EXERCISE 23.18

1. Find the equation of the straight lines passing through the origin and making an angle of 45° with the straight line √3x + y = 11.

Solution:

Given:

Equation passes through (0, 0) and make an angle of 45° with the line √3x + y = 11. We know that, the equations of two lines passing through a point x_1, y_1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,

$x_1 = 0$, $y_1 = 0$, $\alpha = 45°$ and $m = -\sqrt{3}$

So, the equations of the required lines are

$$y - 0 = \frac{-\sqrt{3} + \tan 45°}{1 + \sqrt{3}\tan 45°}(x - 0) \text{ and } y - 0$$

$$= \frac{-\sqrt{3} - \tan 45°}{1 - \sqrt{3}\tan 45°}(x - 0)$$

$$= \frac{-\sqrt{3} + 1}{1 + \sqrt{3}}x \text{ and } y = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}x$$

$$= -\frac{3 + 1 - 2\sqrt{3}}{3 - 1}x \text{ and } y = \frac{3 + 1 + 2\sqrt{3}}{3 - 1}x$$

$$= (\sqrt{3} - 2)x \text{ and } y = (\sqrt{3} + 2)x$$

∴ The equation of given line is $y = (\sqrt{3} - 2)x$ and $y = (\sqrt{3} + 2)x$

2. Find the equations to the straight lines which pass through the origin and are inclined at an angle of 75° to the straight line x + y + √3(y - x) = a.

Solution:

Given:

The equation passes through (0,0) and make an angle of 75° with the line x + y + √3(y - x) = a.

We know that the equations of two lines passing through a point x_1, y_1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here, equation of the given line is,

$$x + y + \sqrt{3}(y - x) = a$$

$$(\sqrt{3} + 1)y = (\sqrt{3} - 1)x + a$$

$$y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}x + \frac{a}{\sqrt{3} + 1}$$

Comparing this equation with y = mx + c

We get,

$$m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$\therefore x_1 = 0, y_1 = 0, \alpha = 75°,$

$m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$ and $\tan 75° = 2 + \sqrt{3}$

So, the equations of the required lines are

$$y - 0 = \frac{2 - \sqrt{3} + \tan 75°}{1 - (2 - \sqrt{3})\tan 75°}(x - 0) \text{ and } y - 0 = \frac{2 - \sqrt{3} - \tan 75°}{1 + (2 - \sqrt{3})\tan 75°}(x - 0)$$

$$y = \frac{2 - \sqrt{3} + 2 + \sqrt{3}}{1 - (2 - \sqrt{3})(2 + \sqrt{3})}x \text{ and } y = \frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}x$$

$$y = \frac{4}{1 - 1}x \text{ and } y = -\sqrt{3}x$$

$$x = 0 \text{ and } \sqrt{3}x + y = 0$$

$\therefore$ The equation of given line is $x = 0$ and $\sqrt{3}x + y = 0$

3. Find the equations of straight lines passing through (2, -1) and making an angle of 45° with the line 6x + 5y – 8 = 0.

Solution:

Given:

The equation passes through (2,-1) and make an angle of 45° with the line 6x + 5y – 8 = 0

We know that the equations of two lines passing through a point x_1, y_1 and making an angle α with the given line y = mx + c are

$$y - 0 = \frac{2 - \sqrt{3} + \tan 75°}{1 - (2 - \sqrt{3})\tan 75°}(x - 0) \text{ and } y - 0 = \frac{2 - \sqrt{3} - \tan 75°}{1 + (2 - \sqrt{3})\tan 75°}(x - 0)$$

$$y = \frac{2 - \sqrt{3} + 2 + \sqrt{3}}{1 - (2 - \sqrt{3})(2 + \sqrt{3})}x \text{ and } y = \frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}x$$

$$y = \frac{4}{1 - 1}x \text{ and } y = -\sqrt{3}x$$

$$x = 0 \text{ and } \sqrt{3}x + y = 0$$

$\therefore$ The equation of given line is $x = 0$ and $\sqrt{3}x + y = 0$

Here, equation of the given line is,
$6x + 5y - 8 = 0$
$5y = - 6x + 8$
$y = -6x/5 + 8/5$
Comparing this equation with $y = mx + c$
We get, $m = -6/5$
Where, $x_1 = 2$, $y_1 = - 1$, $\alpha = 45°$, $m = -6/5$
So, the equations of the required lines are

$$y + 1 = \frac{\left(-\frac{6}{5} + \tan 45°\right)}{\left(1 + \frac{6}{5}\tan 45°\right)}(x - 2) \text{ and } y + 1 = \frac{\left(-\frac{6}{5} - \tan 45°\right)}{\left(1 - \frac{6}{5}\tan 45°\right)}(x - 2)$$

$$y + 1 = \frac{\left(-\frac{6}{5} + 1\right)}{\left(1 + \frac{6}{5}\right)}(x - 2) \text{ and } y + 1 = \frac{\left(-\frac{6}{5} - 1\right)}{\left(1 - \frac{6}{5}\right)}(x - 2)$$

$$y + 1 = -\frac{1}{11}(x - 2) \text{ and } y + 1 = -\frac{11}{-1}(x - 2)$$

$x + 11y + 9 = 0$ and $11x - y - 23 = 0$
$\therefore$ The equation of given line is $x + 11y + 9 = 0$ and $11x - y - 23 = 0$

4. Find the equations to the straight lines which pass through the point (h, k) and are inclined at angle $\tan^{-1} m$ to the straight line $y = mx + c$.

Solution:

Given:
The equation passes through (h, k) and make an angle of $\tan^{-1} m$ with the line $y = mx + c$
We know that the equations of two lines passing through a point x_1, y_1 and making an angle α with the given line $y = mx + c$ are
$m' = m$
So,

$$y - 0 = \frac{2 - \sqrt{3} + \tan 75°}{1 - (2 - \sqrt{3})\tan 75°}(x - 0) \text{ and } y - 0 = \frac{2 - \sqrt{3} - \tan 75°}{1 + (2 - \sqrt{3})\tan 75°}(x - 0)$$

$$y = \frac{2 - \sqrt{3} + 2 + \sqrt{3}}{1 - (2 - \sqrt{3})(2 + \sqrt{3})}x \text{ and } y = \frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}x$$

$$y = \frac{4}{1 - 1}x \text{ and } y = -\sqrt{3}x$$

$$x = 0 \text{ and } \sqrt{3}x + y = 0$$

$\therefore$ The equation of given line is $x = 0$ and $\sqrt{3}x + y = 0$

Here,
$x_1 = h$, $y_1 = k$, $\alpha = \tan^{-1} m$, $m' = m$.
So, the equations of the required lines are

$$y - k = \frac{m + m}{1 - m^2}(x - h) \text{ and } y - k = \frac{m - m}{1 + m^2}(x - h)$$

$$y - k = \frac{2m}{1 - m^2}(x - h) \text{ and } y - k = 0$$

$$(y - k)(1 - m^2) = 2m(x - h) \text{ and } y = k$$

∴ The equation of given line is $(y - k)(1 - m^2) = 2m(x - h)$ and $y = k$.

5. Find the equations to the straight lines passing through the point (2, 3) and inclined at an angle of 45^0 to the lines 3x + y – 5 = 0.

Solution:

Given:

The equation passes through (2, 3) and make an angle of $45^0$with the line 3x + y – 5 = 0.

We know that the equations of two lines passing through a point x_1,y_1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,

Equation of the given line is,

3x + y – 5 = 0

y = - 3x + 5

Comparing this equation with y = mx + c we get, m = - 3

$x_1 = 2$, $y_1 = 3$, α = 45∘, m = - 3.

So, the equations of the required lines are

$$y - 3 = \frac{-3 + \tan 45°}{1 + 3\tan 45°}(x - 2) \text{ and } y - 3 = \frac{-3 - \tan 45°}{1 - 3\tan 45°}(x - 2)$$

$$y - 3 = \frac{-3 + 1}{1 + 3}(x - 2) \text{ and } y - 3 = \frac{-3 - 1}{1 - 3}(x - 2)$$

$$y - 3 = \frac{-1}{2}(x - 2) \text{ and } y - 3 = 2(x - 2)$$

x + 2y – 8 = 0 and 2x – y – 1 = 0

∴ The equation of given line is x + 2y – 8 = 0 and 2x – y – 1 = 0