NCERT Solutions for Class-XII Maths

Chapter-12
Exercise- Miscellaneous

NCERT Chemistry Class 12

1. Refer to Example 9. How many packets of each food should be used to maximize the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

1. let x and y be the number of packets of food P and Q respectively. Obviously, $x \geq 0$, $y \geq 0$. Mathematical formulation of given problem is as follows:

Maximize $Z = 6x + 3y$ ….(1)

Subject to the constraints,

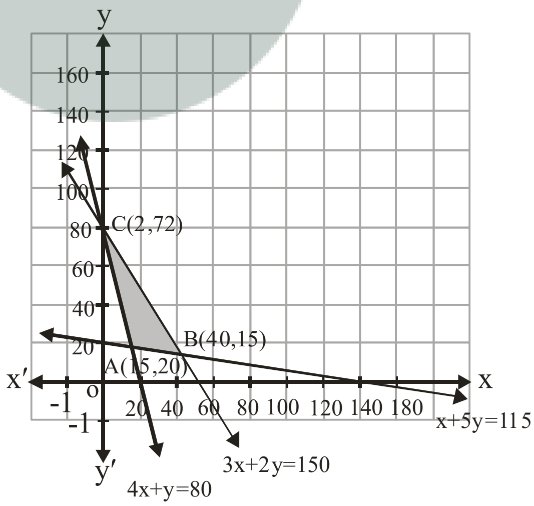
$12x + 3y \geq 240$ (constraint on calcium) i.e. $4x + y \geq 80$ …..(2)

$4x + 20y \geq 460$ (constraint on iron) i.e. $x + 5y \geq 115$ …..(3)

$6x + 4y \leq 300$ (constraint on cholesterol) i.e. $3x + 2y \leq 150$ ….(4)

$x \geq 0$, $y \geq 0$ ….(5)

Now let us graph the feasible region of the system of inequalities (2) to (5). The feasible region (shaded) is shown in the fig. Here, we can observe that the feasible region is bounded.



The coordinates of the corner points A (15,20), B(40,15), and C(2,72).

Corner points	Corresponding value of Z= 6x + 3y
A (15,20)	150
B (40,15)	285 ⟶ Maximum
C (2,72)	228

Now, we find the maximum value of Z. According to table the maximum value of Z = 285 at point B (40,15).
Hence, to get the maximum amount of vitamin A in the diet, the packets of food P should be 40 and the packets of food Q should be 15. The maximum amount of vitamin A will be 285.

2. A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

2. Let the farmer mix x bags of brand P and y bags of brand Q.
The given information can be compiled in a table as follows.

	Vitamin A (units/kg)	**Vitamin B (units/kg)**	**Cost (Rs/kg)**
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	8	11	

The given problem can be formulated as follows.
Minimize $z = 250x + 200y \ldots$ (1)
subject to the constraints,

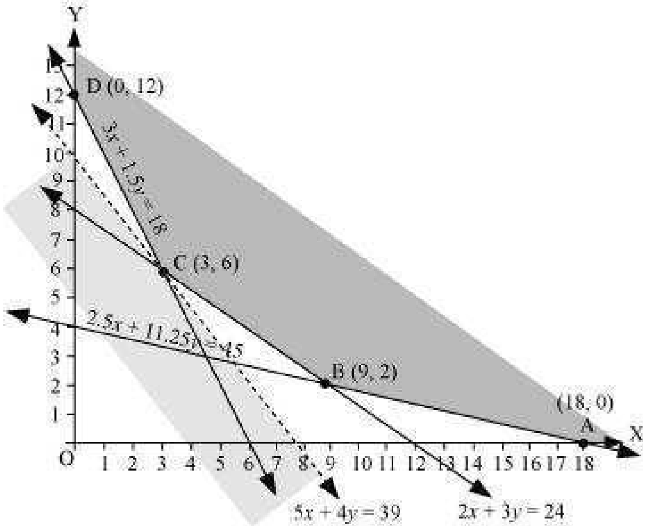
$3x + 1.5y \geq 18$ …(2)

$2.5x + 11.25y \geq 45$ … (3)

$2x + 3y \geq 24$ … (4)

$x, y \geq 0$ … (5)

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (18, 0), B (9, 2), C (3, 6), and D (0, 12).
The values of z at these corner points are as follows.

Corner point	**z = 250x + 200y**	
A (18, 0)	4500	
B (9, 2)	2650	
C (3, 6)	1950	→ Minimum
D (0, 12)	2400	

As the feasible region is unbounded, therefore, 1950 may or may not be the minimum
For this, we draw a graph of the inequality, $250x + 200y < 1950$ or $5x + 4y < 39$, and check whether the resulting half plane has points in common with the feasible region or not.
It can be seen that the feasible region has no common point with $5x + 4y < 39$.
Therefore, the minimum value of z is 2000 at (3, 6).
Thus, 3 bags of brand P and 6 bags of brand Q should be used in the mixture to minimize the cost to Rs 1950.

3. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below:

Food	**Vitamin A**	**Vitamin B**	**Vitamin C**
X	1	2	3
Y	2	2	1

One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet?

3. let x kg and y kg be amount of food X and Y respectively. Obviously $x \geq 0$, $y \geq 0$. Mathematical formulation of given problem is as follows:

Minimize Z = 16x + 20y ….(1)

Subject to the constraints,

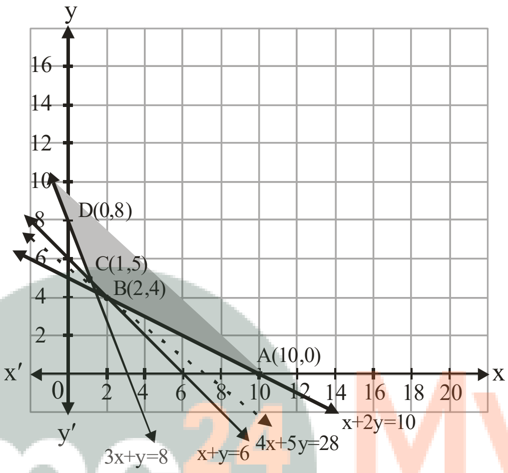
$x + 2y \geq 10$ …..(2)

$x + y \geq 6$ …..(3)

$3x + y \geq 8$ ….(4)

$x \geq 0, y \geq 0$ ….(5)

Now let us graph the feasible region of the system of inequalities (2) to (5). The feasible region (shaded) is shown in the fig. Here, we can observe that the feasible region is unbounded.



The coordinates of the corner points A(10,0),B(2,4), C(1,5)and D(0,8).

Corner points	Corresponding value of Z= 16x + 20y
A (10,0)	160
B (2,4)	112 → Minimum
C (1,5)	116
D (0,8)	160

As, feasible region is unbounded so 112 may or may not be minimum value of Z.

So, 112 is the minimum value of Z, if the open half plane is determined by $16x + 20y < 112$ has no point in common with the feasible region. Otherwise z has no minimum value.

For this we will draw the graph of inequality $16x + 20y < 112$ or $4x + 5y < 28$. Here we find that $16x + 20y < 112$ has no point in common with the feasible region. Hence the minimum value of Z is 112 at B(2,4).

So, 2kg and 4kg be the amount of two foods P and Q respectively which are mixed to get minimum cost to Rs 112.

4. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Types of Toys	Machines		
	I	II	III

A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

4. Let x and y toys of type A and type B respectively be manufactured in a day. The given problem can be formulated as follows. Maximize $z = 7.5x + 5y \ldots$ (1) subject to the constraints,

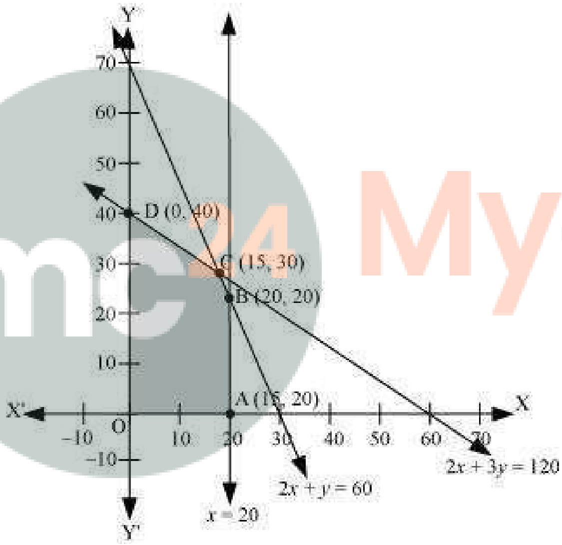
$2x + y \leq 60$ … (1)

$x \leq 20$ … (3)

$2x + 3y \leq 120$ … (4)

$x, y \geq 0$ … (5)

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 0), B (20, 20), C (15, 30), and D (0, 40).

The values of z at these corner points are as follows.

Corner point	**Z = 7.5x + 5y**	
A (20, 0)	150	
B (20, 20)	250	
C (15, 30)	262.5	→ Maximum
O (0, 40)	200	

The maximum value of z is 262.5 at (15, 30).

Thus, the manufacturer should manufacture 15 toys of type A and 30 toys of type B to maximize the profit.

5. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

5. let x and y be the number of tickets of executive class and economy class respectively. Obviously $x \geq 0, y \geq 0$. Mathematical formulation of given problem is as follows:

Maximize $Z = 1000x + 600y$ ….(1)

Subject to the constraints,

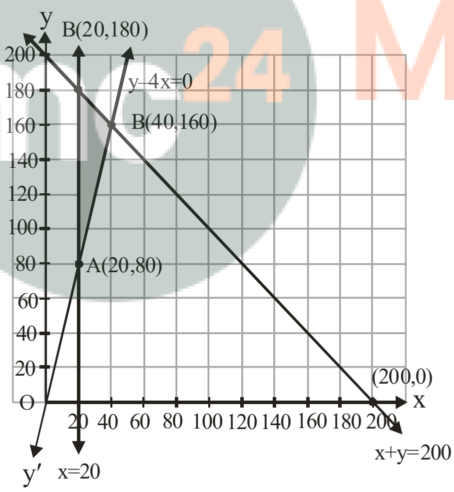
$x + y \leq 200$ …..(2)

$x \geq 20$ …..(3)

$y - 4x \geq 0$ ….(4)

$x \geq 0, y \geq 0$ ….(5)

Now let us graph the feasible region of the system of inequalities (2) to (5). The feasible region (shaded) is shown in the fig. Here, we can observe that the feasible region is bounded.



The coordinates of the corner points A(20,80),B(40,160) and C(20,180).

Corner points	**Corresponding value of Z= 1000x + 600y**
A(20,80)	68000
B(40,160)	136000 → Maximum
C(20,180)	128000

Now, we find the maximum value of Z. According to table the maximum value of Z = 136000 at point B (40,160).

Hence, 40 and 160 be the number of tickets of executive class and economy class respectively to get the maximum profit.

6. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Rs)		
From/To	**A**	**B**
D	6	4
E	3	2
F	2.50	3

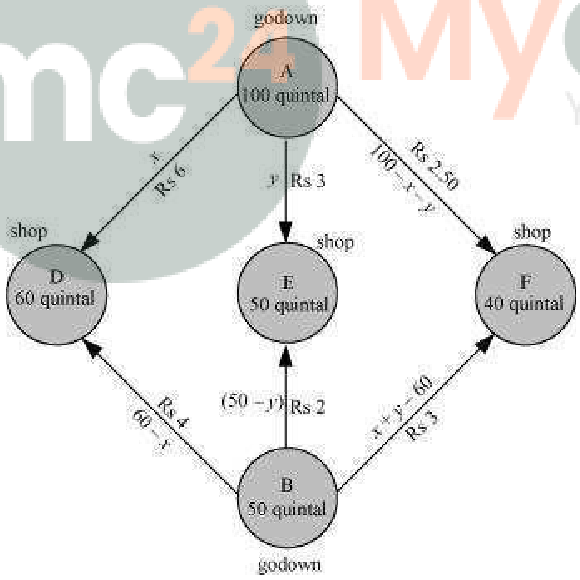
How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

6. Let godown A supply x and y quintals of grain to the shops D and E respectively. Then, (100 − x − y) will be supplied to shop F.

The requirement at shop D is 60 quintals since x quintals are transported from go down A. Therefore, the remaining (60 −x) quintals will be transported from godown B.

Similarly, (50 − y) quintals and 40 − (100 − x − y) = (x + y − 60) quintals will be transported from godown B to shop E and F respectively.

The given problem can be represented diagrammatically as follows.



$x \ge 0, y \ge 0$, and $100 - x - y \ge 0$

$\Rightarrow x \ge 0, y \ge 0$, and $x + y \le 100$

$60 - x \ge 0, 50 - y \ge 0$, and $x + y - 60 \ge 0$

$\Rightarrow x \le 60, y \le 50$, and $x + y \ge 60$

Total transportation cost z is given by,

$z = 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60)$

$= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180$

$= 2.5x + 1.5y + 410$

The given problem can be formulated as

Minimize $z = 2.5x + 1.5y + 410 \ldots$ (1)

subject to the constraints,

$x + y \leq 100$ … (2)

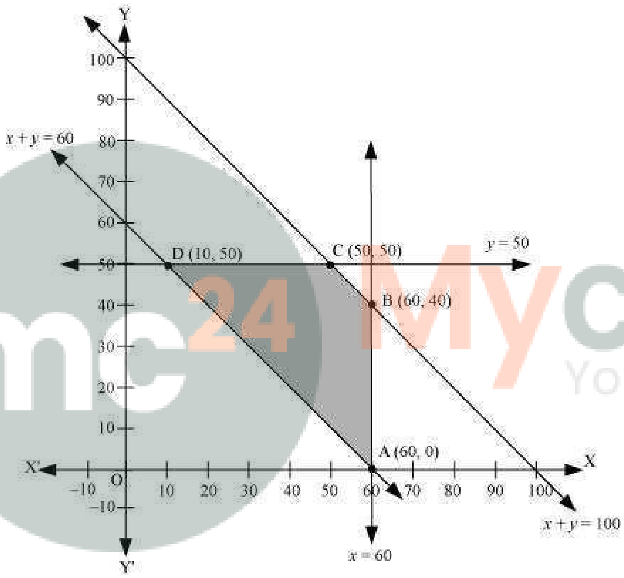
$x \leq 60$ … (3)

$y \leq 50$ … (4)

$x + y \geq 60$ … (5)

$x, y \geq 0$ … (6)

The feasible region determined by the system of constraints is as follows.



The corner points are A (60, 0), B (60, 40), C (50, 50), and D (10, 50).

The values of z at these corner points are as follows.

Corner point	z = 2.5x + 1.5y + 410	
A (60, 0)	560	
B (60, 40)	620	
C (50, 50)	610	
D (10, 50)	510	→ Minimum

The minimum value of z is 510 at (10, 50).

Thus, the amount of grain transported from A to D, E, and F is 10 quintals, 50 quintals,

and 40 quintals respectively and from B to D, E, and F is 50 quintals, 0 quintals, and 0 quintals respectively.

7. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

Distance in (km.)		
From/To	**A**	**B**
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Re 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

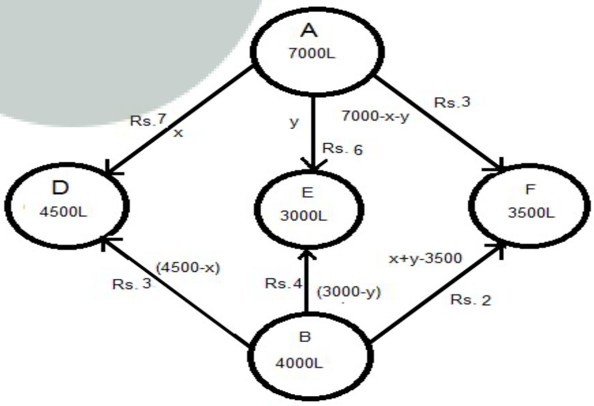
7. let x and y litres of oil be supplied by A to shop D and E respectively. Then 7000 - x - y will be supplied to F. Obviously $x \geq 0$, $y \geq 0$.

Since requirement at D is 4500L but supplied x quintal hence, 4500 - x is sent form B to D.

Similarly requirement at E is 3000L but supplied y quintal hence, 3000 - y is sent form B to E.

Similarly requirement at E is 3500 quintal but supplied 7000 - x - y quintal hence, 3500 - (7000 - x - y) i.e. x + y - 3500 is sent form B to E.

Diagrammatically it can be explained as:



As $x \geq 0$, $y \geq 0$ and $7000 - x - y \geq 0 \Rightarrow x + y \leq 7000$

$4500 - x \geq 0$, $3000 - y \geq 0$ and $x + y - 3500 \geq 0$

$\Rightarrow x \leq 4500$, $y \leq 3000$ and $x + y \geq 3500$

Cost of delivering 10L of petrol = Re.1

Then Cost of delivering 1L of petrol = Rs.1/10

Total transportation cost Z can be calculated as:

$$Z = \frac{7}{10}x + \frac{6}{10}y + \frac{3}{10}(7000 - x - y) + \frac{3}{10}(4500 - x) + \frac{4}{10}(3000 - y) + \frac{2}{10}(x + y - 3500)$$

Z = 0.3x + 0.1y + 3950

Mathematical formulation of given problem is as follows:

Minimize Z = 0.3x + 0.1y + 3950 ….(1)

Subject to the constraints,

$x + y \le 7000$ …..(2)

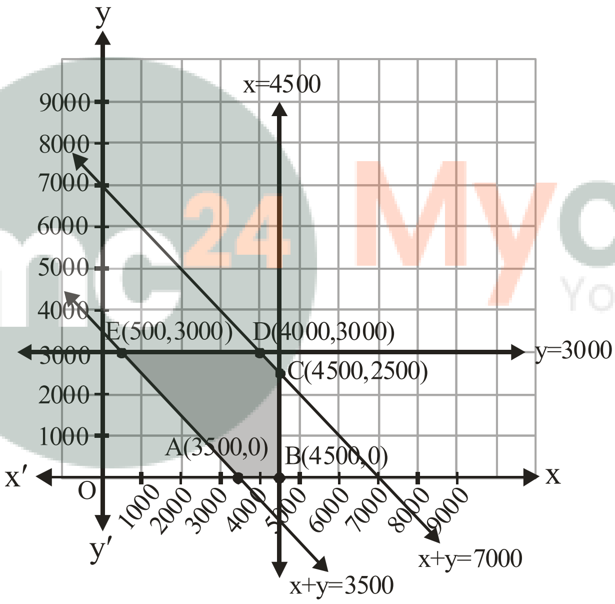
$x \le 4500$ …..(3)

$y \le 3000$ ….(4)

$x + y \ge 3500$ …(5)

$x \ge 0, y \ge 0$ ….(6)

Now let us graph the feasible region of the system of inequalities (2) to (6). The feasible region (shaded) is shown in the fig. Here, we can observe that the feasible region is bounded.



The coordinates of the corner points A(3500,0),B(4500,0),C(4500,2500), D(4000,3000) and E(500,3000) .

Corner points	Corresponding value of Z= 0.3x + 0.1y + 3950
A (3500,0)	5000
B (4500,0)	5300
C (4500,2500)	5550
D (4000,3000)	5450
E (500,3000)	4400 → Minimum

Now, we find the minimum value of Z. According to table the minimum value of Z = 4400 at point E (500,3000).

Hence, the amount of oil delivered from A to D, E and F is 500L, 3000L and 3500L quintals respectively and from B to D,E and F is 4000L, 0 L and 0 L respectively.

8. A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

kg per bag		
	Brand P	**Brand Q**
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

8. Let the fruit grower use x bags of brand P and y bags of brand Q.
The problem can be formulated as
follows. Minimize z = 3x + 3.5y … (1)
subject to the constraints,

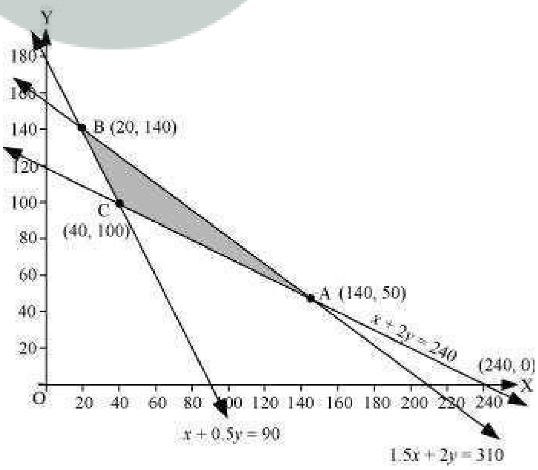
$x+2y\geq 240$ (2)

$x+0.5y\geq 90$ (3)

$1.5x+2y\leq 310$ (4)

$x,y\geq 0$ (5)

The feasible region determined by the system of constraints is as follows.



The corner points are A (240, 0), B (140, 50), and C (20, 140).
The values of z at these corner points are as follows.

Corner point	**z = 3x + 3.5y**	

A (140, 50)	595	
B (20, 140)	550	
C (40, 100)	470	→ Minimum

The maximum value of z is 470 at (40, 100).
Thus, 40 bags of brand P and 100 bags of brand Q should be added to the garden to minimize the amount of nitrogen.
The minimum amount of nitrogen added to the garden is 470 kg.

9. Refer to Question 8. If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen added?

9. let x and y be the bags of brands P and Q respectively. Obviously $x \geq 0$, $y \geq 0$. Mathematical formulation of given problem is as follows:

Maximize $Z = 3x + 3.5y$ ….(1)

Subject to the constraints,

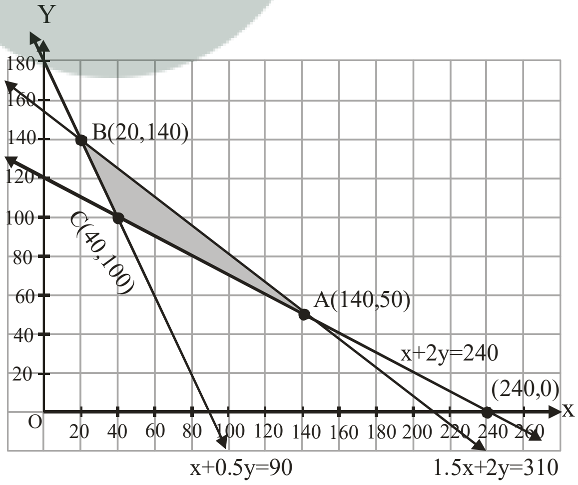
$x + 2y \leq 240$ …..(2)

$3x + 1.5y \leq 270 \Rightarrow x + 0.5y = 90$ …..(3)

$1.5x + 2y \leq 310$ ….(4)

$x \geq 0, y \geq 0$ ….(5)

Now let us graph the feasible region of the system of inequalities (2) to (5). The feasible region (shaded) is shown in the fig. Here, we can observe that the feasible region is bounded.



The coordinates of the corner points A(140,50),B(20,40) and C(40,100).

Corner points	Corresponding value of Z= 3x + 3.5y
A (140,50)	595 → Maximum
B (20,40)	550
C (40,100)	470

Now, we find the maximum value of Z. According to table the maximum value of Z = 595 at point A (140,50). Hence, 140 and 50 be the bags of brands P and Q respectively to be added to the garden to maximize the amount of nitrogen.

10. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

10. Let x and y be the number of dolls of type A and B respectively that are produced per week.

The given problem can be formulated as follows. Maximize z = 12x + 16y … (1) subject to the constraints,

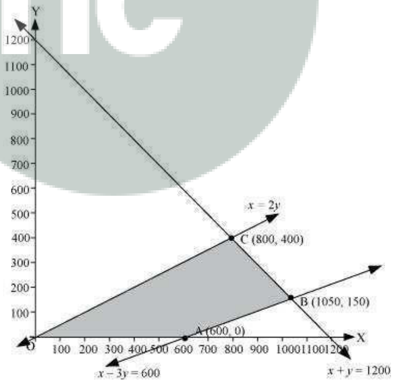
$x + y \leq 1200$ … (2)

$y \leq \frac{x}{2} \Rightarrow x \geq 2y$ … (3)

$x - 3y \leq 600$ … (4)

$x, y \geq 0$ … (5)

The feasible region determined by the system of constraints is as follows.



The corner points are A (600, 0), B (1050, 150), and C (800, 400).

The values of z at these corner points are as follows.

Corner point	z = 12x + 16y	
A (600, 0)	7200	
B (1050, 150)	15000	
C (800, 400)	16000	→ Maximum

The maximum value of z is 16000 at (800, 400).

Thus, 800 and 400 dolls of type A and type B should be produced respectively to get the maximum profit of Rs 16000.