
= log (2 x 2 x 2 x 2 x 2 x 2)/ log (2 x 2 x 2)
= log 2^6/log 2^3
= (6log 2)/(3log 2) [Using $\log_a m^n = n\log_a m$]
= 6/3
= 2
So, log x = 2
Hence, x = 10^2 = 100

(iv) We have, log 225/log 15 = log x
log x = log 225/log 15
= log (15 x 15)/ log 15
= log 15^2/log 15
= (2log 15)/log 15 [Using $\log_a m^n = n\log_a m$]
= 2
So, log x = 2
Hence, x = 10^2 = 100

9. Given log x = m + n and log y = m - n, express the value of log 10x/y^2 in terms of m and n.
Solution:

Given, log x = m + n and log y = m – n
Now consider log 10x/y^2,
log 10x/y^2 = log 10x – log y^2 [Using $\log_a m/n = \log_a m - \log_a n$]
= log 10x – 2log y
= log 10 + log x – 2 log y
= 1 + (m + n) – 2 (m - n)
= 1 + m + n – 2m + 2n
= 1 + 3n - m

10. State, true or false:
(i) log 1 × log 1000 = 0
(ii) log x/log y = log x – log y
(iii) If log 25/log 5 = log x, then x = 2
(iv) log x × log y = log x + log y
Solution:

(i) We have, log 1 × log 1000 = 0
Now,
log 1 = 0 and
log 1000 = log 10^3 = 3log 10 = 3 [Using $\log_a m^n = n\log_a m$]
So,
log 1 × log 1000 = 0 x 3 = 0
Thus, the statement log 1 × log 1000 = 0 is true

(ii) We have, log x/log y = log x – log y
We know that,
log x/y = log x – log y
So,
log x/log y ≠ log x – log y
Thus, the statement log x/log y = log x – log y is false

(iii) We have, log 25/log 5 = log x
log (5 x 5)/log 5 = log x
log 5^2/ log 5 = log x
2log 5/log 5 = log x [Using $\log_a m^n = n\log_a m$]
2 = log x
So, x = 10^2
x = 100
Thus, the statement x = 2 is false

(iv) We know, log x + log y = log xy
So,
log x + log y ≠ log x × log y
Thus, the statement log x + log y = log x × log y is false

11. If $\log_{10} 2$ = a and $\log_{10} 3$ = b; express each of the following in terms of 'a' and 'b':
(i) log 12
(ii) log 2.25

(iii) log $2\frac{1}{4}$

(iv) log 5.4
(v) log 60

(iv) log $3\frac{1}{8}$

Solution:

Given that $\log_{10} 2$ = a and $\log_{10} 3$ = b … (1)
(i) log 12 = log (2 x 2 x 3)
= log (2 x 2) + log 3 [Using $\log_a mn = \log_a m + \log_a n$]
= log 2^2 + log 3
= 2log 2 + log 3 [Using $\log_a m^n = n\log_a m$]
= 2a + b [From 1]

(ii) log 2.25 = log 225/100
= log (25 x 9)/(25 x 4)
= log 9/4
= log $(3/2)^2$
= 2log 3/2 [Using $\log_a m^n = n\log_a m$]
= 2(log 3 – log 2) [Using $\log_a m/n = \log_a m - \log_a n$]
= 2(b - a) [From 1]
= 2b – 2a

(iii) log $2\frac{1}{4}$ = log 9/4
= log $(3/2)^2$

= 2log 3/2 [Using $\log_a m^n = n\log_a m$]
= 2(log 3 – log 2) [Using $\log_a m/n = \log_a m - \log_a n$]
= 2(b - a) [From 1]
= 2b – 2a

(iv) log 5.4 = log 54/10
= log (2 x 3 x 3 x 3)/10
= log (2 x 3^3) – log 10 [Using $\log_a m/n = \log_a m - \log_a n$]
= log 2 + log 3^3 – 1 [Using $\log_a mn = \log_a m + \log_a n$ and log 10 = 1]
= log 2 + 3log 3 – 1 [Using $\log_a m^n = n\log_a m$]
= a + 3b – 1 [From 1]

(v) log 60 = log (10 x 3 x 2)
= log 10 + log 3 + log 2 [Using $\log_a lmn = \log_a l + \log_a m + \log_{10} n$]
= 1 + b + a [From 1]

(vi) log $3\frac{1}{8}$ = log 25/8
= log $5^2/2^3$
= log 5^2 – log 2^3 [Using $\log_a m/n = \log_a m - \log_a n$]
= 2log 5 – 3log 2 [Using $\log_a m^n = n\log_a m$]
= 2log 10/2 – 3log 2
= 2(log 10 – log 2) – 3log 2 [Using $\log_a m/n = \log_a m - \log_a n$]
= 2log 10 – 2log 2 – 3log 2
= 2(1) – 2a – 3a [From 1]
= 2 – 5a

12. If log 2 = 0.3010 and log 3 = 0.4771; find the value of:
(i) log 12
(ii) log 1.2
(iii) log 3.6
(iv) log 15
(v) log 25
(vi) 2/3 log 8
Solution:

Given, log 2 = 0.3010 and log 3 = 0.4771
(i) log 12 = log (4 x 3)
= log 4 + log 3 [Using $\log_a mn = \log_a m + \log_a n$]
= log 2^2 + log 3
= 2log 2 + log 3 [Using $\log_a m^n = n\log_a m$]
= 2 x 0.3010 + 0.4771
= 1.0791

(ii) log 1.2 = log 12/10
= log 12 – log 10 [Using $\log_a m/n = \log_a m - \log_a n$]
= log (4 x 3) – log 10

= log 4 + log 3 – log 10 [Using $\log_a mn = \log_a m + \log_a n$]
= log 2^2 + log 3 – log 10
= 2log 2 + log 3 – log 10 [Using $\log_a m^n = n\log_a m$]
= 2 x 0.3010 + 0.4771 – 1 [As log 10 = 1]
= 0.6020 + 0.4771 – 1
= 1.0791 – 1
= 0.0791

(iii) log 3.6 = log 36/10
= log 36 – log 10 [Using $\log_a m/n = \log_a m - \log_a n$]
= log (2 x 2 x 3 x 3) – 1 [As log 10 = 1]
= log (2^2 x 3^2) – 1
= log 2^2 + log 3^2 – 1 [Using $\log_a mn = \log_a m + \log_a n$]
= 2log 2 + 2log 3 – 1 [Using $\log_a m^n = n\log_a m$]
= 2 x 0.3010 + 2 x 0.4771 – 1
= 0.6020 + 0.9542 – 1
= 1.5562 – 1
= 0.5562

(iv) log 15 = log (15/10 x 10)
= log 15/10 + log 10 [Using $\log_a mn = \log_a m + \log_a n$]
= log 3/2 + 1 [As log 10 = 1]
= log 3 – log 2 + 1 [Using $\log_a m/n = \log_a m - \log_a n$]
= 0.4771 - 0.3010 + 1
= 1.1761

(v) log 25 = log (25/4 x 4)
= log 100/4
= log 100 – log 4 [Using $\log_a m/n = \log_a m - \log_a n$]
= log 10^2 – log 2^2
= 2log 10 – 2log 2 [Using $\log_a m^n = n\log_a m$]
= (2 x 1) – (2 x 0.3010)
= 2 – 0.6020
= 1.398

13. Given 2 $\log_{10} x$ + 1 = $\log_{10} 250$, find:
(i) x
(ii) $\log_{10} 2x$
Solution:

(i) Given equation, 2$\log_{10} x$ + 1 = $\log_{10} 250$
$\log_{10} x^2 + \log_{10} 10 = \log_{10} 250$ [Using $n\log_a m = \log_a m^n$ and $\log_{10} 10 = 1$]
$\log_{10} 10x^2 = \log_{10} 250$ [Using $\log_a m + \log_a n = \log_a mn$]
Removing log on both sides, we have
$10x^2 = 250$
$x^2 = 25$

$x = \pm 5$
As x cannot be a negative value, $x = -5$ is not possible
Hence, $x = 5$

(ii) Now, from (i) we have $x = 5$
So,
$\log_{10} 2x = \log_{10} 2(5)$
$= \log_{10} 10$
$= 1$

14. Given 3log x + ½ log y = 2, express y in term of x.
Solution:

We have, $3\log x + \frac{1}{2} \log y = 2$
$\log x^3 + \log y^{1/2} = 2$ [Using $\log_a m + \log_a n = \log_a mn$]
$\log x^3y^{1/2} = 2$
Removing logarithm, we get
$x^3y^{1/2} = 10^2$
$y^{1/2} = 100/x^3$
On squaring on both sides, we get
$y = 10000/x^6$
$y = 10000x^{-6}$

15. If $x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$, find $\log 10\sqrt{y}/x^2z^3$ in terms of a, b and c.
Solution:

We have,
$x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$
So,
$\log x = a \log 100$, $\log y = b \log 10000$ and $\log z = c \log 10$
$\Rightarrow \log x = a \log 10^2$, $\log y = b \log 10^4$ and $\log z = c \log 10$
$\Rightarrow \log x = 2a \log 10$, $\log y = 4b \log 10$ and $\log z = c \log 10$
$\Rightarrow \log x = 2a$, $\log y = 4b$ and $\log z = c$ … (i)
Now,
$\log 10\sqrt{y}/x^2z^3 = \log 10\sqrt{y} - \log x^2z^3$ [Using $\log_a m/n = \log_a m - \log_a n$]
$= (\log 10 + \log \sqrt{y}) - (\log x^2 + \log z^3)$ [Using $\log_a mn = \log_a m + \log_a n$]
$= 1 + \log y^{1/2} - \log x^2 - \log z^3$
$= 1 + \frac{1}{2} \log y - 2 \log x - 3 \log z$ [Using $\log_a m^n = n\log_a m$]
$= 1 + \frac{1}{2}(4b) - 2(2a) - 3c$ … [Using (i)]
$= 1 + 2b - 4a - 3c$

16. If 3(log 5 – log 3) – (log 5 – 2 log 6) = 2 – log x, find x.
Solution:

We have,
$3(\log 5 - \log 3) - (\log 5 - 2 \log 6) = 2 - \log x$

3log 5 – 3log 3 – log 5 + 2log 6 = 2 – log x
3log 5 – 3log 3 – log 5 + 2log (3 x 2) = 2 – log x
2log 5 – 3log 3 + 2(log 3 + log 2) = 2 – log x [Using $\log_a mn = \log_a m + \log_a n$]
2log 5 – log 3 + 2log 3 + 2log 2 = 2 – log x
2log 5 - log 3 + 2log 2 = 2 – log x
2log 5 - log 3 + 2log 2 + log x = 2
$\log 5^2 - \log 3 + \log 2^2 + \log x = 2$ [Using $n\log_a m = \log_a m^n$]
log 25 - log 3 + log 4 + log x = 2
log (25 × 4 × x)/3 = 2 [Using $\log_a m + \log_a n = \log_a mn$ & $\log_a m - \log_a n = \log_a m/n$]
log 100x/3 = 2
On removing logarithm,
$100x/3 = 10^2$
100x/3 = 100
Dividing by 100 on both sides, we have
x/3 = 1
Hence, x = 3