EXERCISE 1.6

Find the smallest set A such that A ∪ {1, 2} = {1, 2, 3, 5, 9}. Solution:
A ∪ {1, 2} = {1, 2, 3, 5, 9}
Elements of A and {1, 2} together give us the result
So smallest set of A can be
A = {1, 2, 3, 5, 9} – {1, 2}
A = {3, 5, 9}

1. Let A = {1, 2, 4, 5} B = {2, 3, 5, 6} C = {4, 5, 6, 7}. Verify the following identities:
(i) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
(ii) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
(iii) A ∩ (B – C) = (A ∩ B) – (A ∩ C)
(iv) A – (B ∪ C) = (A – B) ∩ (A – C)
(v) A – (B ∩ C) = (A – B) ∪ (A – C)
(vi) A ∩ (B △ C) = (A ∩ B) △ (A ∩ C)
Solution:
(i) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
Firstly let us consider the LHS
(B ∩ C) = {x: x ∈ B and x ∈ C}
= {5, 6}
A ∪ (B ∩ C) = {x: x ∈ A or x ∈ (B ∩ C)}
= {1, 2, 4, 5, 6}

Now, RHS
(A ∪ B) = {x: x ∈ A or x ∈ B}
= {1, 2, 4, 5, 6}.
(A ∪ C) = {x: x ∈ A or x ∈ C}
= {1, 2, 4, 5, 6, 7}
(A ∪ B) ∩ (A ∪ C) = {x: x ∈ (A ∪ B) and x ∈ (A ∪ C)}
= {1, 2, 4, 5, 6}
∴ LHS = RHS
Hence Verified.

(ii) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
Firstly let us consider the LHS
(B ∪ C) = {x: x ∈ B or x ∈ C}

= {2, 3, 4, 5, 6, 7}
(A ∩ (B ∪ C)) = {x: x ∈ A and x ∈ (B ∪ C)}
= {2, 4, 5}

Now, RHS
(A ∩ B) = {x: x ∈ A and x ∈ B}
= {2, 5}
(A ∩ C) = {x: x ∈ A and x ∈ C}
= {4, 5}
(A ∩ B) ∪ (A ∩ C) = {x: x ∈ (A∩B) and x ∈ (A∩C)}
= {2, 4, 5}
∴ LHS = RHS
Hence verified.

(iii) A ∩ (B – C) = (A ∩ B) – (A ∩ C)
B–C is defined as {x ∈ B: x ∉ C}
B = {2, 3, 5, 6}
C = {4, 5, 6, 7}
B–C = {2, 3}
Firstly let us consider the LHS
(A ∩ (B – C)) = {x: x ∈ A and x ∈ (B – C)}
= {2}

Now, RHS
(A ∩ B) = {x: x ∈ A and x ∈ B}
= {2, 5}
(A ∩ C) = {x: x ∈ A and x ∈ C}
= {4, 5}
(A ∩ B) – (A ∩ C) is defined as {x ∈ (A ∩ B): x ∉ (A ∩ C)}
= {2}
∴ LHS = RHS
Hence Verified.

(iv) A – (B ∪ C) = (A – B) ∩ (A – C)
Firstly let us consider the LHS
(B ∪ C) = {x: x ∈ B or x ∈ C}
= {2, 3, 4, 5, 6, 7}.
A – (B ∪ C) is defined as {x ∈ A: x ∉ (B ∪ C)}
A = {1, 2, 4, 5}

(B ∪ C) = {2, 3, 4, 5, 6, 7}
A – (B ∪ C) = {1}

Now, RHS
(A – B)
A – B is defined as {x ∈ A: x ∉ B}
A = {1, 2, 4, 5}
B = {2, 3, 5, 6}
A – B = {1, 4}

(A – C)
A – C is defined as {x ∈ A: x ∉ C}
A = {1, 2, 4, 5}
C = {4, 5, 6, 7}
A – C = {1, 2}
(A – B) ∩ (A – C) = {x: x ∈ (A – B) and x ∈ (A – C)}.
= {1}
∴ LHS = RHS
Hence verified.

(v) A – (B ∩ C) = (A – B) ∪ (A – C)
Firstly let us consider the LHS
(B ∩ C) = {x: x ∈ B and x ∈ C}
= {5, 6}
A – (B ∩ C) is defined as {x ∈ A: x ∉ (B ∩ C)}
A = {1, 2, 4, 5}
(B ∩ C) = {5, 6}
(A – (B ∩ C)) = {1, 2, 4}

Now, RHS
(A – B)
A – B is defined as {x ∈ A: x ∉ B}
A = {1, 2, 4, 5}
B = {2, 3, 5, 6}
A–B = {1, 4}

(A – C)
A – C is defined as {x ∈ A: x ∉ C}
A = {1, 2, 4, 5}

C = {4, 5, 6, 7}
A – C = {1, 2}
(A – B) ∪ (A – C) = {x: x ∈ (A – B) OR x ∈ (A – C)}.
= {1, 2, 4}
∴ LHS = RHS
Hence verified.

(vi) A ∩ (B △ C) = (A ∩ B) △ (A ∩ C)
A = {1, 2, 4, 5} B = {2, 3, 5, 6} C = {4, 5, 6, 7}.
Firstly let us consider the LHS
A ∩ (B △ C)
B △ C = (B – C) ∪ (C - B) = {2, 3} ∪ {4, 7} = {2, 3, 4, 7}
A ∩ (B △ C) = {2, 4}

Now, RHS
A ∩ B = {2, 5}
A ∩ C = {4, 5}
(A ∩ B) △ (A ∩ C) = [(A ∩ B) - (A ∩ C)] ∪ [(A ∩ C) - (A ∩ B)]
= {2} ∪ {4}
= {2, 4}
∴ LHS = RHS
Hence, Verified.

2. If U = {2, 3, 5, 7, 9} is the universal set and A = {3, 7}, B = {2, 5, 7, 9}, then prove that:
(i) (A ∪ B)' = A' ∩ B'
(ii) (A ∩ B)' = A' ∪ B'
Solution:
(i) (A ∪ B)' = A' ∩ B'
Firstly let us consider the LHS
A ∪ B = {x: x ∈ A or x ∈ B}
= {2, 3, 5, 7, 9}
(A∪B)' means Complement of (A∪B) with respect to universal set U.
So, (A∪B)' = U – (A∪B)'
U – (A∪B)' is defined as {x ∈ U: x ∉ (A∪B)'}
U = {2, 3, 5, 7, 9}
(A∪B)' = {2, 3, 5, 7, 9}
U – (A∪B)' = ϕ

Now, RHS
A' means Complement of A with respect to universal set U.
So, A' = U – A
(U – A) is defined as {x ∈ U: x ∉ A}
U = {2, 3, 5, 7, 9}
A = {3, 7}
A' = U – A = {2, 5, 9}

B' means Complement of B with respect to universal set U.
So, B' = U – B
(U – B) is defined as {x ∈ U: x ∉ B}
U = {2, 3, 5, 7, 9}
B = {2, 5, 7, 9}
B' = U – B = {3}
A' ∩ B' = {x: x ∈ A' and x ∈ C'}.
= ϕ
∴ LHS = RHS
Hence verified.

(ii) (A ∩ B)' = A' ∪ B'
Firstly let us consider the LHS
(A ∩ B)'
(A ∩ B) = {x: x ∈ A and x ∈ B}.
= {7}
(A∩B)' means Complement of (A ∩ B) with respect to universal set U.
So, (A∩B)' = U – (A ∩ B)
U – (A ∩ B) is defined as {x ∈ U: x ∉ (A ∩ B)'}
U = {2, 3, 5, 7, 9}
(A ∩ B) = {7}
U – (A ∩ B) = {2, 3, 5, 9}
(A ∩ B)' = {2, 3, 5, 9}

Now, RHS
A' means Complement of A with respect to universal set U.
So, A' = U – A
(U – A) is defined as {x ∈ U: x ∉ A}
U = {2, 3, 5, 7, 9}
A = {3, 7}
A' = U – A = {2, 5, 9}

B' means Complement of B with respect to universal set U.
So, B' = U – B
(U – B) is defined as {x ∈ U: x ∉ B}
U = {2, 3, 5, 7, 9}
B = {2, 5, 7, 9}
B' = U – B = {3}
A' ∪ B' = {x: x ∈ A or x ∈ B}
= {2, 3, 5, 9}
∴ LHS = RHS
Hence verified.

3. For any two sets A and B, prove that
(i) B ⊂ A ∪ B
(ii) A ∩ B ⊂ A
(iii) A ⊂ B ⇒ A ∩ B = A
Solution:
(i) B ⊂ A ∪ B
Let us consider an element 'p' such that it belongs to B
∴ p ∈ B
p ∈ B ∪ A
B ⊂ A ∪ B

(ii) A ∩ B ⊂ A
Let us consider an element 'p' such that it belongs to B
∴ p ∈ A ∩ B
p ∈ A and p ∈ B
A ∩ B ⊂ A

(iii) A ⊂ B ⇒ A ∩ B = A
Let us consider an element 'p' such that it belongs to A ⊂ B.
p ∈ A ⊂ B
Then, x ∈ B

Let and p ∈ A ∩ B
x ∈ A and x ∈ B
x ∈ A and x ∈ A (since, A ⊂ B)
∴ (A ∩ B) = A

4. For any two sets A and B, show that the following statements are equivalent:
(i) A ⊂ B
(ii) A – B = ϕ
(iii) A ∪ B = B
(iv) A ∩ B = A
Solution:
(i) A ⊂ B
To show that the following four statements are equivalent, we need to prove
(i)=(ii), (ii)=(iii), (iii)=(iv), (iv)=(v)
Firstly let us prove (i)=(ii)
We know, A–B = {x ∈ A: x ∉ B} as A ⊂ B,
So, Each element of A is an element of B,
∴ A–B = ϕ
Hence, (i)=(ii)

(ii) A – B = ϕ
We need to show that (ii)=(iii)
By assuming A–B = ϕ
To show: A∪B = B
∴ Every element of A is an element of B
So, A ⊂ B and so A∪B = B
Hence, (ii)=(iii)

(iii) A ∪ B = B
We need to show that (iii)=(iv)
By assuming A ∪ B = B
To show: A ∩ B = A.
∴ A⊂ B and so A ∩ B = A
Hence, (iii)=(iv)

(iv) A ∩ B = A
Finally, now we need to show (iv)=(i)
By assuming A ∩ B = A
To show: A ⊂ B
Since, A ∩ B = A, so A⊂B
Hence, (iv)=(i)

5. For three sets A, B, and C, show that
(i) A ∩ B = A ∩ C need not imply B = C.

(ii) A ⊂ B ⇒ C – B ⊂ C – A
Solution:
(i) A ∩ B = A ∩ C need not imply B = C.
Let us consider, A = {1, 2}
B = {2, 3}
C = {2, 4}
Then,
A ∩ B = {2}
A ∩ C = {2}
Hence, A ∩ B = A ∩ C, where, B is not equal to C

(ii) A ⊂ B ⇒ C – B ⊂ C – A
Given: A ⊂ B
To show: C–B ⊂ C–A
Let us consider x ∈ C– B
⇒ x ∈ C and x ∉ B [by definition C–B]
⇒ x ∈ C and x ∉ A
⇒ x ∈ C–A
Thus x ∈ C–B ⇒ x ∈ C–A. This is true for all x ∈ C–B.
∴ A ⊂ B ⇒ C – B ⊂ C – A

6. For any two sets, prove that:
(i) A ∪ (A ∩ B) = A
(ii) A ∩ (A ∪ B) = A
Solution:
(i) A ∪ (A ∩ B) = A
We know union is distributive over intersection
So, A ∪ (A ∩ B)
(A ∪ A) ∩ (A ∪ B) [Since, A ∪ A = A]
A ∩ (A ∪ B)
A

(ii) A ∩ (A ∪ B) = A
We know union is distributive over intersection
So, (A ∩ A) ∪ (A ∩ B)
A ∪ (A ∩ B) [Since, A ∩ A = A]
A