EXERCISE 13.2

Write True or False and justify your answer in each of the following:

1. The volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere.

Solution:

True

Justification:

Let the radius of the sphere = r.

According to the question,

height and diameter of cylinder = diameter of sphere.

So, the radius of the cylinder = r

And, the height of the cylinder = 2r

We know that,

Volume of sphere = 2/3 volume of cylinder

$$\Rightarrow \frac{4}{3}\pi r^3 = \frac{2}{3}(\pi r^2 \times 2r) = \frac{4}{3}\pi r^3$$

Hence, the given statement “the volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere” is true.

2. If the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged.

Solution:

False

Justification:

Let the original radius of the cone = r

Let height of the cone = h.

The volume of cone = 1/3 $\pi r^2 h$

Now, when radius of a height circular cone is halved and height is doubled, then

$$V = \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times 2h = \frac{1}{3}\pi \times \frac{r^2}{4} \times 2h = \frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right)$$

We can observe that the new volume = half of the original volume.

Hence, the given statement “if the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged” is false.

3. In a right circular cone, height, radius and slant height do not always be sides of a right triangle.

Solution:

Consider a right circular cone, with

Height = h

Radius = r

And slant height = l

We know, right triangle = one angle 90°

Using Pythagoras theorem in
⇒ $l^2 = h^2 + r^2$
This justifies that height, radius and slant height of cone can always be the sides of a right triangle.
Hence, the given statement "in a right circular cone, height, radius and slant height do not always be sides of a right triangle" is true.

4. If the radius of a cylinder is doubled and its curved surface area is not changed, the height must be halved.
Solution:

True
Justification:
Let radius of the cylinder = r
Height of the cylinder = h
Then, curved surface area of the cylinder, CSA = 2πrh
According to the question,
Radius is doubled and curved surface area is not changed.
New radius of the cylinder, R = 2r
New curved surface area of the cylinder, CSA' = 2πrh …(i)
Alternate case:
When R = 2r and CSA' = 2πrh
But curved surface area of cylinder in this case, CSA'= 2πRh = 2π(2r)h = 4πrh …(ii)
Comparing equations (i) and (ii),
We get,
Since, 2πrh ≠ 4πrh
equation (i) ≠ equation (ii)
Thus, if h = h/2 (height is halved)
Then,
CSA' = 2π(2r)(h/2) = 2πrh
Hence, the given statement "If the radius of a cylinder is doubled and its curved surface area is not changed, the height must be halved" is true.

5. The volume of the largest right circular cone that can be fitted in a cube whose edge is 2r equals to the volume of a hemisphere of radius r.
Solution:

According to the question,
Edge of cube, l = 2r
Then, diameter of the cone = 2r
⇒ radius of the cone = 2r/2
= r
Height of the cone, h = height of the cube
= 2r
Volume of the cone is given by,
Volume of cone = $1/3\ \pi r^2 h$
= $1/3\ \pi r^2(2r)$
= $2/3\ \pi r^3$

= Volume of hemisphere of radius r

Hence, the given statement "the volume of the largest right circular cone that can be fitted in a cube whose edge is 2r equals to the volume of a hemisphere of radius r" is true.