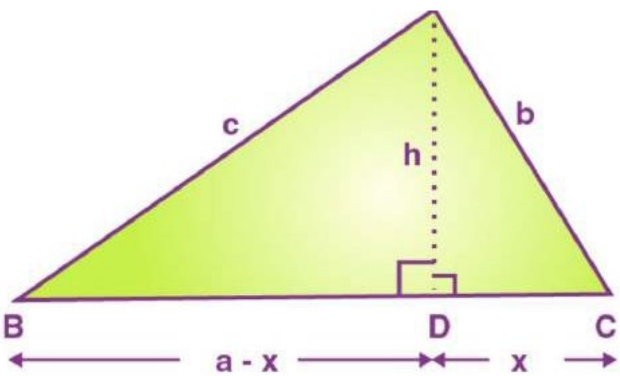
EXERCISE 13B

1. In the figure, given below, AD parallel to BC. Prove that: $c^2 = a^2 + b^2 - 2ax$



Solution:

Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the triangle ABD and applying Pythagoras theorem we get,

$AB^2 = AD^2 + BD^2$

$c^2 = h^2 + (a – x)^2$

$h^2 = c^2 – (a - x)^2$ ……… (1)

First, we consider the triangle ACD and applying Pythagoras theorem we get

$AC^2 = AD^2 + CD^2$

$b^2 = h^2 + x^2$

$h^2 = b^2 – x^2$…………… (2)

from 1 and 2

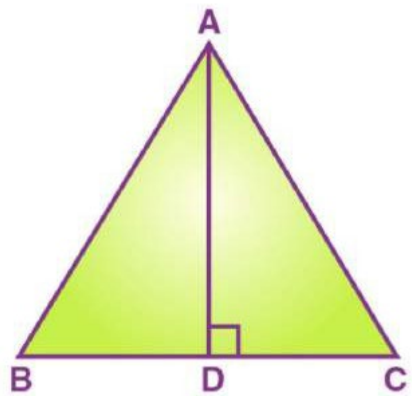
$c^2 – (a – x)^2 = b^2 – x^2$

$c^2 – a^2 – x^2 + 2ax = b^2 – 2ax$

$c^2 = a^2 + b^2 – 2ax$

hence the proof.

2. In equilateral Δ ABC, AD parallel to BC and BC = x cm. Find, in terms of x, the length of AD.

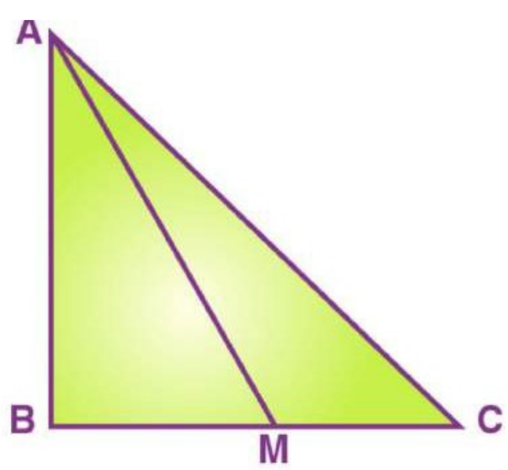


Solution:

In equilateral Δ ABC, AD parallel to BC.
Therefore, BD = DC = x/2 cm.
Applying Pythagoras theorem, we get
In right angled triangle ADC
$AC^2 = AD^2 + DC^2$
$x^2 = AD^2 + (x/2)^2$
$AD^2 = (x)^2 – (x/2)^2$
$AD^2 = (x/2)^2$
AD = (x/2) cm

3. ABC is a triangle, right-angled at B. M is a point on BC. Prove that:
$AM^2 + BC^2 = AC^2 + BM^2$.

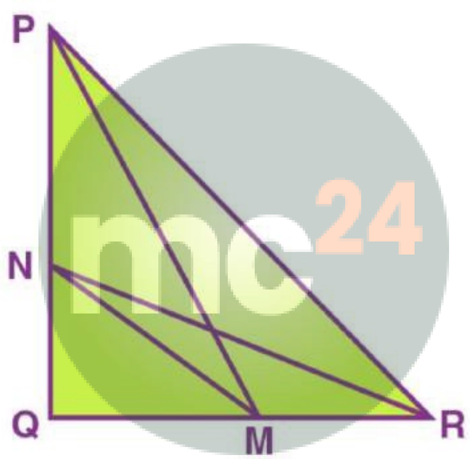
Solution:
The pictorial form of the given problem is as follows,
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
First, we consider the triangle ABM and applying Pythagoras theorem we get,



$AM^2 = AB^2 + BM^2$

$AB^2 = AM^2 - BM^2$ .... (1)
Now we consider the triangle ABC and applying Pythagoras theorem we get
$AC^2 = AB^2 + BC^2$
$AB^2 = AC^2 - BC^2$ …... (2)
From 1 and 2 we get
$AM^2 - BM^2 = AC^2 + BM^2$
$AM^2 + BC^2 = AC^2 + BM^2$
Hence the proof.

4. M and N are the mid-points of the sides QR and PQ respectively of a triangle PQR, right-angled at Q. Prove that:
(i) $PM^2 + RN^2 = 5\ MN^2$
(ii) $4\ PM^2 = 4\ PQ^2 + QR^2$
(iii) $4\ RN^2 = PQ^2 + 4\ QR^2$
(iv) $4\ (PM^2 + RN^2) = 5\ PR^2$



Solution:

Draw, PM, MN, NR
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Since, M and N are the mid-points of the sides QR and PQ respectively, therefore, PN = NQ, QM = RM
(i)
First, we consider the triangle PQM and applying Pythagoras theorem we get,
$PM^2 = PQ^2 + MQ^2$
$= (PN + NQ)^2 + MQ^2$
$= PN^2 + NQ^2 + 2\ PN.\ NQ + MQ^2$
$= MN^2 + PN^2 + 2\ PN.\ NQ$ [we know $MN^2 = NQ^2 + MQ^2$]...............(1)
Now we consider the triangle RNQ and applying Pythagoras theorem,
$RN^2 = NQ^2 + RQ^2$
$= NQ^2 + (QM + RM)^2$
$= NQ^2 + QM^2 + 2\ QM.\ RM + RM^2$ ....(2)

Adding 1 and 2 we get

$PM^2 + RN^2 = MN^2 + PN^2 + 2PN.\ NQ + MN^2 + RM^2 + 2QM.\ RM$

$PM^2 + RN^2 = 2MN^2 + PN^2 + RM^2 + 2PN.\ NQ + 2QM.\ RM$

$PM^2 + RN^2 = 2MN^2 + NQ^2 + QM^2 + 2(QN)^2 + 2\ (QM)^2$

$PM^2 + RN^2 = 2MN^2 + MN^2 + 2MN^2$

$PM^2 + RN^2 = 5MN^2$

Hence the proof.

(ii) Now consider the triangle PQM and apply Pythagoras theorem we get

$PM^2 = PQ^2 + MQ^2$

$4PM^2 = 4PQ^2 + 4\ MQ^2$ [multiplying both sides by 4]

$4PM^2 = 4PQ^2 + 4\ (½\ QR^2)$ [$MQ = ½\ QR$]

$4PM^2 = 4PQ^2 + QR^2$

Hence the proof.

(iii) now consider triangle RQN and apply Pythagoras theorem we get

$RN^2 = NQ^2 + RQ^2$

$4RN^2 = 4NQ^2 + 4\ QR^2$ [multiplying both sides by 4]

$4RN^2 = 4QR^2 + 4\ (½\ PQ^2)$ [$NQ = ½\ PQ$]

$4RN^2 = PQ^2 + 4QR^2$

Hence the proof.

(iv) now consider the triangle PQM and apply Pythagoras theorem,

$PM^2 = PQ^2 + MQ^2$

$= (PN + NQ)^2 + MQ^2$

$= PN^2 + NQ^2 + 2\ PN.\ NQ + MQ^2$

$= MN^2 + PN^2 + 2\ PN.\ NQ$ [we know $MN^2 = NQ^2 + MQ^2$]..............(1)

Now we consider the triangle RNQ and applying Pythagoras theorem,

$RN^2 = NQ^2 + RQ^2$

$= NQ^2 + (QM + RM)^2$

$= NQ^2 + QM^2 + 2\ QM.\ RM + RM^2$

$= MN^2 + RM^2 + 2\ QM.\ RM$............ (2)

Adding 1 and 2 we get

$PM^2 + RN^2 = MN^2 + PN^2 + 2PN.\ NQ + MN^2 + RM^2 + 2QM.\ RM$

$PM^2 + RN^2 = 2MN^2 + PN^2 + RM^2 + 2PN.\ NQ + 2QM.\ RM$

$PM^2 + RN^2 = 2MN^2 + NQ^2 + QM^2 + 2(QN)^2 + 2\ (QM)^2$

$PM^2 + RN^2 = 2MN^2 + MN^2 + 2MN^2$

$PM^2 + RN^2 = 5MN^2$

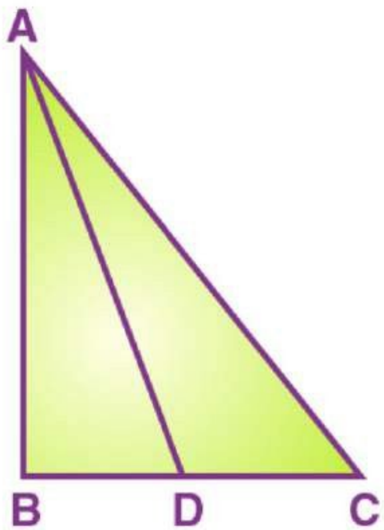
$4\ (PM^2 + RN^2) = 4.\ 5\ (NQ^2 + MQ^2]$

$4\ (PM^2 + RN^2) = 4.\ 5\ [(½\ PQ)^2 + (½\ QR)^2]$

$4\ (PM^2 + RN^2) = 5PR^2$

Hence the proof.

5. In triangle ABC, ∠B = 90° and D is the mid-point of BC. Prove that: $AC^2 = AD^2 + 3CD^2$.

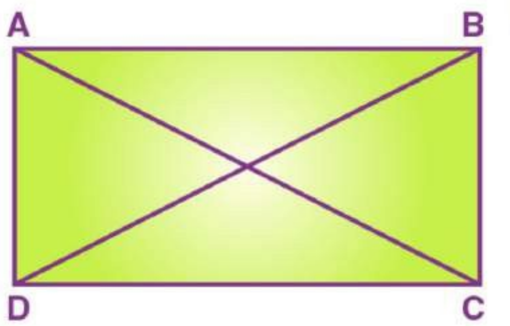


Solution:

Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
In triangle ABC, ∠B = 90° and D is the mid-point of BC. Join AD. Therefore, BD=DC
First, we consider the triangle ADB and applying Pythagoras theorem we get,
$AD^2 = AB^2 + BD^2$
$AB^2 = AD^2 – BD^2$ …. (1)
Similarly, we get from rt. angle triangles ABC we get,
$AC^2 = AB^2 + BC^2$
$AB^2 = AC^2 – BC^2$ …. (1)
From 1 and 2 we get
$AC^2 – BC^2 = AD^2 – BD^2$
$AC^2 = AD^2 – BD^2 + BC^2$
$AC^2 = AD^2 – CD^2 + 4CD^2$ [BD = CD = ½ BC]
$AC^2 = AD^2 + 3CD^2$
Hence the proof.

6. In a rectangle ABCD, prove that: $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$.

Solution:



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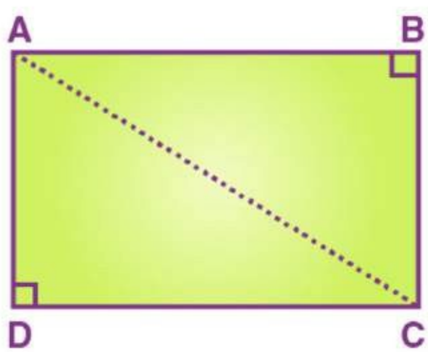
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Since, ABCD is a rectangle angles A, B, C and D are rt. angles.
First, we consider the triangle ACD and applying Pythagoras theorem we get,
$AC^2 = DA^2 + CD^2$ ……. (1)
Similarly, we get from rt. angle triangle BDC we get,
$BD^2 = BC^2 + CD^2$
$= BC^2 + AB^2$ [In a rectangle opposite sides are equal CD = AB]
Adding (i) and (ii),
$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$
Hence the proof.

7.

8. In a quadrilateral ABCD, $\angle B = 90^0$ and $\angle D = 90^0$. Prove that: $2AC^2 - AB^2 = BC^2 + CD^2 + DA^2$

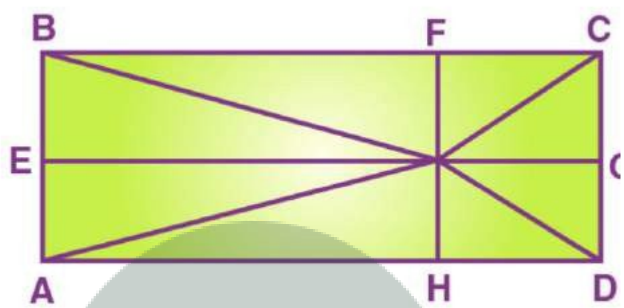
Solution:



In quadrilateral ABCD $\angle B = 90^0$ and $\angle D = 90^0$
So triangle ABC and triangle ADC are right angles.
For triangle ABC, apply Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$AB^2 = AC^2 – BC^2$……………… (i)
For triangle ADC, apply Pythagoras theorem,
$AC^2 = AD^2 + DC^2$……………… (ii)

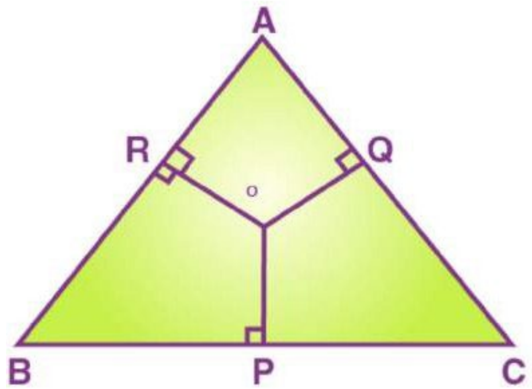
LHS = $2AC^2 - AB^2$
= $2AC^2 - (AC^2 - BC^2)$ from 1
= $2AC^2 - AC^2 + BC^2$
= $AC^2 + BC^2$
= $AD^2 + DC^2 + BC^2$ from 2
= RHS

9. O is any point inside a rectangle ABCD. Prove that: $OB^2 + OD^2 = OC^2 + OA^2$.
Solution:



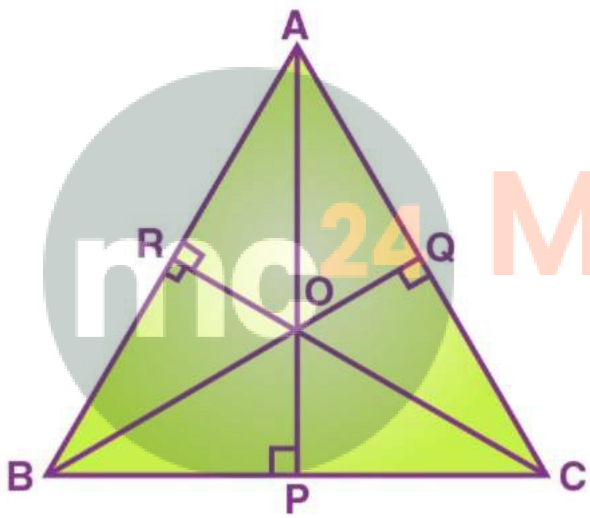
Draw rectangle ABCD with arbitrary point O within it, and then draw lines OA, OB, OC, OD. Then draw lines from point O perpendicular to the sides: OE, OF, OG, OH.
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Using Pythagorean theorem, we have from the above diagram:
$OA^2 = AH^2 + OH^2 = AH^2 + AE^2$
$OC^2 = CG^2 + OG^2 = EB^2 + HD^2$
$OB^2 = EO^2 + BE^2 = AH^2 + BE^2$
$OD^2 = HD^2 + OH^2 = HD^2 + AE^2$
Adding these equalities, we get:
$OA^2 + OC^2 = AH^2 + HD^2 + AE^2 + EB^2$
$OB^2 + OD^2 = AH^2 + HD^2 + AE^2 + EB^2$
From which we prove that for any point within the rectangle there is the relation
$OA^2 + OC^2 = OB^2 + OD^2$
Hence Proved.

10. In the following figure, OP, OQ and OR are drawn perpendiculars to the sides BC, CA and AB respectively of triangle ABC. Prove that: $AR^2 + BP^2 + CQ^2 = AQ^2 + CP^2 + BR^2$



Solution:

Here, we first need to join OA, OB, and OC after which the figure becomes as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides. First, we consider the ΔARO and applying Pythagoras theorem we get,

$AO^2 = AR^2 + OR^2$

$AR^2 = AO^2 - OR^2$ ……. (1)

Similarly, from triangles, BPO, COQ, AOQ, CPO and BRO we get the following results,

$BP^2 = BO^2 - OP^2$ ……. (2)

$CQ^2 = OC^2 - OQ^2$ ……. (3)

$AQ^2 = AO^2 - OQ^2$ ……. (4)

$CP^2 = OC^2 - OP^2$ ……. (5)

$BR^2 = OB^2 - OR^2$ ……. (6)

Adding 1, 2 and 3 we get

$AR^2 + BP^2 + CQ^2 = AO^2 - OR^2 + BO^2 - OP^2 + OC^2 - OQ^2$ ……. (7)

Adding 4, 5 and 6 we get

$AQ^2 + CP^2 + BR^2 = AO^2 - OQ^2 + OC^2 - OP^2 + OB^2 - OR^2$………………(8)

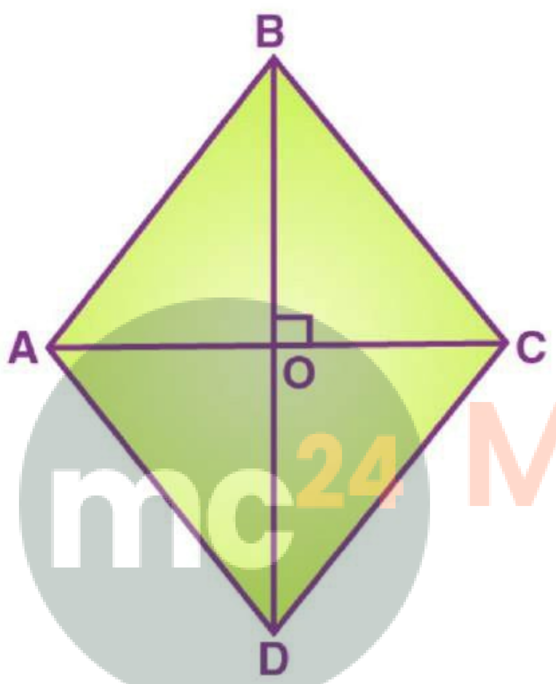
From 7 and 8, we get,

$AR^2 + BP^2 + CQ^2 = AQ^2 + CP^2 + BR^2$

Hence proved.

11. Diagonals of rhombus ABCD intersect each other at point O. Prove that: $OA^2 + OC^2 = 2AD^2 – BD^2/2$

Solution:



We know diagonals of the rhombus are perpendicular to each other.

In quadrilateral ABCD, $\angle AOD = \angle COD = 90^o$

We know triangle AOD and COD are right angle triangle.

In triangle AOD, apply Pythagoras theorem,

$AD^2 = OA^2 + OD^2$

$OA^2 = AD^2 – OD^2$………………(1)

In triangle COD, apply Pythagoras theorem,

$CD^2 = OC^2 + OD^2$

$OC^2 = CD^2 – OD^2$……………… (2)

LHS = $OA^2 + OC^2$

= $AD^2 – OD^2 + CD^2 – OD^2$ from 1 and 2

= $AD^2 – AD^2 – 2(BD/2)^2$ [AD = CD and OD = BD/2]

= $2AD^2 – BD^2/2$

= RHS