Class 11 Physics Chapter 9: Mechanical Properties of Fluid

Long Answers

21. a) Pressure decreases as one ascends the atmosphere. If the density of air is ρ, what is the change in pressure dp over a differential height dh?

b) Considering the pressure p to be proportional to the density, find the pressure p at a height h if the pressure on the surface of the earth is p_0.

c) If $p_0 = 1.03 \times 10^5$ N/m², $\rho_0 = 1.29$ kg/m³ and g = 9.8 m/s² at what height will the pressure drop to (1/10) the value at the surface of the earth?

d) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model.

Solution:

a) Consider a horizontal layer of air at height h with thickness dh and cross-sectional area A:

- Weight of air layer = ρgA dh
- Pressure difference = Weight/Area = ρg dh
- dp = -ρg dh (negative because pressure decreases with height)

b) Given p ∝ ρ, we have $p/p_0 = \rho/\rho_0$ Substituting in dp = -ρg dh: $dp = -(p/p_0)\rho_0 g\, dh = -(\rho_0 g/p_0)p\, dh$

Separating variables: $dp/p = -(\rho_0 g/p_0)\, dh$

Integrating: $\ln(p/p_0) = -(\rho_0 g/p_0)h$

Therefore: $p = p_0 e^{(-\rho_0 gh/p_0)}$

c) For $p = p_0/10$: $p_0/10 = p_0 e^{(-\rho_0 gh/p_0)}$ $1/10 = e^{(-\rho_0 gh/p_0)}$ $\ln(1/10) = -\rho_0 gh/p_0$ $-2.303 = -(1.29 \times 9.8 \times h)/(1.03 \times 10^5)$

$h = (2.303 \times 1.03 \times 10^5)/(1.29 \times 9.8) = 18.43$ km

d) The model assumes isothermal conditions (constant temperature), which is only valid for small heights near Earth's surface.

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22. Surface tension is exhibited by liquids due to force of attraction between molecules of the liquid. The surface tension decreases with increase in temperature and vanishes at boiling point. Given that the latent heat of vaporisation for water L_v = 540 kcal/kg, the mechanical equivalent of heat J = 4.2 J/cal, density of water ρw = 10³ kg/m³, Avogadro's number $N_a = 6.0 \times 10^{23}$ mol⁻¹, and the molecular weight of water M_a = 18 kg/kmol.

a) Estimate the energy required for one molecule of water to evaporate.

b) Show that the inter-molecular distance for water is $d = [M_a/(N_a\rho w)]^{(1/3)}$ and find its value.

c) 1 g of water in the vapour state at 1 atm occupies 1601 cm³. Estimate the intermolecular distance at boiling point, in the vapour state.

d) During vaporisation a molecule overcomes a force F, assumed constant to go from an inter-molecular distance d to d'. Estimate the value of F.

e) Calculate F/d which is a measure of the surface tension.

Solution:

a) Energy for one molecule:

- L_v = 540 kcal/kg = 540 × 4200 J/kg = 2.268×10^6 J/kg
- Energy per kmol = $2.268 \times 10^6 \times 18 = 4.0824 \times 10^7$ J/kmol
- Energy per molecule = $(4.0824 \times 10^7)/(6.0 \times 10^{26}) = 6.8 \times 10^{-20}$ J

b) Volume per molecule in liquid state:

- Volume per kmol = $M_a/\rho w = 18/10^3 = 1.8 \times 10^{-2}$ m³/kmol
- Volume per molecule = $(1.8 \times 10^{-2})/(6.0 \times 10^{26}) = 3.0 \times 10^{-29}$ m³
- If each molecule occupies volume d^3: $d^3 = M_a/(N_a \rho w)$
- $d = [M_a/(N_a\rho w)]^{(1/3)} = [18/(6.0 \times 10^{26} \times 10^3)]^{(1/3)} = 3.1 \times 10^{-10}$ m

c) In vapor state:

- Volume of 1 g = 1601 cm³ = 1601×10^{-6} m³
- Number of molecules in 1 g = $(1 \times 6.0 \times 10^{26})/18000 = 3.33 \times 10^{22}$
- Volume per molecule = $(1601 \times 10^{-6})/(3.33 \times 10^{22}) = 4.8 \times 10^{-26}$ m³
- $d' = (4.8 \times 10^{-26})^{(1/3)} = 3.63 \times 10^{-9}$ m

d) Work done against force F:

- $U = F(d' - d) = 6.8 \times 10^{-20}$ J
- $F = (6.8 \times 10^{-20})/(3.63 \times 10^{-9} - 3.1 \times 10^{-10})$
- $F = (6.8 \times 10^{-20})/(3.32 \times 10^{-9}) = 2.05 \times 10^{-11}$ N

e) Surface tension measure: $F/d = (2.05 \times 10^{-11})/(3.1 \times 10^{-10}) = 6.6 \times 10^{-2}$ N/m

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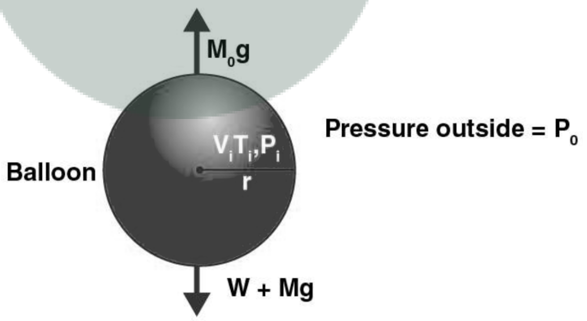
23. A hot air balloon is a sphere of radius 8 m. The air inside is at a temperature of 60°C. How large a mass can the balloon lift when the outside temperature is 20°C?

Solution:



Given:

- R = 8 m, $V = (4/3)\pi R^3 = 2.144 \times 10^3$ m³
- T_i = 60 + 273 = 333 K
- T_0 = 20 + 273 = 293 K
- Atmospheric pressure $p_0 = p_i = 1.013 \times 10^5$ N/m²
- Molar mass of air M_a = 0.029 kg/mol

Using ideal gas law:

- Mass of hot air inside: $m_i = (p_0 V M_a)/(R T_i)$
- Mass of cold air displaced: $m_0 = (p_0 V M_a)/(R T_0)$

Net upward force: $W = (m_o - m_i)g = (p_o V M_a g/R)[(1/T_o) - (1/T_i)]$

$W = (1.013 \times 10^5 \times 2.144 \times 10^3 \times 0.029 \times 9.8)/(8.314) \times [(1/293) - (1/333)]$

$W = 1.885 \times 10^7 \times [(0.003413) - (0.003003)]$

$W = 1.885 \times 10^7 \times 0.00041 = 773$ kg

Therefore, the balloon can lift approximately 773 kg.