Exercise 4(B)

1. Find the cube of:
(i) 3a - 2b
(ii) 5a + 3b
(iii) 2a + 1/2a
(iv) 3a – 1/a (a ≠ 0)
Solution:

Using the identities,
$(a + b)^3 = a^3 + 3ab (a + b) + b^3$ and
$(a - b)^3 = a^3 - 3ab (a - b) + b^3$

(i) $(3a – 2b)^3 = (3a)^3 – 3 \times 3a \times 2b (3a – 2b) - (2b)^3$
$= 27a^3 – 18ab (3a – 2b) – 8b^3$
$= 27a^3 – 54a^2b + 36ab^2 – 8b^3$

(ii) $(5a + 3b)^3 = (5a)^3 + 3 \times 5a \times 3b (5a + 3b) + (3b)^3$
$= 125a^3 + 45ab (5a + 3b) + 27b^3$
$= 125a^3 + 225a^2b + 135ab^2 + 27b^3$

(iii) $(2a + 1/2a)^3 = (2a)^3 + 3 \times 2a \times 1/2a (2a + 1/2a) + (1/2a)^3$
$= 8a^3 + 3 (2a + 1/2a) + 1/8a^3$
$= 8a^3 + 6a + 3/2a + 1/8a^3$

(iv) $(3a – 1/a)^3 = (3a)^3 – 3 \times 3a \times 1/a (3a – 1/a) - (1/a)^3$
$= 27a^3 – 9 (3a – 1/a) – 1/a^3$
$= 27a^3 – 27a + 9a – 1/a^3$

2. If $a^2 + 1/a^2 = 47$ and $a \neq 0$ find:
(i) a + 1/a
(ii) $a^3 + 1/a^3$
Solution:

(i) Given, $a^2 + 1/a^2 = 47$
We know that,
$(a + 1/a)^2 = a^2 + 1/a^2 + 2 \times a \times 1/a$
$= (a^2 + 1/a^2) + 2$
$= 47 + 2$
$= 49$
So,
$a + 1/a = \sqrt{49}$
$= \pm 7$ … (1)

(ii) Using the identity
$(a + b)^3 = a^3 + 3ab (a + b) + b^3$
Now,

$(a + 1/a)^3 = a^3 + 1/a^3 + 3(a + 1/a)$
$a^3 + 1/a^3 = (a + 1/a)^3 - 3(a + 1/a)$
$= (\pm7)^3 – 3(\pm7)$ … [From (1)]
$= \pm343 - \pm21$
Hence, $a^3 + 1/a^3 = \pm322$

3. If $a^2 + 1/a^2 = 18$; $a \neq 0$ find:
(i) $a – 1/a$
(ii) $a^3 – 1/a^3$
Solution:

(i) Given, $a^2 + 1/a^2 = 18$
Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$
Now,
$(a - 1/a)^2 = a^2 + 1/a^2 - 2(a)(1/a)$
$= (a^2 + 1/a^2) - 2$
$= 18 - 2$
$= 16$
Hence,
$a – 1/a = \sqrt{16}$
$= \pm4$ … (1)

(ii) Using the identity,
$(a - b)^3 = a^3 - 3ab (a - b) + b^3$
Now,
$(a – 1/a)^3 = a^3 - 3a(1/a) (a – 1/a) + (1/a)^3$
$= a^3 - 3 (a – 1/a) + 1/a^3$
$a^3 + 1/a^3 = (a – 1/a)^3 + 3 (a – 1/a)$
$= (\pm4)^3 + 3(\pm4)$
$= \pm64 \pm 12$
Hence,
$a^3 + 1/a^3 = \pm76$

4. If $a + 1/a = p$ and $a \neq 0$; then show that:
$a^3 + 1/a^3 = p (p^2 - 3)$
Solution:

Given, $a + 1/a = p$ … (1)
Now, cubing on both sides
$(a + 1/a)^3 = p^3$
$a^3 + 1/a^3 + 3(a + 1/a) = p^3$
$a^3 + 1/a^3 = p^3 - 3(a + 1/a)$
$= p^3 – 3(p)$ [From (1)]
$= p (p^2 - 3)$

- Hence proved

5. If a + 2b = 5; then show that:
$a^3 + 8b^3 + 30ab = 125$.
Solution:

Given, a + 2b = 5
Let's cube it on both sides,
$(a + 2b)^3 = 5^3$
$a^3 + 3(a)(2b)(a + 2b) + (2b)^3 = 125$
$a^3 + 6ab(a + 2b) + 8b^3 = 125$
$a^3 + 8b^3 = 125 - 6ab (a + 2b)$
$= 125 - 6ab (5)$ … [Given]
$= 125 – 30ab$
So,
$a^3 + 8b^3 + 30ab = 125$
- Hence showed

6. If $(a + 1/a)^2 = 3$ and $a \neq 0$, then show: $a^3 + 1/a^3 = 0$.
Solution:

Given, $(a + 1/a)^2 = 3$
$\Rightarrow a + 1/a = \pm\sqrt{3}$ … (1)
We know the identity,
$(a + 1/a)^3 = a^3 + 1/a^3 + 3(a + 1/a)$
$a^3 + 1/a^3 = (a + 1/a)^3 - 3(a + 1/a)$
$= (\pm\sqrt{3})^3 – 3(\pm\sqrt{3})$
$= \pm3\sqrt{3} – (\pm3\sqrt{3})$
$= 0$
Thus, $a^3 + 1/a^3 = 0$

7. If a + 2b + c = 0; then show that:
$a^3 + 8b^3 + c^3 = 6abc$
Solution:

We have, a + 2b + c = 0
a + 2b = -c
Now, on cubing it on both sides we get
$(a + 2b)^3 = (-c)^3$
$a^3 + (2b)^3 + 3(a)(2b)(a + 2b) = -c^3$
$a^3 + 8b^3 + 6ab (a + 2b) = -c^3$
$a^3 + 8b^3 + 6ab (-c) = -c^3$
$a^3 + 8b^3 - 6abc = -c^3$
Hence,
$a^3 + 8b^3 + c^3 = 6abc$

8. Use property to evaluate:
(i) $13^3 + (-8)^3 + (-5)^3$
(ii) $7^3 + 3^3 + (-10)^3$
(iii) $9^3 - 5^3 - 4^3$
(iv) $38^3 + (-26)^3 + (-12)^3$
Solution:

The property is if a + b + c = 0 then
$a^3 + b^3 + c^3 = 3abc$
Now,
(i) a = 13, b = -8 and c = -5
$\Rightarrow 13^3 + (-8)^3 + (-5)^3 = 3(13)(-8)(-5)$ … [Since, 13 + (-8) + (-5) = 0]
= 1560

(ii) a = 7, b = 3, c = -10
$\Rightarrow 7^3 + 3^3 + (-10)^3 = 3(7)(3)(-10)$ … [Since, 7 + 3 + (-10) = 0]
= -630

(iii)a = 9, b = -5, c = -4
$\Rightarrow 9^3 - 5^3 - 4^3 = 9^3 + (-5)^3 + (-4)^3$ … [Since, 9 + (-5) + (-4) = 0]
= 3(9) (-5) (-4) = 540

(iv) a = 38, b = -26, c = -12
$\Rightarrow 38^3 + (-26)^3 + (-12)^3 = 3(38)(-26)(-12)$ … [Since, 38 + (-26) + (-12) = 0]
= 35568

9. If a ≠ 0 and a – 1/a = 3; find:
(i) $a^2 + 1/a^2$
(ii) $a^3 – 1/a^3$
Solution:

(i) We have, a - 1/a = 3
On squaring on both sides, we get
$(a – 1/a)^2 = 3^2$
$a^2 + 1/a^2 – 2 = 9$
$a^2 + 1/a^2 = 9 + 2$
Hence,
$a^2 + 1/a^2 = 11$

(ii) We have, a - 1/a = 3
On cubing on both sides, we get
$(a – 1/a)^3 = 3^3$
$a^2 - 1/a^3 – 3(a – 1/a) = 27$
$a^2 - 1/a^3 = 27 + 3(a – 1/a)$
= 27 + 3(3)
= 27 + 9

Hence,
$a^3 - 1/a^3 = 36$

10. If a ≠ 0 and a – 1/a = 4; find:
(i) $a^2 + 1/a^2$
(ii) $a^4 + 1/a^4$
(iii) $a^3 – 1/a^3$
Solution:

(i) We have, $a – 1/a = 4$ … (a)
On squaring it on both sides, we get
$(a – 1/a)^2 = 4^2$
$a^2 + 1/a^2 – 2(a)(1/a) = 16$
$a^2 + 1/a^2 – 2 = 16$
$a^2 + 1/a^2 = 16 + 2 = 18$ … (1)
Hence, $a^2 + 1/a^2 = 18$

(ii) Now, we know that
$a^4 + 1/a^4 = (a^2 + 1/a^2)^2 – 2$
$= 18^2 – 2$ … [From (1)]
$= 324 – 2$
Hence, $a^4 + 1/a^4 = 322$

(iii) On cubing (i) on both sides, we get
$(a – 1/a)^3 = 4^3$
$a^3 – 1/a^3 – 3(a – 1/a) = 64$
$a^3 – 1/a^3 = 64 + 3(a – 1/a)$
$= 64 + 3(4)$ … [Given]
$= 64 + 12$
Hence, $a^3 – 1/a^3 = 76$

11. If x ≠ 0 and x + 1/x = 2; then show that:
$x^2 + 1/x^2 = x^3 + 1/x^3 = x^4 + 1/x^4$
Solution:

We have, $x + 1/x = 2$
We know that,
$(x + 1/x)^2 = x^2 + 1/x^2 + 2$
$(2)^2 = x^2 + 1/x^2 + 2$
$x^2 + 1/x^2 = 4 – 2$
$= 2$ … (i)

Next, calculating
$(x + 1/x)^3 = x^3 + 1/x^3 + 3(x + 1/x)$
$(2)^3 = x^3 + 1/x^3 + 3(2)$
$x^3 + 1/x^3 = 2^3 – 3(2)$

$= 8 - 6$
$= 2$ … (ii)

Next, we know that
$x^4 + 1/x^4 = (x^2 + 1/x^2) - 2$
$= 2^2 - 2$ … [From (i)]
$= 4 - 2$
$= 2$ … (iii)

Therefore, from (i), (ii) and (iii) we have
$x^2 + 1/x^2 = x^3 + 1/x^3 = x^4 + 1/x^4$

12. If $2x - 3y = 10$ and $xy = 16$; find the value of $8x^3 - 27y^3$.
Solution:

Given,
$2x - 3y = 10$ … (i) and
$xy = 16$ … (ii)
Now, on cubing (i) on both sides
$(2x - 3y)^3 = 10^3$
$(2x)^3 - 3(2x)(3y)(2x - 3y) - (3y)^3 = 1000$ []
$8x^3 - 18(xy)(2x - 3y) - 27y^3 = 1000$
$8x^3 - 18 \times 16 \times 10 - 27y^3 = 1000$
$8x^3 - 2880 - 27y^3 = 1000$
$8x^3 - 27y^3 = 1000 + 2880$
$8x^3 - 27y^3 = 3880$

13. Expand:
(i) $(3x + 5y + 2z)(3x - 5y + 2z)$
(ii) $(3x - 5y - 2z)(3x - 5y + 2z)$
Solution:

(i) We have, $(3x + 5y + 2z)(3x - 5y + 2z)$
$= \{(3x + 2z) + (5y)\}\{(3x + 2z) - (5y)\}$ … [By grouping]
$= (3x + 2z)^2 - (5y)^2$ … [As $(a + b)(a - b) = a^2 - b^2$]
$= 9x^2 + 4z^2 + (2 \times 3x \times 2z) - 25y^2$
$= 9x^2 + 4z^2 + 12xz - 25y^2$
$= 9x^2 + 4z^2 - 25y^2 + 12xz$

(ii) We have, $(3x - 5y - 2z)(3x - 5y + 2z)$
$= \{(3x - 5y) - (2z)\}\{(3x - 5y) + (2z)\}$ … [By grouping]
$= (3x - 5y)^2 - (2z)^2$ … [As $(a + b)(a - b) = a^2 - b^2$]
$= 9x^2 + 25y^2 - 2 \times 3x \times 5y - 4z^2$
$= 9x^2 + 25y^2 - 30xy - 4z^2$
$= 9x^2 + 25y^2 - 4z^2 - 30xy$

14. The sum of two numbers is 9 and their product is 20. Find the sum of their (i) Squares (ii) Cubes

Solution:

Given, the sum of two numbers is 9 and their product is 20
Let's assume the numbers to 'a' and 'b'
So, we have
$a + b = 9$ … (1) and
$ab = 20$ … (2)
Now,
On squaring (1) on both sides gives, we get
$(a + b)^2 = 9^2$
$a^2 + b^2 + 2ab = 81$
$a^2 + b^2 + 2(20) = 81$ … [From (2)]
$a^2 + b^2 + 40 = 81$
$a^2 + b^2 = 81 – 40 = 41$
(i) Hence, the sum of their squares is 41

Next,
On cubing (1) on both sides, we get
$(a + b)^3 = 9^3$
$a^3 + b^3 + 3ab (a + b) = 729$
$a^3 + b^3 + 3 \times (20) \times (9) = 729$ … [From (1) and (2)]
$a^3 + b^3 = 729 - 540 = 189$
(ii) Hence, the sum of their cubes is 189.

15. Two positive numbers x and y are such that x > y. If the difference of these numbers is 5 and their product is 24, find:
(i) Sum of these numbers
(ii) Difference of their cubes
(iii) Sum of their cubes.
Solution:

Given $x - y = 5$ and $xy = 24$ $(x>y)$
$(x + y)^2 = (x - y)^2 + 4xy = 25 + 96 = 121$
So, $x + y = 11$; sum of these numbers is 11.
Cubing on both sides gives
$(x - y)^3 = 5^3$
$x^3 - y^3 - 3xy(x - y) = 125$
$x^3 - y^3 - 72(5) = 125$
$x^3 - y^3 = 125 + 360 = 485$
So, difference of their cubes is 485.
Cubing both sides, we get
$(x + y)^3 = 11^3$
$x^3 + y^3 + 3xy(x + y) = 1331$
$x^3 + y^3 = 1331 - 72(11) = 1331 - 792 = 539$

So, sum of their cubes is 539.

16. If $4x^2 + y^2 = a$ and $xy = b$, find the value of $2x + y$.
Solution:

Given, $xy = b$ … (i) and $4x^2 + y^2 = a$ … (ii)
Now,
$(2x + y)^2 = (2x)^2 + 4xy + y^2$
$= (4x^2 + y^2) + 4xy$
$= a + 4b$ … [Using (i) and (ii)]
Hence,
$2x + y = \pm\sqrt{(a + 4b)}$